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Theoretical Analysis of the Impact of CSO and CTB Distortions on BER Performances in Analog/*M*-QAM HFC/CATV Transmission Systems

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Introduction

The co-transmission of AM-VSB and *M*-QAM signals in the HFC/CATV networks involves some unfavorable effects of the AM-VSB channels on the *M*-QAM channels (Fig. 1), or the so-called cross-effects [1–3]. They find expression in worsening C/N, i.e. a great BER and getting nonlinear products (composite distortions: CSO; CTB) as a result of the laser "clipping", the Rayleigh backscattering and reflection noises.

In the majority of cases composite distortions are treated as noises because their unfavorable effect involves a BER increase.

Further on the article treats the factors causing the rise of composite distortions and the methods of their reduction and limitation. The BER has been defined by means of a mathematical analysis, on account of the arising CTB and CSO in the receiving-transmitting link, by means of Weibull and Rayleigh (Gaussian) distribution for several AM-VSB signals modulation indexes. The results of the analysis are presented in a graphic mode.

Factors causing the 2nd and 3rd order composite distortions

Many disturbing factors decreasing the quality of the digital channels are proved to be present within the HFC systems, whereas some of them are linked to the topology of the network, and others to the means of mixing the AM-VSB and *M*-QAM signals and parameters of the optical devices – optical receivers, optical transmitters and optical amplifiers.

One of the most important problems in contemporary HFC networks is maintaining a high level of electric parameters of the transmitted signals with respect to CSO and CTB.

The solution of this problem is connected to determining the causes and factors for the rise of composite distortions, and their nature as well. On the other hand, choosing an appropriate mathematical method giving a description of the composite distortions is of great importance to the correctness and the complexity of defining the above-mentioned parameters.

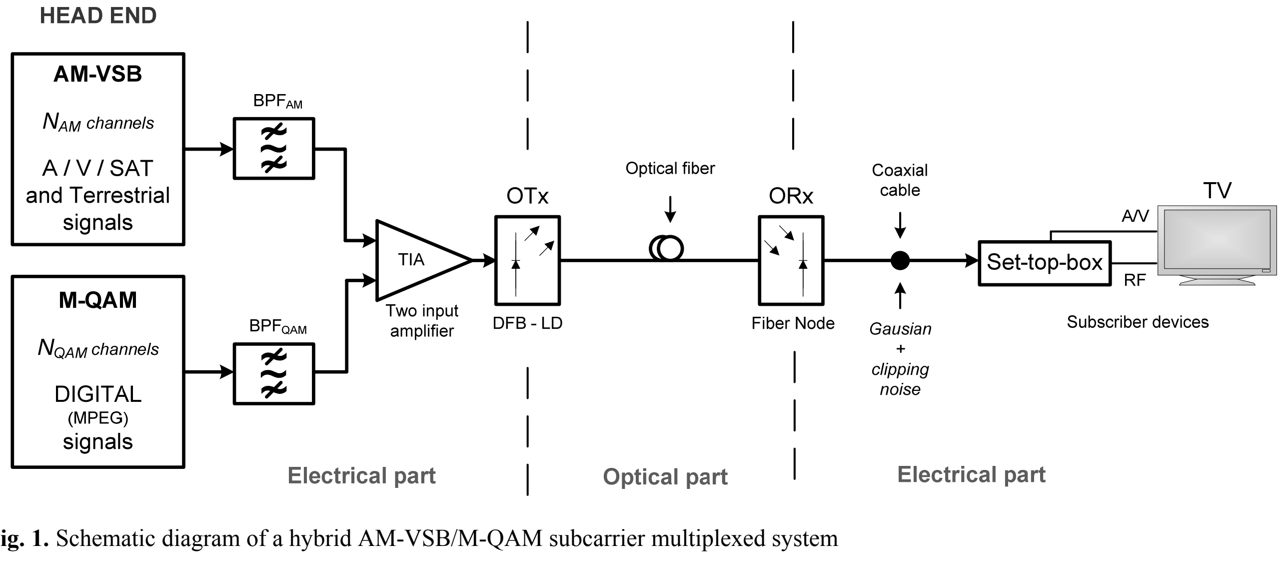

Fig. 1. Schematic diagram of a hybrid AM-VSB/M-QAM subcarrier multiplexed system

Each HFC network trunk line is composed of optical fibers and branches of coaxial cables. It is necessary to consider the interferences in each of the two network fields: the coaxial and the optical.

