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Statistics of Macro SC Diversity System with Two Micro EGC Diversity Systems and Fast Fading

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Introduction

Diversity methods have long been used in wireless systems to mitigate the detrimental effects of multipath fading and co-channel interference. Diversity schemes can be classified according to the type of combining employed at the receiver, namely, maximal-ratio combining (MRC), equal gain combining (EGC), selection diversity combining (SDC) and generalized selection combining (GSC). In an EGC combiner, the outputs of different diversity branches are first co-phased and weighted equally before being summed to give the resultant output. EGC combiner does not require the estimation of the channel gains, and hence it results in reduced receiver complexity relative to the MRC scheme. However, the performance of EGC is inferior to that of MRC since the branch weights are all set to unity. The Rayleigh distribution is the most widely used distribution to describe the received envelope value. The Rayleigh flat fading channel model assumes that all make up the resultant received signal are reflected or scattered and there is no direct path from the transmitter to the receiver. The Rayleigh distributed envelope of a received signal is given by :

$$Px(x)=\begin{Bmatrix} \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right), & 0\le x<\infty \\ 0 \qquad\qquad , & x<0 \end{Bmatrix}, \qquad (1)$$

where σ – the received root-mean-square (rms) envelope level; σ^2 – the average power of the received signal.

As mentioned above, diversity has long been recognized as a powerful communication receiver technique for mitigating the detrimental effects of channel fading and co-channel interference. The underlying premise is that if several uncorrelated replicas of a signal are received over multiple diversity paths with comparable signal strengths, then it is improbable that these signals will experience simultaneous deep fades. Diversity methods can be employed either at the base station (macroscopic diversity) or at the mobile station (microscopic diversity), although the antenna separation required differs for each case. In practice, microscopic diversity reception techniques are employed to combat the fast fading variations in the received signal strength caused by fast fading, whereas macroscopic diversity is used to mitigate the slower fading variations caused by shadowing.

Model of the receiver

Macro diversity system with two micro diversity systems is being disscused in this paper. The model of the discussed system is shown in Fig. 1. Signals at the first micro diversity combiner input are x_1 and x_2, and signals at the second micro diversity combiner input are y_1 and y_2. The signal at the first micro diversity system output is x, and the signal at the second micro diversity system output is y. Micro diversity combiners are EGC and the macrocombiner is SC. At the macro diversity combiners input there is a substantive Rayleigh fading. Micro diversity combiners are used to reduce the influence of fast Rayleigh fading to the system performances.

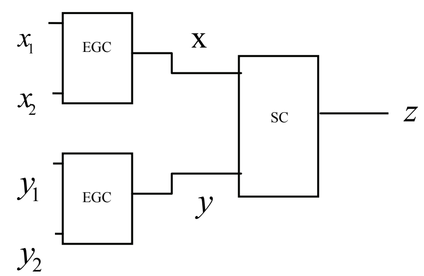

Fig. 1. The model of the system

Probability density of random variable z after macrolevel combining is:

$$p_z(z)=\int_0^\infty d\Omega_1\int_0^{\Omega_1} d\Omega_2 p_x\left(z/\Omega_1\right)p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)+$$
$$+\int_0^\infty d\Omega_2\int_0^{\Omega_2} d\Omega_1 p_y\left(z/\Omega_2\right)p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right), \qquad (2)$$

where

$$p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)=\frac{\rho^{-\frac{1}{2}(C-1)}}{\Gamma(C)(1-\rho)y_0^{C+1}}\left(\Omega_1\Omega_2\right)^{\frac{1}{2}(C-1)}e^{-\frac{\Omega_1+\Omega_2}{y_0(1-\rho)}}\times$$
$$\times I_{C-1}\left(\frac{\sqrt{4\rho\Omega_1\Omega_2}}{y_0(1-\rho)}\right) \qquad (3)$$

and $c \geq ½$ denotes the fading severity index, $Ic(.)$ is the modified Bessel function of the first kind of order c, $\Gamma(.)$ is the Gamma function, notation $E[.]$ denotes the statistical average of its argument, ρ $(0 < \rho < 1)$ corresponds to the power correlation coefficient between the signals received on the two closely spaced antennas.

