Performance of Two-Hop Relay Assisted Decode-and-Forward Transmission under Mixed Fading Environments

Nuri Kapucu[1], Mehmet Bilim[1], Ibrahim Develi[1], Yasin Kabalci[2]
[1]*Department of Electrical and Electronics Engineering, Erciyes University, 38039, Kayseri, Turkey*
[2]*Nigde Vocational College of Technical Sciences, Nigde University, 51200, Nigde, Turkey*
nurikapucu@erciyes.edu.tr

***Abstract*—This study analyses the performance of two-hop decode-and-forward (DF) relaying protocol under mixed faded channels by evaluating outage probability (P_{out}) and average bit-error probability (ABEP). It is assumed that the channel between source and relay is Rayleigh fading, whereas the relay-destination link is subject to Weibull fading. First, a closed-form expression is formulated for P_{out} and then it is used to obtain ABEP for the considered system model. The ABEP performance of two-hop DF relaying is presented for *M*-ary phase shift keying (*M*-PSK) signaling with different constellation sizes. Moreover, the proposed analytical expressions are verified by simulations that present the correctness of the derived formulas.**

***Index Terms*—Decode-and-forward, two-hop relaying, Weibull distribution.**

I. INTRODUCTION

Two-hop transmission technologies have attracted much interest in wireless communications due to its connectivity when direct transmission is impractical because of possible shadowing effects or path loss attenuation. In real wireless communication environments, the links between cooperating nodes can experience asymmetric fading conditions. Several works have analysed two-hop transmission systems which use Amplify-and-Forward (AF) or Decode-and-Forward (DF) technique operating over fading channels [1]–[10]. In [1] and [2], the authors investigated dual-hop DF communication models with beamforming over Nakagami-*m* faded channels. Ikki and Ahmed presented closed form formulations in order to analyse two-hop transmissions under Weibull and Gamma faded conditions, respectively [3], [4]. The works on two-hop transmission mentioned previously were considered over symmetric fading conditions. However, symmetric fading model is practically unrealistic since it does not include the real wireless environment characteristics.

On the other hand, some papers have also presented the performance of two-hop relaying systems for asymmetric channel models which mean the wireless medium conditions of each hop is not the same [5]–[10]. It is due to the fact that the received signals are different for each relay link. Xu *et al.* [5] proposed exact expressions of two-hop AF relay system for outage probability (P_{out}) and average symbol error probability (ASEP) in mixed Nakagami-*m* and Rician faded wireless environments. This study was achieved by using the cumulative distribution function (CDF) of the total signal-to-noise ratio (SNR). Similar to [5], in [6], a dual-hop nonregenerative transmission system where the source-relay and relay-destination channels are subject to Rayleigh and Rician distributions was considered. The authors in [6] obtained exact and approximate formulas for P_{out} and ABEP. In [7], an AF relaying system in which the links experience Rayleigh/Rician and Rician/Rayleigh fading environments was examined. Ouyang *et al.* [8] found some approximations on the analytical and asymptotic average symbol error rate (ASER) expressions for dual-hop AF relaying under Rayleigh/Rician asymmetric fading conditions. The proposed expression in [8] was derived based on moment generating function (MGF) of the total SNR. In [9], P_{out} of two-hop DF relaying scheme was evaluated for Rayleigh/Gamma fading conditions. An exact P_{out} expression for the cooperation model was introduced in [9]. Lee *et al.* [10] derived a closed-form formulation for P_{out} of a two-hop relaying system which consists of radio-frequency (RF) and free-space optics (FSO) channels. In [11], an AF relaying transmission model was investigated based on formulated expressions for P_{out} and ABEP under mixed Rayleigh/Hoyt fading.

Few works have focused on the dual-hop transmission models under asymmetric fading conditions, *i.e.*, Rayleigh/Rician, Rician/Rayleigh, Rayleigh/Nakagami-*m*, Rayleigh/generalized Gamma, Rayleigh/Hoyt and RF/FSO. However, there is not any work which considers deriving new formulas for the P_{out} and ABEP of dual-hop DF relay network in Rayleigh/Weibull asymmetric faded links. The Weibull fading has a pliable distribution to describe wireless mediums, especially outdoor multipath fading environment [12]. It is also reported that the Weibull fading model

Manuscript received January 9, 2014; accepted July 9, 2014.

provides good fit of the statistical characteristics of wireless fading channels for theoretical studies [13]. Motivated by these observations, we study DF technique based a dual-hop communication system where the channels of two hops experience Rayleigh and Weibull faded conditions for the first time. Moreover, we derived the exact P_{out} expression for the considered system and illustrated some numerical results for different scenarios. Furthermore, an analytical expression of ABEP is obtained for different modulation schemes. This task is done by evaluating the difficult integral part of the ABEP expression. We also provide computer simulations and analytical results on ABEP to show the validity of the derived expressions.

