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Robot Localization under Dynamic Uncertainty

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Introduction

This paper is an extension of authors' previous work presented in [1], and is an effort to view critical robot tasks by making a clear separation between different inference levels, e.g. vector marks method belongs to higher inference level and is described in literature [2]. This work is dedicated for Maximum relative Entropy (*MrE*) method using simultaneous updating of model combination and data. Practically all Simultaneous localization and mapping (SLAM) using Bayesian inference perform sequential updating. We derive the approach which can perform validation and estimation (*VE*) of data using simultaneous updating and model combination.

Moreover, it embodies Occam's razor, which states that "entities must not be multiplied beyond necessity". In other words, in addition to VE of observed data we must know for how long sensors did not report correctly based on other sensors which link to the failing sensors through associations, i.e. formalism which can be expressed by formulas.

Maximum relative Entropy

Maximum relative Entropy method uses widely known relative entropy formula (Kullback-Leibler divergence):

$$S[P, P_{\text{old}}] = -\int_{\theta}\int_{x} P(x,\theta)\log\frac{P(x,\theta)}{P_{\text{old}}(x,\theta)}\mathrm{d}x\mathrm{d}\theta, \qquad (1)$$

where P stands for probability density function, x is observed data as $x=\{l_{\text{left}}, l_{\text{right}}, \alpha\}$ with l_{left} - left optical wheel encoder's measured distance's increment, l_{right} - right optical wheel encoder's measured distance's increment, α - robot direction angle's increment during robot's motion; $\theta=\{\theta', \theta'', \theta'''\}$ is the selection of observation models with θ' representing robot motion model, when left wheel returns correct observation, and right wheel slides (as the result of dynamic uncertainty such as manual hit or sliding because of slippery ground), θ'' representing a model when right wheel reports correct observation, and left wheel slides and finally θ''' representing model when both optical encoders report correct observation values.

Joint prior $P_{\text{old}}(x,\theta)$ consists of likelihood $P_{\text{old}}(x\,|\,\theta)$ and Bayesian prior $P_{\text{old}}(\theta)$ through relationship

$$P_{\text{old}}(x,\theta) = P_{\text{old}}(\theta)P_{\text{old}}(x\,|\,\theta). \qquad (2)$$

Several important notes have to be clarified at this point.

a) Robot direction shift angle α is assumed to be valid for the sake of clarity when explaining this approach to a reader. In fact, it can be shown that validation of α can be performed at higher abstraction layer where data associations come into effect after we validated optical wheel encoders' observations.

b) Another assumption about the angle shift α is that it is based on the feature drifts when inferring it while tracking environment features in perpendicular direction compared to robot's motion direction. A reader can find the distance measurement in 3D environment in literature [3].

c) It is assumed that l_{left} has been read from the optical encoder, which was on the wheel that truly traveled shorter distance than the distance truly traveled by the right wheel. The approach of this paper can be easily extended to incorporate any combination of these true distances.

d) It is worth of mentioning that any model combination is possible and supported by this method. In fact, the simplest and most intuitive model combination is inferred. Moreover, what distinguishes this method from other Bayesian Model Combination methods is that MrE can validate observations, estimate model combination and infer robot's position (location) *simultaneously* based on environment data associations and dynamic constraints.

Bayesian Model Averaging is sometimes used in solving localization tasks too. Its interpretation is that one model is responsible for generating the whole data set. Its serious drawbacks are performance due to its sequential nature and not treating of combinations of the models. Moreover, *performance* should receive a special attention, when mission critical tasks are to be sought.

Dynamic constraints

The following dynamic constraints will be used when maximizing (1) using Lagrangian:

$$P(x) = \int_\theta P(x,\theta)\mathrm{d}\theta = \delta(x - x_{\text{new}})\,, \tag{3}$$

$$\int_\theta \int_x P(x,\theta)\mathrm{d}x\mathrm{d}\theta = 1\,, \tag{4}$$

$$\int_\theta \int_x P(x,\theta) f(\theta)\mathrm{d}x\mathrm{d}\theta = \langle \Delta \rangle\,, \tag{5}$$

where $\delta(x - x_{new})$ – the Dirac delta function representing constraint on observed data [4], formula (4) embodies normal distribution's normalization constraint stating that "robot is somewhere", $\langle \Delta \rangle$ – statistical expectation of the line segment modulus traveled by robot's center during observation time and function $f(\theta)$ – expressed in terms of

$$f(\theta) = \frac{2\left(l^{'}_{left} + l^{''}_{right} + d\alpha\left(\theta^{'} - \theta^{''}\right) + \frac{l^{'''}_{left} + l^{'''}_{right}}{2} \right)\sin\frac{\alpha}{2}}{\alpha}\,. \tag{6}$$