The probability density function of the EGC micro diversity output in the presence of Rayleigh fading is given with [8]

$$p_r\left(r\right)=\frac{r^{M-1}e^{-\frac{r}{\Omega}}}{\Omega^M\left(M-1\right)!}, \qquad (4)$$

where M – denotes the number of microcombiner branches; Ω – denotes equivalent signal power at the microcombiner' s output which is also defined with [8]:

Since we are considering cases with two-branches microcombiners, our pdfs at the macrocombiner inputs are:

$$p_x\left(x/\Omega_1\right)=\frac{xe^{-\frac{x}{\Omega_1}}}{\Omega_1^{\,2}} \qquad (5)$$

and

$$p_y\left(y/\Omega_2\right)=\frac{ye^{-\frac{y}{\Omega_2}}}{\Omega_2^{\,2}}, \qquad (6)$$

after substituting previous expressions into (2) with the respect to (3) we obtain following expression:

$$p_z(z)=\int_0^\infty d\Omega_1\int_0^{\Omega_1} d\Omega_2\frac{ze^{-\frac{z}{\Omega_1}}}{\Omega_1^2}\frac{\rho^{-\frac{1}{2}(C-1)}}{\Gamma(C)(1-\rho)y_0^{C+1}}(\Omega_1\Omega_2)^{\frac{1}{2}(C-1)}e^{-\frac{\Omega_1+\Omega_2}{y_0(1-\rho)}}\times$$
$$\times I_{C-1}\left(\frac{\sqrt{4\rho\Omega_1\Omega_2}}{y_0(1-\rho)}\right)+$$
$$+\int_0^\infty d\Omega_2\int_0^{\Omega_2} d\Omega_1\frac{ze^{-\frac{z}{\Omega_2}}}{\Omega_2^2}\frac{\rho^{-\frac{1}{2}(C-1)}}{\Gamma(C)(1-\rho)y_0^{C+1}}(\Omega_1\Omega_2)^{\frac{1}{2}(C-1)}\cdot e^{-\frac{\Omega_1+\Omega_2}{y_0(1-\rho)}}\times$$
$$\times I_{C-1}\left(\frac{\sqrt{4\rho\Omega_1\Omega_2}}{y_0(1-\rho)}\right)=J_1+J_2\,. \qquad (7)$$

The integrals from the previous expressions can be solved by developing modified Bessel function of the first kind of order c by using well-known transformation:

$$I_n\left(x\right)=\sum_{k=0}^{\infty}\frac{x^{2k+n}}{2^{2k}k!\Gamma\left(k+n+1\right)}. \qquad (8)$$

Now expression for J_1 can be written

$$J_1=\sum_{k=0}^{+\infty}\int_0^\infty d\Omega_1\frac{ze^{-\frac{z}{\Omega_1}}\Omega_1^{k+c-3}\rho^k e^{-\frac{\Omega_1}{y_0(1-\rho)}}\gamma\left(k+c,\frac{\Omega_1}{y_0(1-\rho)}\right)}{\Gamma(c)(1-\rho)^k y_0^{\,k}k!\Gamma(c+k)}, \qquad (9)$$

where $\gamma(a,x)$ denotes lower incomplete Gamma function defined with

$$\gamma\left(a,x\right)=\int_0^x t^{a-1}e^{-t}dt, \qquad (10)$$

which can be exspressed trough more general hypergeometric function as

$$\gamma\left(a,x\right)=\frac{x^a}{a}e^{-x}{}_1F_1\left(1,1+a,x\right). \qquad (11)$$

After developing hypergeometrical function using

$${}_1F_1(a,b,x)=\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}\frac{x^n}{n!} \qquad (12)$$

and some mathematical manipulations, by regarding well-known expression related with modified K-Bessel function of second order

$$\int_0^\infty x^{v-1}e^{-\frac{a}{x}-bx}dx=2\left(\frac{a}{b}\right)^{\frac{v}{2}}K_v\left(2\sqrt{ab}\right) \qquad (13)$$

J_1 can be written as

$$J_1=\sum_{k=0}^{+\infty}\sum_{n=0}^{+\infty}\frac{2z\rho^k\left(\frac{zy_0(1-\rho)}{2}\right)^{\frac{2k+2c+n-2}{2}}K_{2k+2c+n-2}\left(2\sqrt{\frac{2x}{y_0(1-\rho)}}\right)}{\Gamma(c)(1-\rho)^{2k+c+n}y_0^{\,2k+c+n}k!n!\Gamma(c+k+n+1)\Gamma(k)}. \qquad (14)$$

In the similar manner we can obtain closed form expression for J_2 and it can be written as:

$$J_2=\sum_{l=0}^{+\infty}\sum_{m=0}^{+\infty}\frac{2z\rho^l\left(\frac{zy_0(1-\rho)}{2}\right)^{\frac{2l+2c+m-2}{2}}K_{2l+2c+m-2}\left(2\sqrt{\frac{2x}{y_0(1-\rho)}}\right)}{\Gamma(c)(1-\rho)^{2l+c+m}y_0^{\,2l+c+m}l!m!\Gamma(c+l+m+1)\Gamma(l)}. \qquad (15)$$

The pdf of macrcombiner output is presented at Fig. 2 for same values of fading parameters