II. General Descriptions

A. System Model

In Fig. 1, a two-hop DF transmission scheme is addressed where no direct transmission occurs between the source (*S*) and the destination (*D*). In such a system, a source communicates with the destination via a relay (*R*) node. The *R* is considered to be operating in the half duplex mode. In DF relaying protocol, the source message is transmitted to the *R* and then, the estimated source message at *R* is transmitted to the *D*.

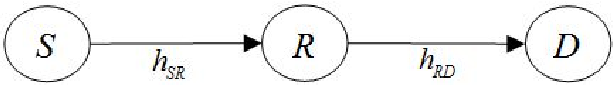

Fig. 1. Illustration of two-hop relaying scheme.

$\gamma_1 = |h_{SR}|^2 P_1 / N_0$ and $\gamma_2 = |h_{RD}|^2 P_2 / N_0$ indicate instantaneous SNRs of *S*-*R* and *R*-*D* links, respectively. P_1 and P_2 are the transmission power at *S* and *R*, respectively. N_0 is the additive white Gaussian noise (AWGN).

B. Channel Model

The channels between the $S-R$ and $R-D$ channels experience Rayleigh and Weibull fading conditions. While h_{SR} denotes flat Rayleigh fading coefficient, h_{RD} represents flat Weibull fading coefficient. The probability density function (PDF) of γ_1 is exponentially distributed with Rayleigh PDF given as

$$f_{\gamma_1}(\gamma_1) = \frac{1}{\bar{\gamma}_1}\exp\left(-\frac{\gamma_1}{\bar{\gamma}_1}\right), \tag{1}$$

where $\bar{\gamma}_1 = E\left(|h_{SR}|^2\right)P_1 / N_0$ is the average value of γ_1 and $E(\cdot)$ is the expectation operator.

The PDF of γ_2 can be given according to Weibull distribution as

$$f_{\gamma_2}(\gamma_2) = \frac{c\,\Omega^{c-1}}{\bar{\gamma}_2^{c}}\gamma_2^{c-1}\exp\left(-\frac{\Omega^{c-1}\gamma_2^{c}}{\bar{\gamma}_2^{c}}\right), \tag{2}$$

where c is parameter that expresses the severity of fading and Ω is the scaling parameter. $\bar{\gamma}_2 = E\left(|h_{RD}|^2\right)P_2 / N_0$ is the average value of γ_2.

III. Performance Analysis

A. Outage Probability

Outage probability, P_{out}, is described as the probability that the received SNR drops below a pre-defined outage level, γ_{th}. In the DF relaying, outage occurs if the instantaneous SNR of one or two of the hops fall below the outage level, γ_{th}. So, P_{out} is formulated according to the total SNR [4] as

$$\begin{aligned} P_{out} &= F_{\gamma_{eq}}(\gamma_{th}) = \Pr\{\min(\gamma_1,\gamma_2) \le \gamma_{th}\} = \\ &= 1 - \Pr(\gamma_1 > \gamma_{th})\Pr(\gamma_2 > \gamma_{th}), \end{aligned} \tag{3}$$

where $F_{\gamma_{eq}}(\gamma_{th})$ is the CDF of the end-to-end SNR. $\Pr(\gamma_1 > \gamma_{th})$ is determined according to Rayleigh fading PDF for *S*-*R* link as

$$\Pr(\gamma_1 > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} p_{\gamma_1}(\gamma_1)\,d\gamma_1 = \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_1}\right). \tag{4}$$

For *R*-*D* link over Weibull fading channel, $\Pr(\gamma_2 > \gamma_{th})$ can be calculated as follows

$$\begin{aligned} \Pr(\gamma_2 > \gamma_{th}) &= \int_{\gamma_{th}}^{\infty} p_{\gamma_2}(\gamma_2)\,d\gamma_2 = \\ &= \frac{c\,\Omega^{c-1}}{\bar{\gamma}_2^{c}}\int_{\gamma_{th}}^{\infty}\gamma_2^{c-1}\exp\left(-\frac{\Omega^{c-1}\gamma_2^{c}}{\bar{\gamma}_2^{c}}\right)d\gamma_2. \end{aligned} \tag{5}$$