As there are numerous publications on CSO and CTB in coaxial networks, I am going to point out only the basic factors having effect on them - thermal noise brought in by the network; nonlinear distortions caused by the active devices; reflections and noise caused by other technical equipment.

Apart from thermal noise and nonlinear distortions, some laser "clipping" impulsive noises are being brought into the optical field [2, 3]. The impulsive noise inherent of the multiplexed signals by a frequency division causes reduction of the laser diode's output power $P_o(t)$ close to the zero, and its input driving current is lower than the current I_b determining its working point Fig.2. So with analog signals amplitudes surpassing the value (I_b - I_{th}) the signal is being limited which leads to the rise of nonlinear products causing an increase of the bit-error rate (BER).

The optical modulation index per AM-VSB channel is given by

$$m_{AM} = I/(I_b - I_{th}), \tag{1}$$

where I_b – the laser bias; I_{th} – the laser threshold current; the total RMS modulation index is

$$\mu = m_{AM}\sqrt{N_{AM}\ /\ 2}\ . \tag{2}$$

In a multichannel system whit $N_{AM} > 10$, the probability distribution for the amplitude of $I(t)$ can be accurately modeled as a Gaussian random process with variance

$$\sigma^2 = \mu^2(I_b - I_{th})^2 \tag{3}$$

and rectangular power spectral density.

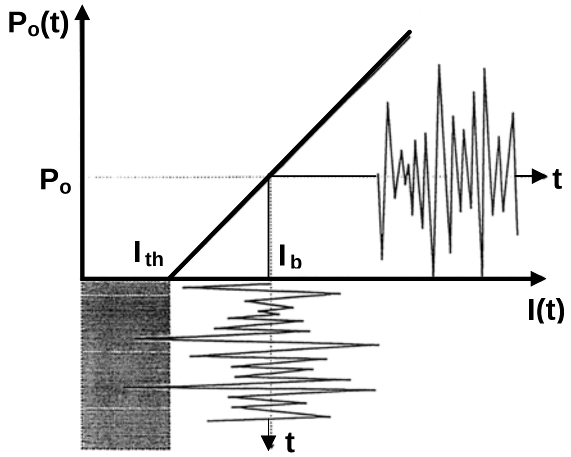

Fig. 2. Threshold clipping induced distortion of SCM

BER performances analysis

In order to provide a set image quality we have to define the bit - error rate of the digital channel (M-QAM signal) in connection to the composite distortions. For the BER to be calculated we need to know the function (pdf) describing the probability density of the distortions. The permissible levels of distortion can be defined by means of the method of BER analysis taking into consideration this function.

The Weibull distribution [4] has been used here in order to examine the composite distortions' statistical features:

$$P(z) = \frac{\alpha}{\beta^{\alpha}}.z^{\alpha-1}.exp\ [-(\frac{z}{\beta})^{\alpha}]\ ;\ z \geq 0, \tag{4}$$

where "α"– the "skewness" setting the form of the distribution; "β" – a "scale factor' determining the amplitude.

Weibull distribution with skewness of 2 has the same form as the Rayleigh distribution, which is the Gaussian random process amplitude distribution. Therefore the Weibull distribution can be applied to all kinds of composite distortions, whether their behavior is suddenly changing or not, by means of selection of skewness level's appropriate values.

At $\alpha = 2$ and $\beta = (2\sigma^2)^{1/2,}$ expression (4) changes to

$$P(z) = \frac{z}{\sigma^2}.exp\,(-\frac{z^2}{2\sigma^2})\,, \tag{5}$$

which are actually a Rayleigh distribution [4] and its probability distortion function looks like

$$P(x,y) = \frac{1}{2\pi\sigma^2}.exp\Big[-\frac{\left(x^2+y^2\right)}{2\sigma^2}\Big], \tag{6}$$

where $z = x + jy$ and σ^2 – the summary power of the Gaussian and the impulsive noises, i.e. $\sigma^2 = \sigma_g^2 + \sigma_i^2$.

