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Radiation of Electric and Magnetic Dipole Antennas in Magnetically Anisotropic Media

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Introduction

The electric and magnetic field intensities satisfy the following system of stationary Maxwell's equations:

$$\begin{cases} \text{rot}\,\boldsymbol{E} - i\omega \cdot \boldsymbol{B} = \boldsymbol{0}, \\ \text{rot}\,\boldsymbol{H} + i\omega \cdot \boldsymbol{D} = \boldsymbol{j}, \end{cases} \tag{1}$$

which is possible to be presented in matrix form

$$\mathbf{MU} = \mathbf{J}, \tag{2}$$

where $\mathbf{M} = \begin{pmatrix} i\omega\varepsilon\varepsilon_0\mathbf{I} & \mathbf{G}_0 \\ \mathbf{G}_0 & -i\omega\,\hat{\boldsymbol{\mu}}\mu_0 \end{pmatrix}$, $\mathbf{U} = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}$, $\boldsymbol{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$,

$\mathbf{G}_0 = \begin{pmatrix} 0 & -\partial_z & \partial_y \\ \partial_z & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix}$, $\boldsymbol{H} = \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}$, $\mathbf{J} = \begin{pmatrix} \boldsymbol{j} \\ \boldsymbol{0} \end{pmatrix}$,

$\boldsymbol{j} = \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}$, $\boldsymbol{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $\hat{\boldsymbol{\mu}} = \begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu_1 \end{pmatrix}$, ω – the constant frequency of electromagnetic field; $\mathbf{M}$ – Maxwell's operator; $\mathbf{I}$ – a unity matrix 3x3; $\boldsymbol{E}$, $\boldsymbol{H}$ – the intensity of electric and magnetic fields, μ a factor of magnetic permeability; $\mathbf{J}$ – vector of current density.

In magnetically anisotropic media the relation between induction and intensity of the magnetic field is:

$$\boldsymbol{B} = \mu_0\hat{\boldsymbol{\mu}}\boldsymbol{H}, \; B_x = \mu_0\mu H_x, \; B_y = \mu_0\mu H_y, \; B_z = \mu_0\mu_1 H_z, \tag{3}$$

and vector of electric induction:

$$\boldsymbol{D} = \varepsilon\varepsilon_0\boldsymbol{E}. \tag{4}$$

The elements of the magnetic permeability tensor $\hat{\boldsymbol{\mu}}$ are chosen so that the axis of anisotropy is directed along axis z. It is required to define the intensities of the electromagnetic field $\boldsymbol{E}$, $\boldsymbol{H}$ in the space of generalized functions.

Solution of the problem

A method based on the theory of the generalized function of the Fourier transformation is used for solving the matrix equation (2) [1]:

$$\tilde{\boldsymbol{E}}(\boldsymbol{k}) = \mathrm{F}[\boldsymbol{E}(\boldsymbol{r})] = \int_{\mathrm{R}^3} \boldsymbol{E}(\boldsymbol{r}) \exp(-i\boldsymbol{kr})dV, \tag{5}$$

$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{F}^{-1}[\tilde{\boldsymbol{E}}(\boldsymbol{k})] = \frac{1}{(2\pi)^3}\int_{\mathrm{R}^3} \tilde{\boldsymbol{E}}(\boldsymbol{k}) \exp(i\boldsymbol{kr})d^3k, \tag{6}$$

where $d^3k = dk_x dk_y dk_z$, $dV = dxdydz$, $\boldsymbol{r} = (x, y, z)$.