Clarification on function parameters: argument d – the distance between robot wheels divided by 2, $l^{'}_{\text{left}}$ – distance traveled by left wheel for trajectory parts where model $\theta^{'}$ was in effect, $l^{''}_{\text{right}}$ – distance traveled by left wheel for trajectory parts where model $\theta^{''}$ was in effect, $l^{'''}_{\text{left}}$ – distance traveled by left wheel for trajectory parts where model $\theta^{'''}$ was in effect and similarly $l^{'''}_{\text{right}}$. It can be easily seen that model $\theta^{'''}$ is a composite of models $\theta^{'}$ and $\theta^{''}$.

Special attention has to be paid to the expectation $\langle \Delta \rangle$. Robot has to interrogate for its incremental distance and angle measurements so that the time Δt between interrogations has to be

a) small enough so that angle shift α is big enough to validate distances using it,
b) big enough so that change of robot kinetic energy is negligible and constraint (5) is satisfied.

A time period was picked $\Delta t = 100ms$ in simulations whose results are presented later in this paper.

Simultaneous updating with model combination and data

Maximization process of formula (1) by incorporating constraints (3–5) through Lagrange multipliers and product rule of conditional probabilities leads to

$$P_{\text{new}}(\theta) = P_{\text{old}}(\theta) P_{\text{old}}(x_{\text{new}} \mid \theta) \frac{e^{\beta f(\theta)}}{\zeta_{\text{new}}(x_{\text{new}}, \beta)}\,, \tag{7}$$

where $P_{\text{old}}(\theta)$ – prior as in Bayes theorem and $P_{\text{old}}(x_{\text{new}} \mid \theta)$ – Bayesian likelihood. Lagrange multiplier β is calculated using

$$\frac{\partial \ln \zeta(x_{\text{new}}, \beta)}{\partial \beta} = \langle \Delta \rangle \tag{8}$$

and normalization factors ζ and ζ_{new} are found using

$$\zeta(x, \beta) = \int_\theta P_{\text{old}}(x,\theta) e^{\beta f(\theta)}\mathrm{d}\theta\,, \tag{9}$$

$$\zeta_{\text{new}}(x_{\text{new}}, \beta) = \int_\theta P_{\text{old}}(x_{\text{new}},\theta) e^{\beta f(\theta)}\mathrm{d}\theta\,. \tag{10}$$

The detailed explanation of MrE approach and its justification can be found in [4].

During the simulation prior $P_{\text{old}}(\theta)$ is treated as flat, i.e. uniform prior distribution. This is justified by the fact that prior dynamic uncertainties or previous x observations have no effect to current calculation of probabilities if we know the expectation $\langle \Delta \rangle$. Markov assumption goes into effect. Bringing it even simpler: if we know that Δt is so small that the change of robot kinetic energy is negligible, we know what is the modulus of line segment traveled by robot's center. So uniform prior distribution becomes proper and our prior is justified.

Special consideration has to be taken when calculating $P_{\text{old}}(x_{\text{new}} \mid \theta)$. In the simulation model, used by this paper the change of robot kinetic energy is negligible too. It means that the $f(\theta)$ has normal (Gaussian) distribution expressed as

$$P_{\text{old}}(x_{\text{new}} \mid \theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(f(\theta) - \langle \Delta \rangle)^2}{2\sigma^2}}\,, \tag{11}$$

where standard deviation σ has to be picked so that it is

a) small enough to make likelihood function sharp enough to validate observation data and infer robot's position,
b) big enough so that calculation errors do not corrupt MLE function with noise.