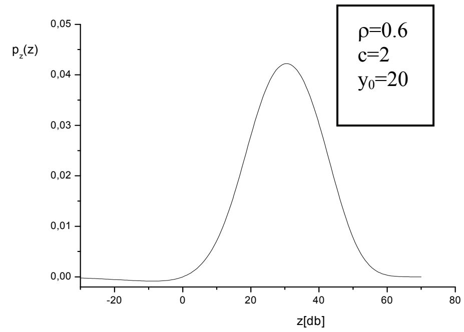

Fig. 2. Probability density function

As above, we calculated Cumulative Probability Density function for random variable z and it is shown in Fig. 3:

$$F_z(z) = \int_0^z \left\{ \int_0^\infty d\Omega_1 \int_0^{\Omega_1} d\Omega_2 p_x\left(\frac{x}{\Omega_1}\right) p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) + \right.$$

$$\left. + \int_0^\infty d\Omega_2 \int_0^{\Omega_2} d\Omega_1 p_y\left(\frac{y}{\Omega_2}\right) p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) \right\} dz. \quad (16)$$

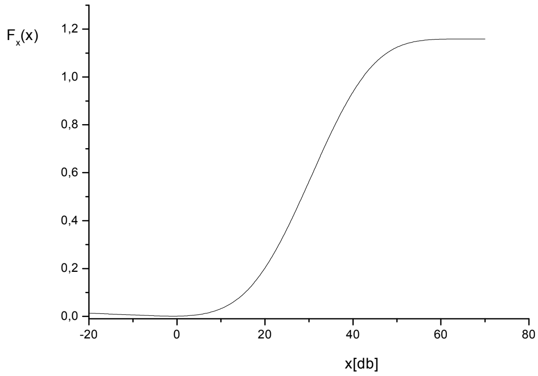

Fig. 3. Cumulative Probability Density Function

First order moment of random variable is calculated by next formula and shown in Fig. 4:

$$m_1 = \int_0^z z \left\{ \int_0^\infty d\Omega_1 \int_0^{\Omega_1} d\Omega_2 p_x\left(\frac{x}{\Omega_1}\right) p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) + \right.$$

$$\left. + \int_0^\infty d\Omega_2 \int_0^{\Omega_2} d\Omega_1 p_y\left(\frac{y}{\Omega_2}\right) p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) \right\} dz. \quad (17)$$

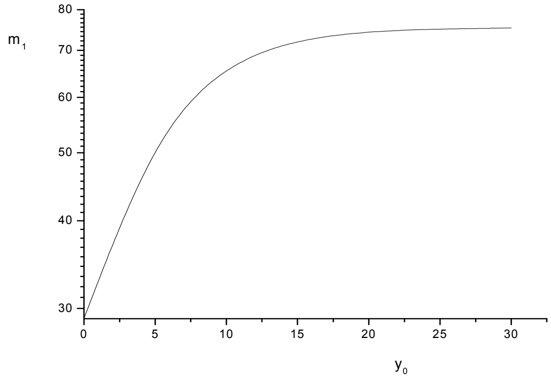

Fig. 4. First order Moment

Second order moment of random variable is calculated by next formula and shown in Fig. 5:

$$m_2 = \int_0^z z^2 \left\{ \int_0^\infty d\Omega_1 \int_0^{\Omega_1} d\Omega_2 p_x\left(\frac{x}{\Omega_1}\right) p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) + \right.$$

$$\left. + \int_0^\infty d\Omega_2 \int_0^{\Omega_2} d\Omega_1 p_y\left(\frac{y}{\Omega_2}\right) p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) \right\} dz. \quad (18)$$

The amount of fading can be calculated by :

$$A_F = \frac{m_2}{m_1^2} - 1. \quad (19)$$

The amount of fading is shown in Fig.6

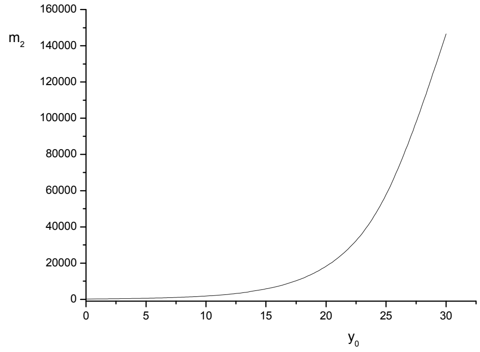

Fig. 5. Second order Moment

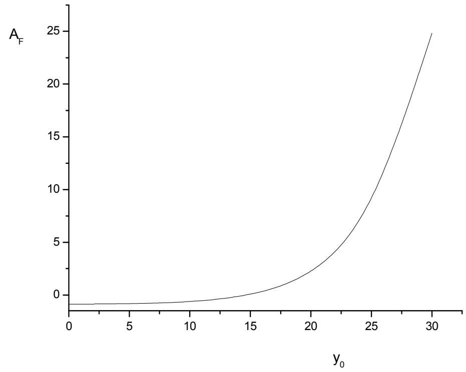

Fig. 6. The Amount of Fading

The channel capacity is given by formula (20) and shown in Fig. 7:

$$C = \int_0^\infty \ln(1 + z^2) p_z(z) dz. \quad (20)$$

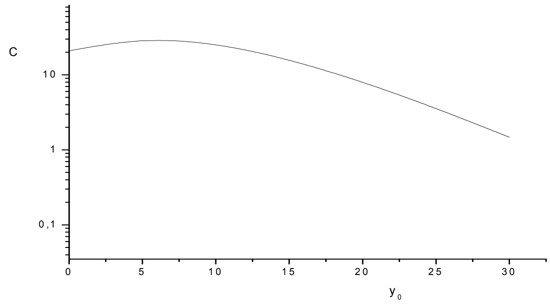

Fig. 7. Channel capacity

The bit error probability for corelated system is shown in Fig. 8 and can be calculated

$$Pe_1 = \int_0^\infty erfc(z^2) p_z(z) dz. \quad (21)$$

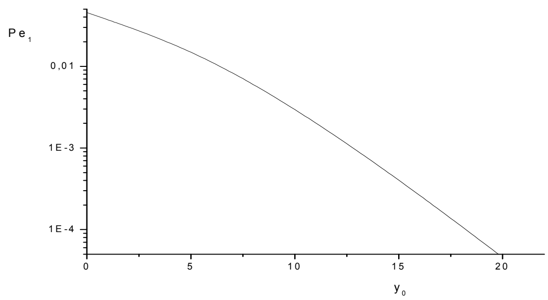

Fig. 8. Bit error probability

The bit error probability for QPSK system can be calculated by (22) and it is shown in Fig. 9:

$$Pe_2 = \int_0^{\infty} (erfc(z^2) + erfc^2(z^2)) p_z(z) dz . \qquad (22)$$

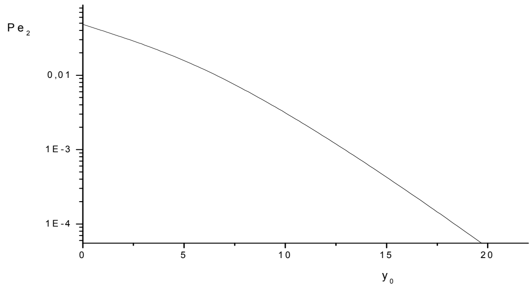

Fig. 9. Bit error probability for QPSK

Conclusion

The Probability density function is calculated by closed form of expression, and so: Cumulative probability density function, First order Moment of random variable, Second order Moment of random variable, Amount of fading, Channel capacity, Bit error rate, are calculated in this paper. The output combiner signal is simply the sum of combiner input signals. It was needed to use the binomial formula for calculating n-order moment. Besides this, the general Rayleigh moments of accidental variables are calculated too. The average signal from output of EGC combiner with two or more branches, the square average output signal and the signal variance can be calculated by obteined formulas. Also, the signal probability density can be calculated by, on that way, obteined output signal moments and some assembly of orthogonal functions.

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In this paper the statistics of macro SC diversity sistems with two micro EGC diversity systems and Rayleigh fading are given. We calculated and simulated: The Probability Density Function, The Cumulative Probability Density Function, The first order Moment of Random Variable, The second order Moment of Random Variable, The Amount of Fading, The Channel Capacity, The Bit Error Rate. Ill. 9, bibl. 8 (in English; abstracts in English, Russian and Lithuanian).

Б. Милошевич, П. Спалевич, М. Петрович. Д. Вушкович, Б. Милошевич. Статистическое исследование системы выборочного сочетания и разниц, а так же быстрого федингa двух микро EGC систем // Электроника и электротехника. – Каунас: Технология, 2009. – № 8(96). – C. 55–58.

Исследованы: система выборочного сочетания, систама двух микро EGC , а также фединг Релея. Расчитанно и смоделированно: функция плотности вероятности;интегральная функция распределения; момент переменных первого порядка; момент переменных второго порядка; сумма фединга; емкость канала и коэффициент двоичных погрешностей. Ил. 9, библ. 8 (на английском языке; рефераты на английском, русском и литовском яз.).

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Ištirta atrankinio derinimo sistema, dviejų EGC mikro sistema bei Reilėjaus fedingas. Apskaičiuota ir sumodeliuota tikimybės tankio funkcija, suminė tikimybės tankio funkcija, pirmos eilės kintamųjų momentas, antros eilės kintamųjų momentas, fedingo suma, kanalo talpa, dvejetainių klaidų koeficientas. Il. 9, bibl. 8 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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