By means of direct Fourier transformation we write down the system of the equations (1) or (2) in matrix form:

$$\tilde{\mathbf{M}}\tilde{\mathbf{U}} = \tilde{\mathbf{J}}. \tag{7}$$

The solution of this problem is reduced to the solution of the system of the linear algebraic equations (7), where $\tilde{\mathbf{U}}$ is defined by means of inverse matrix $\tilde{\mathbf{M}}^{-1}$:

$$\tilde{\mathbf{U}} = \tilde{\mathbf{M}}^{-1}\tilde{\mathbf{J}}. \tag{8}$$

By introducing new functions according to

$$\tilde{\Psi}_0 \stackrel{\text{def}}{=} \frac{1}{k_0^2 - k_x^2 - k_y^2 - k_z^2}, \tag{9}$$

$$\widetilde{\Psi}_1^m \overset{\text{def}}{=} \frac{1}{k_n^2 - k_x^2 - k_y^2 - \dfrac{\mu_1}{\mu} k_z^2}, \tag{10}$$

$$\widetilde{\Psi}_2^m \overset{\text{def}}{=} \left(\frac{\mu_1}{\mu} - 1\right)\widetilde{\Psi}_1^m \widetilde{\Psi}_0. \tag{11}$$

The components of the electromagnetic field after transformations in image space can be written as follows:

$$\begin{cases} \widetilde{E}_x = -\dfrac{i}{\varepsilon\varepsilon_0\omega}(k_0^2(\widetilde{j}_x\widetilde{\Psi}_0 + k_y[\boldsymbol{k},\widetilde{\boldsymbol{j}}]_z\widetilde{\Psi}_2^m) - k_x\boldsymbol{k}\widetilde{\boldsymbol{j}}\widetilde{\Psi}_0), \\ \widetilde{E}_y = -\dfrac{i}{\varepsilon\varepsilon_0\omega}(k_0^2(\widetilde{j}_y\widetilde{\Psi}_0 - k_x[\boldsymbol{k},\widetilde{\boldsymbol{j}}]_z\widetilde{\Psi}_2^m) - k_y\boldsymbol{k}\widetilde{\boldsymbol{j}}\widetilde{\Psi}_0), \\ \widetilde{E}_z = -\dfrac{i}{\varepsilon\varepsilon_0\omega}(k_0^2\widetilde{j}_z - k_z\boldsymbol{k}\widetilde{\boldsymbol{j}})\widetilde{\Psi}_0. \end{cases} \tag{12}$$

$$\begin{cases} \widetilde{H}_x = -i\cdot([\boldsymbol{k},\widetilde{\boldsymbol{j}}]_x\widetilde{\Psi}_0 + k_x k_z[\boldsymbol{k},\widetilde{\boldsymbol{j}}]_z\widetilde{\Psi}_2^m), \\ \widetilde{H}_y = -i\cdot([\boldsymbol{k},\widetilde{\boldsymbol{j}}]_y\widetilde{\Psi}_0 + k_y k_z[\boldsymbol{k},\widetilde{\boldsymbol{j}}]_z\widetilde{\Psi}_2^m), \\ \widetilde{H}_z = -i[\boldsymbol{k},\widetilde{\boldsymbol{j}}]_z\widetilde{\Psi}_1^m. \end{cases} \tag{13}$$

$$([\boldsymbol{k},\widetilde{\boldsymbol{j}}]_z \equiv k_x\widetilde{j}_y - k_y\widetilde{j}_x). \tag{14}$$

It is possible to represent the electromagnetic fields in (12) and (13) in vector form as following

$$\widetilde{\boldsymbol{E}} = -\frac{i}{\varepsilon\varepsilon_0\omega}\left(k_0^2\left\{\widetilde{\boldsymbol{j}}\widetilde{\Psi}_0 + \left[\boldsymbol{k},\mathbf{e}_z[\boldsymbol{k},\widetilde{\boldsymbol{j}}_\perp]_z\right]\widetilde{\Psi}_2^m\right\} - \boldsymbol{k}(\boldsymbol{k}\widetilde{\boldsymbol{j}})\widetilde{\Psi}_0\right) \tag{15}$$