With CTB and CSO present, arisen as a result of a Gaussian noise and a "clipping" noise (impulsive noise), the BER is calculated below by means of solving the probability density function (pdf) regarding the phase and the quadrature component of the Rayleigh distribution variable quantity z.

$$P_e = K_M.exp\left(\gamma\right).\iint\limits_D P\left(x,y\right)dxdy\,, \tag{7}$$

where γ – a clipping index (impulsive index); γ<<1; K_M – a ratio depending on the M-ary quadrature amplitude modulation and the Gray's code (see Table 1), [5].

Replacing $P(x,y)$ with expression (6), we get

$$P_e = K_M.exp\left(\gamma\right).\iint\limits_D \frac{1}{2\pi\sigma^2}.exp\Big[-\frac{\left(x^2+y^2\right)}{2\sigma^2}\Big]\,dxdy\ . \tag{8}$$

The D area of the pdf function integration is between -∞, +∞ and half of the minimum distance d_{min} between two adjacent states at the QAM modulation. From [5]

$$d_{min} = m_q F\sqrt{\frac{6}{M-1}}T\ , \tag{9}$$

where m_q – the optical modulation index for M-QAM signal; F – the receiving-transmitting link efficiency; M – multiplicity of QAM modulation; T – symbol time of the M-QAM signal; $T = 1/B$; B is the bandwidth of the channel.

Table 1. Values of K_M at quadrature amplitude modulation

M	16	32	64	128	256
K_M	0,375	0,329	0,292	0,26	0,234

We put $x = u\sqrt{2\sigma^2}$, $y = v\sqrt{2\sigma^2}$ and replace in (8).

$$P_e = K_M.exp(\gamma). \int_{\frac{d_{min}}{2\sqrt{2\sigma^2}}}^{+\infty} \int_{-\infty}^{+\infty} \left\{ \frac{1}{2\pi\sigma^2}.exp\left[-\frac{\left(u^2+v^2\right).2\sigma^2}{2\sigma^2} \right].2\sigma^2 \right\} dudv . \quad (10)$$

After making the necessary cancellations, turning to polar co-ordinates and changing limits, its final look is

$$P_e = \frac{1}{2}\cdot K_M \cdot exp(\gamma)\cdot exp\left\{ -\left[\sqrt{\frac{3}{\alpha}\cdot\frac{\Gamma\left(\frac{2}{a}\right)}{M-1}\cdot C/N} \right]^{\alpha} \right\}, \quad (11)$$

where Γ(•) – Gamma function [4].

BER's graphic dependency on the C/N variation in cases of Gaussian noise and impulsive noise from

$$P_e = \frac{1}{2}\cdot K_M \cdot exp\,[-\frac{3}{2}\cdot\frac{C/N}{M-1}+\gamma] \quad (12)$$

is shown in Fig.3 with $m_{AM} = 7\,\%$ and five values of the *M*, causing the rise of CSO and CTB as a result of the "clipping" in laser diode. The BER and C/N values in Fig.3 are in a scale of logarithms. After comparing the BER values, for a different *M*, is watched determinate dependence, lying on the next outcomes:

- By increasing M= 2^n , where *n* = 4, 5, 6, 7 and 8, for the BER value to be kept the same (for example 1,E-11, i.e. 1.10^{-11}) is necessary to be increased C/N from 24 dB (for M=16) to 36 dB (for M=256);
- To be kept BER as a constant at increasing *n* with 1 is necessary C/N to grow up with 3 dB.

The parameters of the optical link in Fig.1 are: *M*=16/32/64/128/256; f_H = 47 MHz; f_K = 470 MHz; *F* = 0,69mA; *RIN* = -150dB/Hz; $i_n = 24.10^{-12}\ A/\sqrt{Hz}$; N_{AM} = 42 and *B* = 8 MHz.

