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Nonlinear Detection of Weak Pseudoperiodic Signals hidden under the Noise Floor

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Introduction

The task of signal detection and separation is a central theme in a wide variety of fields. Many techniques exist to improve the capability of detecting or enhancement a weak target signal corrupted by the additive Gaussian noise or the background noise generated by an unknown nonlinear dynamical mechanism. Due to its simplicity in implementation and efficiency in computation the nonlinear phase-space projection technique together with singular value decomposition or approximate joint diagonalization a set of time-delayed covariance matrices procedure is an essential tool in noise reduction [1–3], signal detection [4, 5], Blind Source Separation (BSS) [6–9] and biomedical signal processing [10, 11] algorithms. Given a noisy time series $\{x_i\}_{i=1}^{L}$ the phase points can be reconstructed by time delay embedding [12] – i.e. $\{\boldsymbol{x}_i\}_{i=1}^{L-(m-1)\tau}$:

$$\boldsymbol{x}_i = \left[x_i, x_{i+\tau}, x_{i+2\tau}, \cdots, x_{i+(m-1)\tau}\right]^T, \qquad (1)$$

where m – the embedding dimension, τ – time delay and $(\cdot)^T$ denotes the transpose of a real matrix. At $\tau = 1$ the reconstructed phase space matrix $\mathbf{X}$ with m rows and $M = L - m + 1$ columns (called a trajectory matrix) is defined by

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{L-m+1} \\ x_2 & x_3 & \cdots & x_{L-m+2} \\ \vdots & & \ddots & \\ x_m & x_{m+1} & & x_L \end{bmatrix}. \qquad (2)$$

It is widely assumed that if the noise is additive white Gaussian zero-mean measurement noise uncorrelated with the signal, the covariance matrix $\mathbf{R}(0)$ of the trajectory matrix $\mathbf{X}$ and time-delayed covariance matrix $\mathbf{R}(d)$ take the following structure respectively:

$$\mathbf{R}(0) = E\{\mathbf{x}(t)\mathbf{x}^{\mathrm{T}}(t)\} = \mathbf{R}_s(0) + \sigma^2\mathbf{I}, \qquad (3)$$

$$\mathbf{R}(d) = E\{\mathbf{x}(t)\mathbf{x}^{T}(t+d)\} = \mathbf{R}_s(d), \qquad (4)$$

where $E\{\cdot\}$ denotes the mathematical expectation value of the bracketed quantity, σ^2 – the variance of the noise, $\mathbf{I}$ – the identity matrix, $d \geq m+1$ (if $\tau = 1$), $\mathbf{R}_s(0)$ and $\mathbf{R}_s(d)$ the covariance matrix and time-delayed covariance matrix of true target signal respectively. However, adopting the delayed covariance matrices resolves the influence of the noise only at low and moderate amount of noise. If the signal-noise ratio (SNR) is low (approximately minus 20 dB and lower), this assumption is violated and the equation (4) is not valid for finite number of data samples – a noise related term still remains in the time-delayed covariance matrix. Moreover, the entries of covariance matrix determined by noise are much larger than that of target signal and the eigenvalues of $\mathbf{R}(d)$ demonstrate the behaviour of the eigenvalues of a pure noise time-delayed covariance matrix. Consequently, we cannot assign the first eigenvectors with the largest eigenvalues to signal subspace and the remaining eigenvectors with the smallest eigenvalues to noise subspace.

The objective of this paper becomes to experimentally investigate the detection performance of algorithm based on nonlinear phase-space reconstruction, a principal components analysis and frequency analysis in the detection of extremely weak pseudoperiodic signals buried in a large white Gaussian background. By pseudoperiodic signals we mean either a representative of a periodic orbit perturbed by dynamical noise or that the time series are produced from nonlinear deterministic systems. This class of time series – (chaotic) pseudoperiodic – has aroused great interest due to their close relation to some important natural and physiological systems.

Throughout the paper, the *x* component of the well-known Rossler system, which is chaotic and contains obvious periodic component, for illustration is used.

Detection scheme

The delayed covariance matrix $\mathbf{R}(d)$ is computed with one matrix $\mathbf{R}_r$ obtained by eliminating the last d columns of centered trajectory matrix $\mathbf{X}$ (with $\mathbf{X} = \mathbf{X} - \overline{\mathbf{X}}$, where $\overline{\mathbf{X}}$ is the column matrix of mean over dimension $1,\cdots,m)$ and another matrix, $\mathbf{R}_s$, obtained by eliminating the first d columns of $\mathbf{X}$:

$$\mathbf{R}(d) = \frac{1}{NN-1}\mathbf{X}_r\mathbf{X}_s^T, \tag{5}$$

where $NN = M - d$. The introduced trajectory matrix, similar to a square Hankel matrix, is matrix with constant (positive sloping) skew-diagonals. To avoid the large entries $\sigma^2\mathbf{I}$ at some descending diagonal of the time-delayed covariance matrix $\mathbf{R}(d)$ time-lag is chosen $d \geq m+1$.