$$\begin{aligned} \widetilde{\boldsymbol{H}} = {} & i(\boldsymbol{k}_z - \boldsymbol{k})k_z[\boldsymbol{k},\widetilde{\boldsymbol{j}}_\perp]_z\widetilde{\Psi}_2^m + i\mathbf{e}_z[\boldsymbol{k},\widetilde{\boldsymbol{j}}_\perp]_z \times \\ & \times(\widetilde{\Psi}_0 - \widetilde{\Psi}_1^m) - i[\boldsymbol{k},\widetilde{\boldsymbol{j}}]\widetilde{\Psi}_0, \end{aligned} \tag{16}$$

where $\widetilde{\boldsymbol{j}} = \widetilde{\boldsymbol{j}}_\perp + \widetilde{\boldsymbol{j}}_0$, $\widetilde{\boldsymbol{j}}_0 = (0,0,\widetilde{j}_z)$, $\widetilde{\boldsymbol{j}}_\perp = (\widetilde{j}_x,\widetilde{j}_y,0)$,

$k_0^2 = \omega^2\varepsilon_0\varepsilon\mu\mu_0$, $k_n^2 = k_0^2\mu_1/\mu$.

It should be noted that the following useful formulae follow from (12):

$$\widetilde{\Psi}_0 - \widetilde{\Psi}_1^m \equiv (k_0^2 - k_z^2)\widetilde{\Psi}_2^m, \tag{17}$$

$$\widetilde{\Psi}_0 - \frac{\mu_1}{\mu}\widetilde{\Psi}_1^m \equiv (k_x^2 - k_y^2)\widetilde{\Psi}_2^m. \tag{18}$$

With the help of identity in (17) and (18), the last equation (16) can be represented as

$$\widetilde{\boldsymbol{H}} = i(k_0^2\mathbf{e}_z - k_z\boldsymbol{k})[\boldsymbol{k},\widetilde{\boldsymbol{j}}_\perp]_z\widetilde{\Psi}_2^m - i[\boldsymbol{k},\widetilde{\boldsymbol{j}}]\widetilde{\Psi}_0. \tag{19}$$

Using the property of convolution:

$$\mathrm{F}^{-1}[\widetilde{\mathbf{M}}^{-1}\widetilde{\mathbf{J}}] = \mathbf{M}^{-1} * \mathbf{J}, \tag{20}$$

and considering inverse Fourier transformation

$$\mathbf{M}^{-1} = \mathrm{F}^{-1}[\widetilde{\mathbf{M}}^{-1}], \qquad \mathbf{J} = \mathrm{F}^{-1}[\widetilde{\mathbf{J}}], \tag{21}$$

it is possible to get the solution of the Maxwell equations (2) as:

$$\mathbf{U} = \mathbf{M}^{-1} * \mathbf{J}, \tag{22}$$

where symbol "$*$" denotes the convolution on coordinates x, y, z.

So, after the inverse Fourier transformations from (15) and (19) we obtain:

$$\begin{aligned} \boldsymbol{E} = -\frac{i}{\varepsilon_0\varepsilon\omega}\Big(& (k_0^2 + \text{graddiv})\boldsymbol{j} * \Psi_0 - k_0^2 \times \\ & \times \text{rot}(\mathbf{e}_z(\mathbf{e}_z\text{rot}\boldsymbol{j}_\perp)) * \Psi_2^m\Big). \end{aligned} \tag{23}$$

$$\boldsymbol{H} = (k_0^2\mathbf{e}_z + \frac{\partial}{\partial z}\text{grad})\text{rot}_z\boldsymbol{j}_\perp * \Psi_2^m - \text{rot}\boldsymbol{j} * \Psi_0. \tag{24}$$

This solution can be written in the form of the sum of (12) two solutions:

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2, \ \boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2. \tag{25}$$