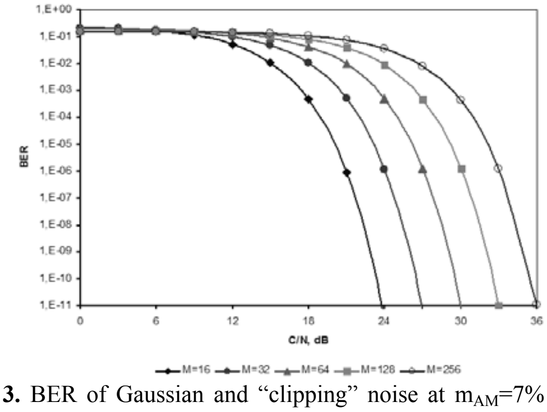

Fig. 3. BER of Gaussian and "clipping" noise at m_{AM}=7% and five values of M

The influence of AM-VSB signals, respectively of the "clipping" noise, onto the BER at changing of $m_{AM} = 0 \div 20\,\%$ (for five values of *M*) is shown on the next graphics. For all of them the C/N value is 24 dB. On Fig.4 are given the BER performances for the up pointed parameters of the optical link in accordance with expression (12). The $m_{AM} < 7\,\%$ values do not cause an alternation in BER when the multiplicity of QAM modulation is the same. At $m_{AM} > 7\,\%$ BER performances aggravates sharply. For example for *M* = 16 BER increases from 1.10^{-12} ($m_{AM} = 2\,\%$) to 1.10^{-5} ($m_{AM} = 20\,\%$).

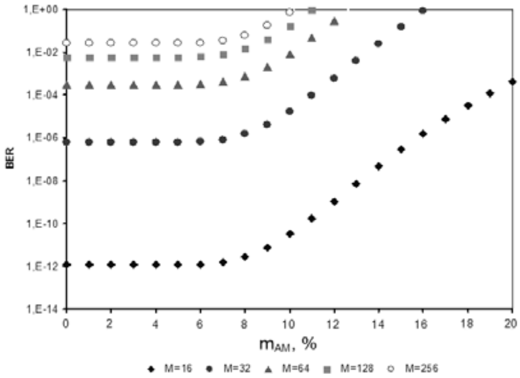

Fig. 4. BER = func(m_{AM}) under the common influence of the composite distortions from Gaussian and "clipping" noise

(12) expresses the dependency of the probability error function on the composite distortions, but without rendering an account on the independent effect of each one of them (CSO и CTB). The probability distortion functions, result of the nonlinearity of the laser diode and trunk line amplifiers, can be defined by means of Weibull distribution.

The probability function of the composite second order (CSO) distortions can be estimated with the following expression:

$$P_e^{CSO} = \frac{1}{2}\cdot K_M \cdot exp\,[-(\frac{5}{4}\cdot\Gamma(\frac{5}{6})\cdot\frac{C/N}{M-1})^{\frac{6}{5}}+\gamma]\,. \quad (13)$$

The probability function of the composite triple beat (CTB) is estimated by the expression:

$$P_e^{CTB} = \frac{1}{2}\cdot K_M \cdot exp\,[-(2\cdot\Gamma(\frac{4}{3})\cdot\frac{C/N}{M-1})^{\frac{3}{4}}+\gamma]\,. \quad (14)$$

BER graphic dependencies on the m_{AM} variation for (13) and (14) are shown in Fig.5 and Fig.6 separately for CSO and CTB with the above mentioned optical link parameters and C/N.

Conclusion

Composite distortions in the HFC systems have an unfavorable effect on the BER values, whereas with low modulation indexes (m_{AM} < 7 %) of AM-VSB signals prevailing is the Gaussian noise. In cases when m_{AM} increases to 7 % and more, the laser diode enters a deeper "clipping" regime (Fig.2), which leads to a significant increase of the second and third order composite distortions.

For m_{AM} > 8 %, in order to have a normally functioning HFC/CATV system, C/N must be greater than 30 dB. This could be achieved by increasing the optical modulation index for QAM signals $m_q \geq 0{,}52\,\%$.