In order to solve (5), we use [14, equation (3.381.9)] and $\Pr(\gamma_2 > \gamma_{th})$ can be derived as

$$\Pr(\gamma_2 > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} p_{\gamma_2}(\gamma_2)\,d\gamma_2 = \Gamma\left(1, \frac{\Omega^{c-1}\gamma_{th}^{c}}{\bar{\gamma}_2^{c}}\right). \tag{6}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [14]. By substituting (4) and (6) into (3), P_{out} is derived given as

$$P_{out} = 1 - \left\{\Gamma\left(1, \frac{\Omega^{c-1}\gamma_{th}^{c}}{\bar{\gamma}_2^{c}}\right)\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_1}\right)\right\}. \tag{7}$$

B. Average Bit Error Probability (ABEP)

This subsection gives the derivation of the ABEP, $P_b(e)$, of the considered dual-hop DF relaying network. By using the derived $F_{\gamma_{eq}}(\gamma_{th})$ expression, ABEP can be given as [6]

$$P_b(e) = \frac{1}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{eq}}\left(\frac{t^2}{\ }\right) e^{-t^2/2}\, dt. \qquad (8)$$

In (8), by setting $\ =1$ and $\ =2$, the performance of quadrature phase-shift keying (QPSK) and binary phase-shift keying (BPSK) can be evaluated, respectively. By substituting (7) into (8), $P_b(e)$ is derived as

$$P_b(e) = \underbrace{\frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t^2/2}\, dt}_{W_1} - \underbrace{\frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t^2/2} e^{-t^2/\gamma_1} \Gamma\left(1, \frac{\Omega^{c-1}\left(t^2/\ \right)^c}{\gamma_2^c}\right) dt}_{W_2}. \qquad (9)$$

The first term (W_1) in (9) can be calculated as [14, equation (3.321.3)]

$$W_1 = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t^2/2}\, dt = \frac{1}{2}. \qquad (10)$$

By using [14, equation (8.352.7)], the second term (W_2) of (9) can be rewritten in the way of well-known exponential function as

$$W_2 = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t^2\left(\frac{\gamma_1+2}{2\gamma_1}\right)} e^{-t^{2c}\left(\frac{\Omega^{c-1}}{\gamma_2^c\, c}\right)} dt. \qquad (11)$$

We are not aware of an exact solution to the integral in (11). So, we considered two different conditions as $c = 0.5$ and $c = 1$. Therefore, we have derived closed form expressions for each special cases ($c = 0.5$ and $c = 1$).

For the case of $c = 0.5$, W_2 can be determined by using [14, equation (3.322.2)] as

$$W_2 = \frac{1}{\sqrt{2\pi}} \left\{ \sqrt{\pi\beta}\, \exp\left(\beta z^2\right) \left[1 - \Phi\left(z\sqrt{\beta}\right)\right] \right\}, \qquad (12)$$

where $\beta = 1 \Big/ 4\left(\frac{\gamma_1+2}{2\gamma}\right)$ and $z = \left[1/(\Omega\gamma_2\ )\right]^{1/2}$. $\Phi(\cdot)$ is the error function which is defined as [14, equation (8.250.1)]

$$\Phi(x) = erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du. \qquad (13)$$

For the case of $c = 1$, W_2 can be obtained by using [14, equation (3.321.3)] as

$$W_2 = \left(2\sqrt{2}\sqrt{N}\right)^{-1}, \qquad (14)$$

where N is expressed as

$$N = \frac{1}{2} + \frac{1}{\gamma_1} + \frac{1}{\gamma_2}. \qquad (15)$$

In order to solve W_1 and W_2, we employed some mathematical manipulations which are not given here because of space constraints.

If (10) and (12) are employed in (9), the ABEP can be expressed for the cases of $c = 0.5$ as

$$P_b(e) = \frac{1}{2} - \left\{ \begin{array}{l} \sqrt{\pi \dfrac{1}{4\left(\frac{\gamma_1+2}{2\gamma_1}\right)}} \exp\left(\dfrac{1}{4}\left(\dfrac{\gamma_1+2}{2\gamma_1}\right)^{-1} (\Omega\gamma_2\ )^{-1}\right) \times \\ \times \left[1 - \Phi\left(\left[\dfrac{1}{(\Omega\gamma_2\ )}\right]^{1/2} \sqrt{\dfrac{1}{4\left(\frac{\gamma_1+2}{2\gamma_1}\right)}}\right)\right] \end{array} \right\}. \qquad (16)$$

For the cases of $c = 1$, by substituting (10) and (14) into (9), the ABEP can be written as

$$P_b(e) = \frac{1}{2} - \frac{1}{2\sqrt{2}} \left\{ \frac{\gamma_1+2}{2\gamma_1} + \frac{1}{\gamma_2} \right\}^{-1/2}. \qquad (17)$$

IV. NUMERICAL RESULTS

In realistic wireless environments, the channel conditions between cooperating nodes are subject to different fading distributions. Therefore, this work is based on such a realistic scenario which is more suitable to show the effects of real wireless environments. Performance evaluation results are presented with the help of the P_{out} and ABEP expressions derived in the previous section for two-hop DF relaying operating in mixed Rayleigh/Weibull faded links. P_{out} and ABEP are plotted versus average SNR per hop in Fig. 2 and Fig. 3, respectively.