It should be noted that the first of them is the 'isotropic' solution. It is defined by Green's function Ψ_0 and the density of the current $\boldsymbol{j}_0$ directed endwise the axis z (of the anisotropy):

$$\begin{cases} \boldsymbol{E}_1 = -\dfrac{i}{\varepsilon_0\varepsilon\omega}(\text{graddiv}+k_0^2)(\Psi_0 * \boldsymbol{j}_0), \\ \boldsymbol{H}_1 = -\text{rot}(\Psi_0 * \boldsymbol{j}_0), \end{cases} \tag{26}$$

where the Green's function Ψ_0 can be defined from (7) by inverse Fourier transformation [1]

$$\Psi_0 = \mathrm{F}^{-1}[\widetilde{\Psi}_0] = -\frac{1}{4\pi}\frac{\exp(ik_0 r)}{r}, \ r = \sqrt{x^2 + y^2 + z^2}. \tag{27}$$

The second solution can be written by using the component of the density of the current $\boldsymbol{j}_\perp$ perpendicular to axis z and the Green's functions Ψ_0 and Ψ_2^m:

$$\begin{cases} \boldsymbol{E}_2 = -\dfrac{i}{\varepsilon_0\varepsilon\omega}\Big((k_0^2 + \text{graddiv})\boldsymbol{j}_\perp * \Psi_0 - k_0^2\text{rot}\times \\ \quad \times(\mathbf{e}_z\text{rot}_z\boldsymbol{j}_\perp) * \Psi_2^m\Big), \\ \boldsymbol{H}_2 = (k_0^2\mathbf{e}_z + \dfrac{\partial}{\partial z}\text{grad})\text{rot}_z\boldsymbol{j}_\perp * \Psi_2^m - \text{rot}\boldsymbol{j}_\perp * \Psi_0. \end{cases} \tag{28}$$

$$\Psi_1^m = \mathrm{F}^{-1}[\widetilde{\Psi}_1^m] = -\frac{1}{4\pi}\sqrt{\frac{\mu}{\mu_1}}\frac{\exp(ik_n r')}{r'}. \tag{29}$$

$$r' = \sqrt{x^2 + y^2 + \frac{\mu}{\mu_1}z^2}. \tag{30}$$

$$\Psi_2^m = (\mu_1/\mu - 1)\Psi_0 * \Psi_1^m. \tag{31}$$

The function Ψ_0, (27), is a fundamental solution of the Helmholtz operator for isotropic medium, while Ψ_1^m in (29) corresponds to the functions Ψ_0 in space deformed along the axis z.

Furthermore, the following transformations (identities) are valid [2]:

$$F^{-1}\left[\frac{\widetilde{\Psi}_0}{k_0^2-k_z^2}\right]=\Psi_0 * F^{-1}\left[\frac{1}{k_0^2-k_z^2}\right]=$$

$$=-\frac{i}{8\pi k_0}[e^{izk_0}\left(\mathrm{Ci}(k_0(r-z))+i\,\mathrm{Si}(k_0(r-z))\right)+$$

$$+e^{-izk_0}(\mathrm{Ci}(k_0(r+z))+i\,\mathrm{Si}(k_0(r+z)))], \quad (32)$$

$$F^{-1}\left[\frac{\widetilde{\Psi}_1^m}{k_0^2-k_z^2}\right]=\Psi_1^m * F^{-1}\left[\frac{1}{k_0^2-k_z^2}\right]=$$

$$-\frac{i}{8\pi k_0}[e^{izk_0}\left(\mathrm{Ci}(k_n r'-k_0 z)+i\,\mathrm{Si}(k_n r'-k_0 z)\right)+$$

$$e^{-izk_0}(\mathrm{Ci}(k_n r'+k_0 z)+i\,\mathrm{Si}(k_n r'+k_0 z))], \quad (33)$$

$$F^{-1}[\widetilde{\Psi}_2^m]=F^{-1}\left[\frac{\widetilde{\Psi}_0}{k_0^2-k_z^2}\right]-F^{-1}\left[\frac{\widetilde{\Psi}_1^m}{k_0^2-k_z^2}\right]. \quad (34)$$