Analyzing Fig.5 and Fig.6 we come to the following conclusions:

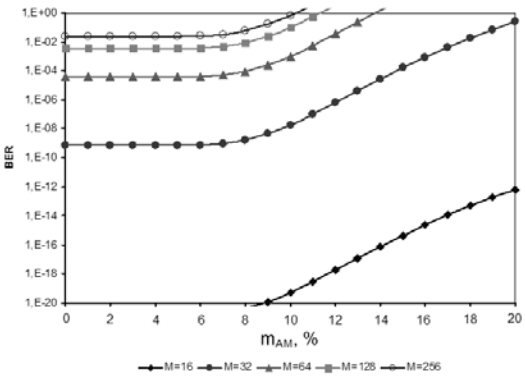

Fig. 5. BER = func(m$_{AM}$) under the influence of CSO Gaussian and “clipping” noise

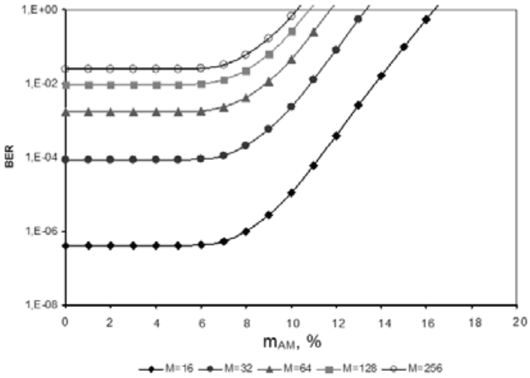

Fig. 6. BER = func(m$_{AM}$) under the influence of CTB from Gaussian and “clipping” noise

- CSO worsen BER lesser than the Gaussian noise and CTB, whereas the last ones are of most essential importance;
- Examining the composite distortions we have to pay greater attention to the CTB levels, rather than Gaussian noise and CSO levels;
- In order to have a normally functioning HFC system there must be a high carrier-to-noise ratio (C/N), when the modulation index of the analog signals m_{AM} is growing, and so do the composite distortions respectively.

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A bit-error-rate (BER) performances analysis for a Hybrid fiber coaxial (HFC) network with subcarrier multiplexed AM-VSB/M-QAM transmission over an optical fiber is presented. It is shown that the BER of M-QAM channels in such systems can be significantly affected due to occasionally laser diode “clipping” of the analog signals. Here we analytically determine the amplitude distribution of composite distortions and the clipping noise by modeling it as a Weibull distribution. The results of the analysis are presented in a graphic mode. Ill. 6, bibl. 5, tabl. 1 (in English; abstracts in English, Russian and Lithuanian).

О. Б. Панагиев. Теоретический анализ влияния CSO и CTB искажения на BER характеристики в аналоговых/ *M*-QAM HFC/CATV транслирующих системах // Электроника и электротехника. – Каунас: Технология, 2009. – № 8(96). – С. 39–42.

Представлен анализ влияния исскажения второго и третьего порядков на коэффициент двоичных ошибок (BER) при совместной передаче аналоговых и цифровых сигналов в гибридной волоконно-оптической сети кабельного телевидения с применением мултиплексирования. Показано что “клиппинг” в лазерном диоде значительно влияет BER. Здесь аналитически определено распределение амплитудны составные искажения и шум от ограничения, моделированный функцией распределения Вейбула. Результаты анализа представлены посредством графиков. Ил. 6, библ. 5, табл. 1 (на английском языке; рефераты на английском, русском и литовском яз.).

O. B. Panagiev. PVO ir CTB iškraipymų poveikio DKK charakteristikoms teorinė analizė analoginėse / M-QAM HFC / CATV transliavimo sistemose // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 8(96). – P. 39–42.

Pateikta antrojo ir trečiojo laipsnio iškraipymų poveikio analizė dvejetainių klaidų koeficientams (DKK), kuri atlikta per mišrius analoginių ir skaitmeninių signalų perdavimus hibridiniais optiniais kabelinės televizijos tinklais taikant multipleksavimą. Nustatyta, kad, lazerio signalo „karpymas“ stipriai veikia DKK. Čia analitiškai buvo apibrėžti amplitudės pasiskirstymo sudėtiniai iškraipymai ir triukšmai, kurie modeliuoti kaip Veibulo pasiskirstymo funkcija. Analizuoti rezultatai pateikti darbe grafiniu pavidalu. Il. 6, bibl. 5, lent. 1 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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