Taking the standard singular value decomposition $\mathbf{R}(d) = \mathbf{W}\Sigma\mathbf{U}^{\mathbf{T}}$, the projected trajectory matrix is computed via the equation [1, 3]

$$\hat{\mathbf{X}} = \mathbf{U}_1 \cdot \mathbf{U}_1^T \cdot \left(\mathbf{X} - \overline{\mathbf{X}}\right) + \overline{\mathbf{X}}, \tag{6}$$

where the eigenvectors U_1, associated with the q largest eigenvalues diag($\mathbf{\Sigma}$), span the signal subspace and $\overline{\mathbf{X}}$ is the mean over dimension $1,\cdots,m$. Finally, a enhanced one-dimensional signal is created from the new space, typically by time-aligning and averaging the columns of the trajectory matrix $\hat{\mathbf{X}}$ (see [1] for more details)

$$\hat{\mathbf{X}}_{al} = \begin{bmatrix} \hat{x}_1 & \cdots & \hat{x}_{1+(m-1)} & \cdots & \hat{x}_{L-m+1} & \\ & \cdots & \hat{x}_{1+(m-1)} & \cdots & \hat{x}_{L-m+1} & \cdots \\ & \ddots & \vdots & \ddots & \vdots & \ddots \\ & & \hat{x}_{1+(m-1)} & \cdots & \hat{x}_{L-m+1} & \hat{x}_L \end{bmatrix} \tag{7}$$

and analyzed using the standard frequency estimation.

Detection performance analyses from simulated data sets

The detection performance of before-mentioned scheme was investigated with the x component of the Rössler system – the generated 200000 data points were divided into 20 equal blocks of 10000 samples. The equation of the Rössler system was numerically integrated with a step, which guaranty oversampling – about 260 data points over period. The additive white Gaussian noise was added with SNR from – 22 dB to –26dB. For such a small value of SNR, Fig. 1 shows that the signal frequency is completely hidden in the noisy broadband spectrum.

The trajectory matrix (2) was constructed by over-embedding [1], [3], [5] with time delay $d = 1$. The data dimension $m = 250$ (approximately one main period of target signal) was chosen. Data processing and time and frequency analyses were performed using software written in Matlab (The MathWorks, Natick, MA). Two eigenvectors, associated with the largest eigenvalues were chosen.

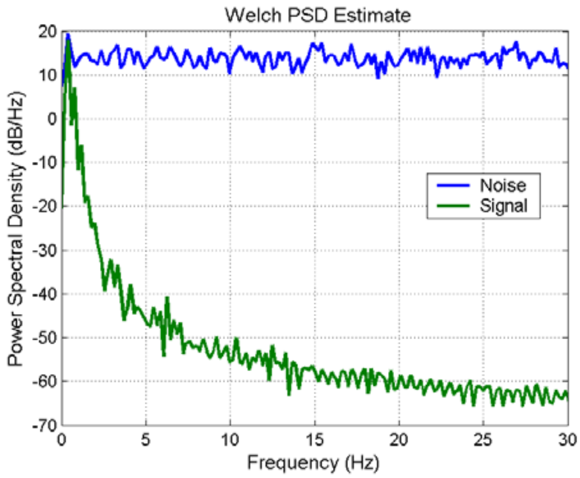

Fig. 1. Frequency spectra of the white Gaussian noise and target Rossler signal with SNR = – 26 dB

Fig. 2 shows frequency spectra of the target signal correctly extracted from additive white Gaussian noise environment. Fig. 3 shows the original Rossler signal and extracted.

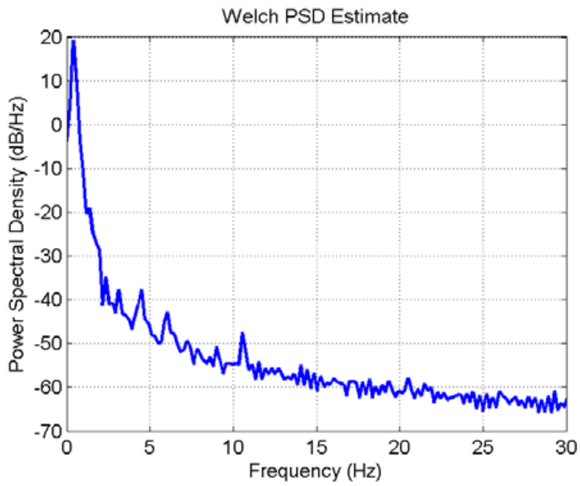

Fig. 2. Frequency spectra of the target signal extracted from additive white Gaussian noise environment, where the main frequency (0,4 Hz) of the signal is clearly observable