We find that the function Ψ_2^m, (31) is given by:

$$\Psi_2^m=\frac{1}{i8\pi k_0}[e^{ik_0 z}(\mathrm{Ci}(k_0(r-z))+$$
$$+i\,\mathrm{Si}(k_0(r-z)))+e^{-ik_0 z}(\mathrm{Ci}(k_0(r+z))+$$
$$+i\,\mathrm{Si}(k_0(r+z)))-e^{ik_0 z}(\mathrm{Ci}(k_n(r'-k_0 z))+$$
$$+i\,\mathrm{Si}(k_n(r'-k_0 z)))-e^{-ik_0 z}\times$$
$$\times(\mathrm{Ci}(k_n(r'+k_0 z))+i\,\mathrm{Si}(k_n(r'+k_0 z)))], \quad (35)$$

where integral cosine and sine are defined by formulae's:

$$\mathrm{Ci}(z)=\gamma+\ln z+\int_0^z\frac{\cos t-1}{t}dt, \quad (36)$$

$$\mathrm{Si}(z)=\int_0^z\frac{\sin t}{t}dt-\frac{\pi}{2}, \quad (37)$$

and Euler's constant $\gamma=0{,}5772$.

Radiation patterns of Hertzian radiator in magnetically anisotropic media

On the basis of the results obtained above, we consider now numerical results for the radiation of the electric dipole.

The moment of point electric dipole is given by

$$\boldsymbol{p}=\boldsymbol{n}pe^{-i\omega t}, \ (\boldsymbol{p}=\boldsymbol{p}_0+\boldsymbol{p}_\perp), \quad (38)$$

where p is a constant. It corresponds to the current density defined by means of the Dirac delta-function:

$$\boldsymbol{j}=-i\omega\cdot\boldsymbol{p}\delta(\boldsymbol{r}). \quad (39)$$

The expression of the electromagnetic field for electric radiator will take the following form as for isotropic medium, when the direction of the dipole moment is parallel to the axis z ($\boldsymbol{p}=\boldsymbol{p}_0$) (Fig. 1):

$$\begin{cases}\boldsymbol{E}_1=-(\varepsilon_0\varepsilon)^{-1}(\mathrm{graddiv}+k_0^2)(\Psi_0\boldsymbol{p}_0),\\ \boldsymbol{H}_1=i\omega\cdot\mathrm{rot}(\Psi_0\boldsymbol{p}_0).\end{cases} \quad (40)$$

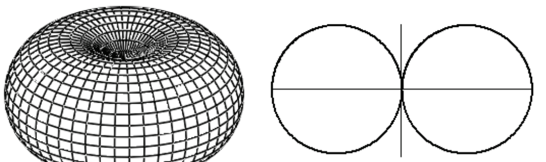
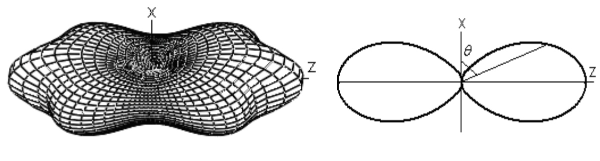


Fig. 1. Directional diagrams (DD**)**. The dipole moment is parallel to the axis z ($\boldsymbol{p}=\boldsymbol{p}_0$)

Also when the direction of the dipole moment is perpendicular to the axis z, we obtain ($\boldsymbol{p}=\boldsymbol{p}_\perp$) (Fig. 2, Fig. 3):