There is an important note that has to be mentioned regarding $P_{new}(\theta)$ as in formula (7). The expectation value is not the same as Maximum Likelihood Estimate (*MLE*). Mathematically it can be expressed as

$$\langle \theta \rangle <> \arg\max_\theta P_{\text{new}}(\theta)\,. \tag{12}$$

Sequential versus simultaneous

Before elaborating on sequential versus simultaneous we need to clarify on commutivity of constraints used in MrE approach. Commuting constraints are such constraints that make no difference whether they are processed at the same time or not, for details see [2] and related work of A. Giffin and A. Caticha. Assume an agent is bypassing the robot. Also assume that robot inferred α while tracking this moving agent. It is clear that we can no longer use simultaneous updating, because we can no surely know the expectation $\langle \Delta \rangle$.

When constraints are non-commuting then sequential updating has to be performed using MrE method. One of the works in sequential updating with data when localizing and mapping the robot under dynamic constraints is FastSLAM approach presented in the monograph [5]. MrE could extend such sequential approaches by adding Validation of data, and dropping uninformative priors.

The word *simultaneous* in acronym SLAM explains that it solves both localization and mapping as such. However its nature is sequential from the point of view of updating with observed data. It takes multiple Bayesian iterations until FastSLAM converges to the next proper robot's position. Meanwhile MrE uses updating with model combination and data and it is simultaneous from the point of view of updating using Bayes theorem. It enables data validation, and helps to avoid position estimate deviations when solving SLAM problem under strong dynamic uncertainty. Which means faster robot reaction and fewer deviations in location.

Simulation results

MrE using simultaneous updating with model combination and data has been confirmed in the simulation experiment. Equations of the robot models are derived, simulation framework is implemented and two simulation experiments were performed.

It is clear $\theta^{''}$ statistics is sufficient for finding the distribution of $\theta^{'}$, so we can find marginal distributions shown in figures (1) and (2) using formulas

$$P(\theta^{'}) = \int_{\theta''} P(\theta^{'}, \theta^{''}) d\theta^{''}, \tag{13}$$

$$P(\theta^{''}) = \int_{\theta'} P(\theta^{'}, \theta^{''}) d\theta^{'}, \tag{14}$$

where $P(\theta^{'}, \theta^{''}) = P_{new}(\theta)$ from formula (7).

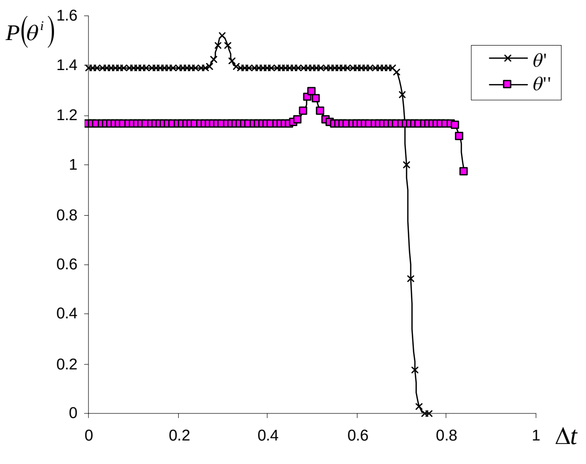

Fig. 1. First simulation experiment's MLE functions with model change estimates at $0.3\Delta t$ and $0.5\Delta t$

At the first experiment the left wheel's observation data is lost for time period $(0.3\Delta t \div 0.5\Delta t]$ and the right wheel's observation data is lost for period $[0 \div 0.3\Delta t]$. MLE functions (Fig. 1) show correct detection of time moments in a relative range where each model was in effect. At the second experiment the right wheel's observation data is lost for time period $(0.03\Delta t \div 0.5\Delta t]$ and the right wheel's observation data is lost for period $[0 \div 0.03\Delta t]$.