When the main period of target signal is approximately known, the shape of the time-delayed covariance matrix of true target signal can be enhanced by averaging the $\mathbf{R}(d)$ and $\mathbf{W}(d) = \mathbf{J}\cdot\mathbf{R}(d)\cdot\mathbf{J}$, where $\mathbf{J}$ is the exchange matrix with ones on its antidiagonal and zeros elsewhere. Factually the matrix $\mathbf{W}(d)$ represents the trajectory matrix with permuted entries, i. e.

$$\mathbf{X} = \begin{bmatrix} x_L & x_{L-1} & \cdots & x_m \\ x_{L-1} & x_{L-2} & \cdots & x_{m-1} \\ \vdots & & \ddots & \\ x_{L-m+1} & x_{L-m} & & x_1 \end{bmatrix}. \tag{8}$$

Therefore, the covariance matrices $\mathbf{R}(d)$ and $\mathbf{W}(d)$ are calculated with opposite lags. For true target signal $\mathbf{R}(d) \approx \mathbf{W}(d)$ at $d \approx T$, where T is the main period of the true source signal, but these matrices are unequal for noise and averaging operation

$$\mathbf{Z}(d) = [\mathbf{R}(d) + \mathbf{W}(d)]/2\,, \tag{9}$$

allow to suppress the influence of additive noise in some degree. The probability of detection by using $\mathbf{R}(d)$ and $\mathbf{Z}(d)$ is shown in Fig. 4.

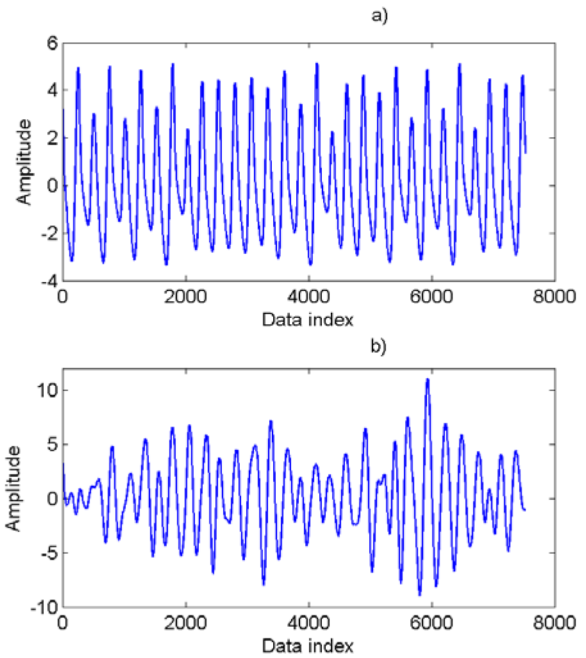

Fig. 3. Signals: a) Original Rossler and b) extracted from additive white Gaussian noise environment at SNR= – 26 dB

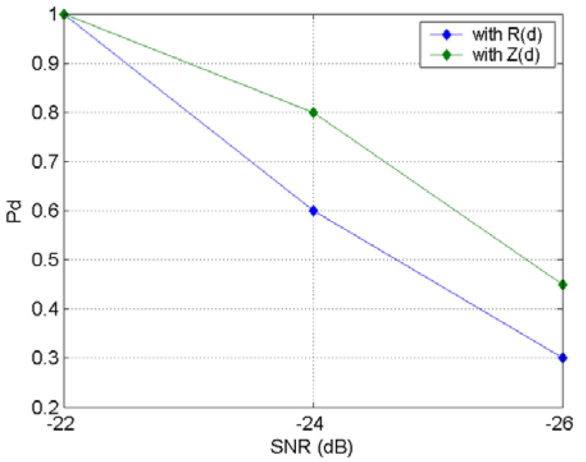

Fig. 4. The probability of detection (Pd) versus signal-noise ratio (SNR) by using single delayed covariance matrix $\mathbf{R}(d)$ and averaged covariance matrix $\mathbf{Z}(d)$

It is found that the detection algorithm by oversampling and appropriate selection of embedding dimension (overembedding) can reliably extract the main signal frequency at $SNR \geq -(22 \div 26) dB$. Naturally, the detection threshold of this algorithm, based on auto covariance, depends on the time series length and its stationarity.