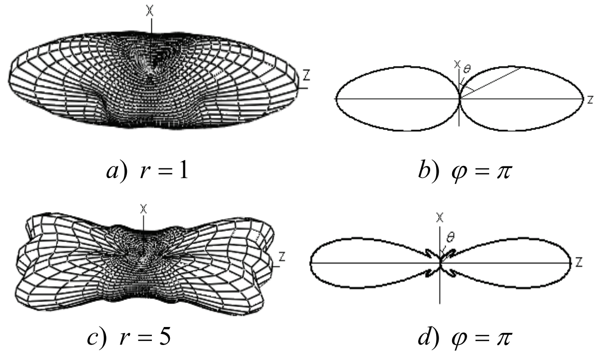
$$\begin{cases}\boldsymbol{E}_2=\dfrac{k_0^2\mathrm{rot}(\mathbf{e}_z\mathrm{rot}_z\boldsymbol{p}_\perp\Psi_2^m)-(k_0^2+\mathrm{graddiv})(\boldsymbol{p}_\perp\Psi_0)}{\varepsilon_0\varepsilon},\\ \boldsymbol{H}_2=(k_0^2\mathbf{e}_z+\dfrac{\partial}{\partial z}\mathrm{grad})\mathrm{rot}_z(\boldsymbol{p}_\perp\Psi_2^m)-\mathrm{rot}(\boldsymbol{p}_\perp\Psi_0).\end{cases} \quad (41)$$



a) $r=3$ *b*) $\varphi=\pi/2$

c) $r=9$ *d*) $\varphi=\pi/2$

Fig. 2. DD. The axis of electric dipole is perpendicular to axis z ($\mathrm{p}=\mathrm{p}_\perp$), $\mu_1/\mu=9$



a) $r=1$ *b*) $\varphi=\pi$

c) $r=5$ *d*) $\varphi=\pi$

Fig. 3. DD. The axis of electric dipole is perpendicular to axis z ($\boldsymbol{p}=\boldsymbol{p}_\perp$), $\mu_1/\mu=15$

Radiation pattern of point magnetic dipole moment in the magnetically anisotropic media

On the basis of the obtained results, we consider now radiation of point magnetic dipole moment. For a point radiator with the oscillating magnetic dipole moment, similarly to the electric dipole case (38), we have:

$$\boldsymbol{m} = \boldsymbol{n}\, m \exp(-i\omega \cdot t)\,,\ (\boldsymbol{m} = \boldsymbol{m}_0 + \boldsymbol{m}_\perp\,,\ m = \text{const}\,) \quad (42)$$

the electric current density is defined by using Dirac's delta-function:

$$\boldsymbol{j} = -[\boldsymbol{m}, \nabla] \cdot \delta(\boldsymbol{r})\,. \quad (43)$$

Components of a current density (43) have the following form:

$$\boldsymbol{j}_0 = \mathbf{e}_z \left(m_y \frac{\partial}{\partial x} - m_x \frac{\partial}{\partial y} \right) \delta(\boldsymbol{r})\,, \quad (44)$$

and

$$\boldsymbol{j}_\perp = \left(\mathbf{e}_x (m_z \frac{\partial}{\partial y} - m_y \frac{\partial}{\partial z}) + \mathbf{e}_y (m_x \frac{\partial}{\partial z} - m_z \frac{\partial}{\partial x}) \right) \delta(\boldsymbol{r}). \quad (45)$$

<u>*Case* $\boldsymbol{m} = \boldsymbol{m}_0$</u>. Relation between density of an electric current $\boldsymbol{j}_\perp$ and the magnetic dipole moment is defined from (45) in case when the magnetic dipole moment $\boldsymbol{m}$ is directed lengthwise *z*:

$$\boldsymbol{j}_\perp = \boldsymbol{m}_0 \left(\mathbf{e}_x \frac{\partial}{\partial y} - \mathbf{e}_y \frac{\partial}{\partial x} \right) \delta(\boldsymbol{r})\,. \quad (46)$$

It should be noted that the following useful formula hold:

$$\operatorname{div} \boldsymbol{j}_\perp = 0\,,\ \boldsymbol{j}_0 = 0\,. \quad (47)$$