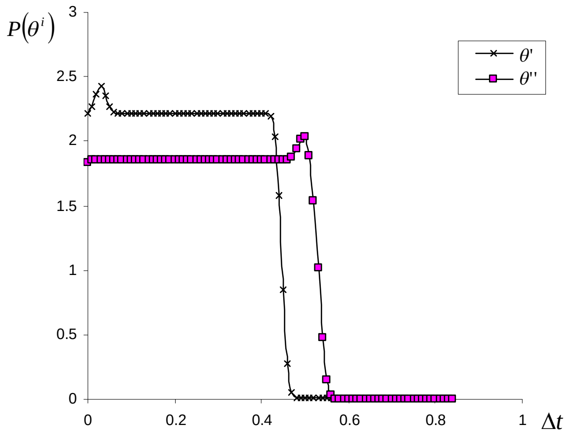

Fig. 2. Second simulation experiment's MLE functions with model changes at $0.3\Delta t$ and $0.5\Delta t$

It is worth of noting that it is not possible to infer of exact time moment of model shift during time period Δt without making more frequent measurements, but the question is whether we really need it. It is clear that for practical intuition it is enough of optimal detection of the duration of how long each model was in effect. This would give us enough information on the ground where robot is moving on and other external disturbances. Moreover, time Δt is selected to be small enough to make us less interested in deeper sampling of the signal.

Conclusions

The following conclusions can be drawn based on theoretical derivations and simulation results:

1. Simulation results confirm that models selected were correct and approach finds optimal solution under.
2. If robot's task is mission critical, and there can be no delays in processing, then either simultaneous updating has to be chosen, or new robotic sensor added. The new sensors should observe static or relatively features, thus enabling simultaneous updating. This would require increasing Δt when higher inference level.
3. Sliding of robots wheel due to manual intervention or kidnapping can be detected by using regular motion sensors, (e.g. wheels' distances and absolute robot's angle).
4. MrE with Simultaneous updating and model combination embodies Occam's razor, performs simultaneous VE of data through MLE and might be a part of any sequential Bayesian approach.

Further research is to extend models for non Gaussian distributions of sensor likelihoods, and apply MrE when all sensors are prone to errors by seeking for static features to infer from.

Acknowledgement

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Maximum relative entropy approach for determination of robot localization principles is developed and confirmed through simulation experiments. The approach updates probability posterior using simultaneous updating of model combination and data. Model combination is performed using simultaneous validation and estimation of observed data. This approach is not restricted to robot localization tasks, but it can be used in a wide range of fields, including econometrics, physics and other technological sciences. This approach can be used in high volume real time systems, including signal analysis. Ill. 2, bibl. 5 (in English; summaries in English, Russian and Lithuanian).

С. Барткевичюс, Р. Урнежюс, С. Жебраускас. Локализация робота при динамической неопределенности // Электроника и электротехника. – Каунас: Технология, 2009. – № 7(95). – С. 3–6.

Развит и подтвержден экспериментальным моделированием подход максимальной относительной энтропии при определении принципов локализации робота. При этом подходе вероятностное распределение обновляется одновременно с обновлением комбинации моделей и данных. Подход не ограничен только задачей локализации робота, он может использоваться в широком диапазоне областей, включая эконометрику, физику, другие технологические науки. Этот подход также может использоваться в системах реального времени большого объема, включая обработку сигнала. Ил. 2 , библ. 5 (на английском языке; рефераты на английском, русском и литовском яз.).

S. Bartkevičius, R. Urniežius, S. Žebrauskas. Robotų lokalizacija dinaminio neapibrėžtumo sąlygomis // Elektronika ir elektrotechnika. –Kaunas: Technologija, 2009. – Nr. 7(95). – P. 3–6.

Pateikta ir modeliavimo eksperimentu įrodyta maksimalios santykinės entropijos metodo modifikacija nustatant roboto lokalizacijos principus. Šiuo metodu atnaujinama aposteriorinė tikimybė tuo pačiu metu įvertinant modelių kombinaciją ir išmatuotus duomenis. Modelio kombinacija aprašoma taip, kad tuo pačiu metu būtų atmetami klaidingi duomenys ir atkuriami prarasti duomenys. Šis būdas neapsiriboja vien tik roboto lokalizacijos uždaviniu, bet gali būti naudojams daugelyje sričių, tarp jų ekonometrijoje, fizikoje ir kituose technologiniuose moksluose. Metodą galima naudoti didelio duomenų srauto realaus laiko sistemoms, įskaitant signalų analizę. Il. 2, bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).