It will be observed, that when the fourth-order cross-cumulant matrix is used instead of delayed covariance matrix, the performance of detection at low SNR noticeably fall – the detection algorithm can reliably extract the main signal frequency in this experiment only at $SNR \geq -18\, dB$. For time series $w(n), x(n), y(n)$ and $z(n)$ estimates of the fourth-order cross-cumulants are obtained as [13]

$$\begin{aligned} &C_4(k,l,r) = \\ &= E\{w(n)\cdot x(n+k)\cdot y(n+l)\cdot z(n+r)\} - \\ &- E\{w(n)\cdot x(n+k)\}\cdot E\{y(n)\cdot z(n+r-l)\} - \\ &- E\{w(n)\cdot y(n+l)\}\cdot E\{x(n)\cdot z(n+r-k)\} - \\ &- E\{w(n)\cdot z(n+r)\}\cdot E\{x(n)\cdot y(n+l-k)\}\quad , \end{aligned} \tag{10}$$

where $E\{\cdot\}$ denotes the ensemble expectation operator and k, l, r are appropriate lags. In this work $w(n), x(n), y(n)$ and $z(n)$ are the rows $i, j, j+1$ and $j+2$ of trajectory matrix (2) respectively and $j = 1:m$ for either $i = 1:m-2$;

Conclusions

In this paper the detection algorithms consisting of nonlinear phase space reconstruction technique, principal components analysis feature selection and frequency analysis are investigated by applying them to high-noisy pseudoperiodic chaotic Rossler signal. It is demonstrated, that algorithm is able to detect weak pseudoperiodic or pseudoperiodic deterministic signals completely hidden beneath the additive Gaussian noise floor at SNR up to –24 dB. The signal‘s main frequency can be extracted accurately and no false frequency peaks occur in spectrum of enhanced signal by the time series with length of over 10000 points. Whereas adopting the fourth-order cross-cumulant matrix instead of delayed covariance matrix leads to noticeably worse results. Therefore, it may be concluded, that detection technique based on singular value decomposition performing on the time-delayed covariance matrix of reconstructed phase space by selection of appropriate embedding (overembedding) and oversampling is a fairly effective method for detecting weak pseudoperiodic signals buried in a Gaussian noisy background.

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K. Pukėnas. Nonlinear Detection of Weak Pseudoperiodic Signals hidden under the Noise Floor // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 4(100). – P. 77–80.

The extraction of weak pseudoperiodic (deterministic) signals buried in a additive Gaussian noisy background is investigated by applying the nonlinear signal detection algorithm, based on phase-space embedding technique, principal component analysis and power spectral analysis. By analyzing Rossler chaotic signals, it is demonstrated that the detection algorithm based on the singular value decomposition of the time-delayed covariance matrix of the reconstructed high-dimensional phase space matrix is able to detect weak pseudoperiodic signals completely hidden beneath the additive Gaussian noise floor at SNR up to –24 dB.. Ill. 4, bibl. 13 (in English; abstracts in English, Russian and Lithuanian).

К. Пукенас. Нелинейное детектирование слабых псевдопериодических хаотических сигналов ниже уровня шумов // Электроника и электротехника. – Каунас: Технология, 2010. – № 4(100). – С. 77–80.

Исследуется выделение частот слабых псевдопериодических хаотических сигналов из адитивного гауссового шума при использовании алгоритма обнаружения сигналов, основанного на реконструкции фазового пространства, анализе главных компонент и спектрального анализа. Путем анализа хаотического сигнала Росслера показывается, что алгоритм, основанный на декомпозиции задержанной по времени ковариационной матрицы данных реконструированного многомерного фазового пространства позволяет обнаружить основную частоту сигнала Росслера на фоне белого гауссового шума при отношении сигнал-шум выше – 24 дБ. Ил. 4, библ. 13 (на английском языке; рефераты на английском, русском и литовском яз.).

K. Pukėnas. Silpnų pseudoperiodinių chaotinių žemesnio nei triukšmo lygio signalų išskyrimas taikant netiesinius metodus // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 4(100). – P. 77–80.

Tiriamas silpnų pseudoperiodinių (chaotinių) signalų išskyrimas iš adityviojo baltojo Gauso triukšmo, signalų detekcijai naudojant algoritmą, pagrįstą fazine erdvės rekonstrukcija, esminių komponenčių analize ir spektrine analize. Atlikus tyrimus su chaotiniu Rosslerio signalu, parodoma, kad algoritmas, rekonstruotos remiantis daugiamatės fazinės erdvės duomenų suvėlintosios kovariacinės matricos dekompozicijos tikriniais vektoriais, įgalina išskirti pagrindinius Rosslerio signalo dažnius iš baltojo Gauso triukšmo, kai signalas ir triukšmo santykis didesnis kaip -24 dB. Il. 4, bibl. 13 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).