Taking into account (47), intensities of the electromagnetic field by the magnetic dipole moment are defined from the solutions (28) in this case, as following :

$$\begin{cases} \boldsymbol{E} = \dfrac{k_n^2 \boldsymbol{m}_0}{i\varepsilon\varepsilon_0\omega} \operatorname{rot}(\mathbf{e}_z \Psi_1^m), \\ \boldsymbol{H} = \boldsymbol{m}_0 \Delta \Psi_1^m + \boldsymbol{m}_0 (\dfrac{\mu_1}{\mu} - 1) \dfrac{\partial^2}{\partial z^2} \Psi_1^m - \boldsymbol{m}_0 \dfrac{\mu_1}{\mu} \dfrac{\partial}{\partial z} \operatorname{grad} \Psi_1^m. \end{cases} \quad (48)$$

We have taken advantage of the next formulae which followed from (17) after inverse Fourier transformation

$$\frac{\mu_1}{\mu} \Psi_1^m = \Psi_0 + (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \Psi_2^m\,, \quad (49)$$

$$\Psi_0 = \Psi_1^m + (\frac{\partial^2}{\partial z^2} + k_0^2) \Psi_2^m\,. \quad (50)$$

Directional diagrams are represented in Fig. 4.

<u>*Case* $\boldsymbol{m} = \boldsymbol{m}_\perp$</u>. For the point magnetic dipole moment $\boldsymbol{m}_\perp$ which is perpendicular to axis *z*, by substituting (43) or (44) and (45) in (26) and (28), we define intensities of electromagnetic field as following (Fig. 5, Fig. 6):

$$\begin{cases} \boldsymbol{E} = i \dfrac{k_0^2}{\varepsilon\varepsilon_0\omega} \{ \dfrac{\partial}{\partial z} ([\boldsymbol{m}_\perp, \mathbf{e}_z] \dfrac{\mu_1}{\mu} \Psi_1^m - \operatorname{grad}_\perp \times \\ \qquad \times \operatorname{rot}_z (\boldsymbol{m}_\perp \Psi_2^m)) - \mathbf{e}_z \operatorname{rot}_z (\boldsymbol{m}_\perp \Psi_0) \}, \quad (51) \\ \boldsymbol{H} = (k_0^2 \mathbf{e}_z + \dfrac{\partial}{\partial z} \operatorname{grad}) \dfrac{\partial}{\partial z} \operatorname{div} \boldsymbol{m}_\perp \Psi_2^m - \operatorname{rot}\operatorname{rot}(\boldsymbol{m}_\perp \Psi_0). \end{cases}$$

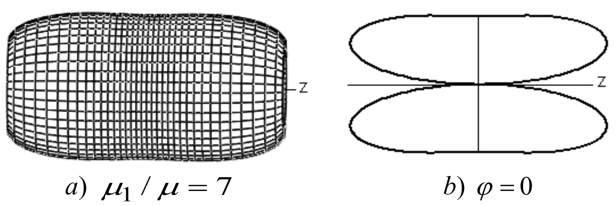


Fig. 4. DD. The axis of magnetic dipole is parallel to axis *z* ($\boldsymbol{m} = \boldsymbol{m}_0$).

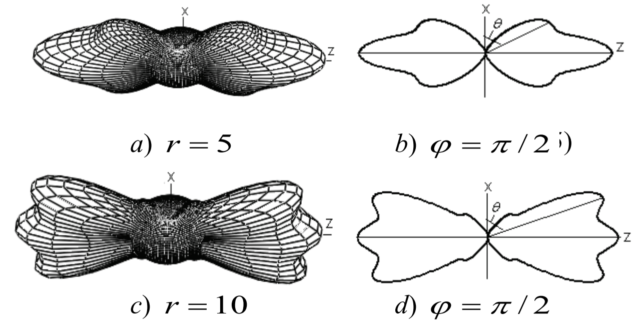


Fig. 5. DD. The axis of magnetic dipole is perpendicular to axis *z*, $\mu_1 / \mu = 7$.

The solution can be written by using the component of the density of the current $\boldsymbol{j}_\perp$ perpendicular to axis *z*.

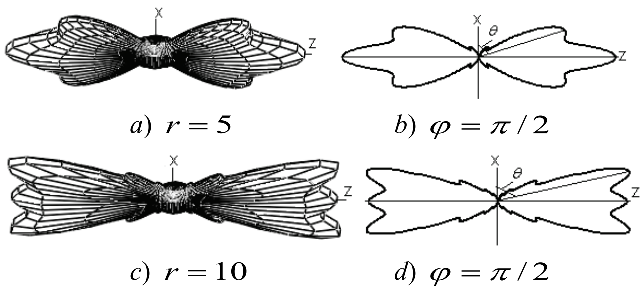


Fig. 6. DD. The axis of magnetic dipole is perpendicular to axis *z*, $\mu_1 / \mu = 15$.

Conclusions

The numerical calculation of the solution of Maxwell equations satisfies the energy conservation law. Numerical computation shows that time average value energy flux on a surface of sphere from a point dipole remains independent from its radius.

As shown in the electric dipole directional diagrams, medium becomes isotropic for such radiator if its moment is directed along anisotropy axis. The dipole pattern in isotropic media is shown in Fig. 1 and directional diagram itself possesses the rotation symmetry. However the point magnetic moment does not possess such property.

When μ_1 approaches μ, the potential Ψ_2 tends to zero and the well-known expressions of electromagnetic field follow from (26)-(28):

$$\boldsymbol{E} = i \frac{\omega}{k_0^2} (\operatorname{grad}\operatorname{div} + k_0^2) \boldsymbol{A}\,, \quad (51)(52)$$

$$\boldsymbol{H} = (\mu\mu_0)^{-1} \operatorname{rot} \boldsymbol{A}\,, \quad (53)$$

where the well - known formula for the vector potential of the electromagnetic field for isotropic mediums is given by:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu\mu_0}{4\pi} \int_V \boldsymbol{j}(\boldsymbol{r}) \frac{\exp(ik\,|\,\boldsymbol{r}-\boldsymbol{r}'\,|)}{|\,\boldsymbol{r}-\boldsymbol{r}'\,|} dV\,. \qquad (54)$$

The obtained generalized solutions of the Maxwell equations are valid for any values of μ_1 and μ, also near the sources of the electromagnetic waves, described by discontinuous and singular functions.

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In this paper the exact analytical solutions of the Maxwell equations in boundless magnetically anisotropic media are obtained in the vector type by the method of generalized functions at the action of the distributed currents. The electrical and magnetic dipole directional diagrams are plotted. Ill. 6, bibl. 4 (in English; summaries in English; Russian and Lithuanian).

С. С. Саутбеков, И. А. Канымгазиева, П. В. Франгос. Метод обобщенных функций в стационарных задачах электродинамики для магнитных анизотропных сред // Электроника и электротехника. – Каунас: Технология, 2010. – № 1(97). – С. 23–27.

В данной статье в векторном виде методом обобщенных функций получены точные решения уравнений Максвелла для безграничных гиротропных сред при действии распределенных токов. Приведены диаграммы направленности электрического и магнитного диполей. Ил. 6, библ. 4 (на английском языке; рефераты на английском, русском и литовском яз.).

S. S. Sautbekov, I. A. Kanymgazieva, P. V. Frangos. Apibendrinto funkcijų metodo taikymas elektrodinamikos magnetinės anizotropinės erdvės uždaviniuose // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 1(97). – P. 23–27.

Vektorine forma pateikti tikslūs būdai Maksvelo lygtims spręsti taikant apibendrintų funkcijų metodą veikiant beribėje anizotropinėje magnetinėje erdvėje paskirstytoms srovėms. Pateiktos elektrinės ir magnetinės dipolio kryptingumo diagramos. Il. 6, bibl. 4 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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