The outage probability is shown for different outage threshold values when $\gamma_{th} = 6$ dB, $\gamma_{th} = 12$ dB and $\gamma_{th} = 18$ dB in Fig. 2. We focus on two values of fading parameter which are chosen as $c = 1$ and $c = 2$. As shown in Fig. 2, outage performance is better when the outage threshold value is low.

It can be clearly seen from Fig. 2 that the performance with $c = 2$ is better than the case of $c = 1$ when the average SNR per hop is greater than 8 dB for $\gamma_{th} = 6$ dB. However, similar behavior is observed when the average SNR per hop is greater than 11 dB and 12 dB for the cases of $\gamma_{th} = 12$ dB and $\gamma_{th} = 18$ dB, respectively. As a final remark from Fig. 2, P_{out} improves with an increase of c or with the decrease of γ_{th} value.

In Fig. 3, ABEP performance is presented based on analytical and simulation results for validating the correctness of the derived formulas where BPSK and QPSK modulation techniques are employed.

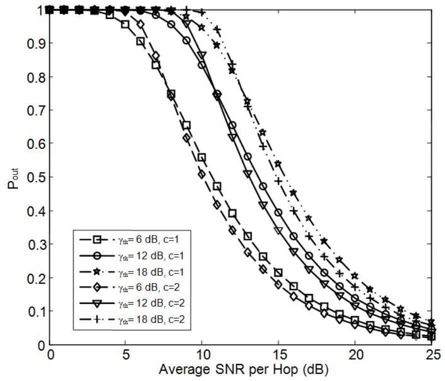

Fig. 2. P_{out} of two-hop relay assisted DF transmission system over mixed Rayleigh/Weibull fading channels for different outage threshold and fading parameters.

The performance results shown in Fig. 3 clearly illustrate that the ABEP of BPSK and QPSK for two-hop DF relaying improves with the larger values of fading parameter, *c*. It can be observed that the analytical results and the computer simulation results are in excellent concurrence for both BPSK and QPSK modulations when $c = 0.5$. However, the difference between analytical and simulation results for the ABEP is very small when $c = 1$ with BPSK modulation. For example, the ABEP value from simulation equal 8×10^{-2}, 2×10^{-3} while the analytical ABEP values are 1×10^{-1}, 1.6×10^{-3} for $SNR = 5$ dB and $SNR = 25$ dB, respectively.

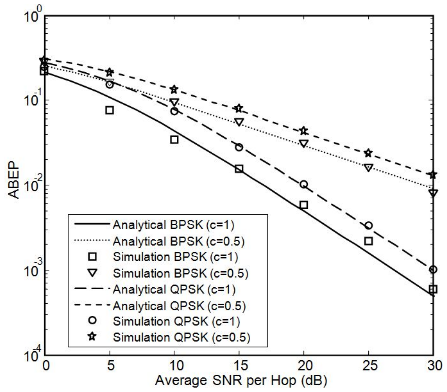

Fig. 3. ABEP of two-hop relay assisted DF transmission over mixed Rayleigh and Weibull fading channels for BPSK and QPSK modulation schemes for the cases of $c = 0.5$ and $c = 1$.

This tendency (the tightness of the proposed expressions) is valid for all values of average SNR per hop as shown in Fig. 3. Besides, there is a trade-off between the signal constellation size and performance gain. This means that the larger signal constellation size such as QPSK causes low performance gain in comparison to BPSK, as expected. As an example, the performance of the considered system is always better with BPSK compared to QPSK when the value of fading parameter is the same for both signalling techniques.

V. CONCLUSIONS

Closed-form P_{out} and ABEP expressions have been derived based on CDF expression of two-hop DF relaying system over mixed Rayleigh/Weibull faded links. This mixed faded channel model is capable of more accurately modeling various realistic two-hop wireless transmissions. In this realistic scenario, ABEP is determined when BPSK and QPSK signaling schemes are employed. The obtained analytical expressions can be employed to appraise the system performance of realistic two-hop relaying applications. The correctness of the expressions derived in this work is verified by computer simulation results.

REFERENCES

[1] D. B. da Costa, S. Aissa “Dual-hop decode-and-forward relaying systems with relay selection and maximal-ratio schemes”, *Electron. Letts*., vol. 45, no. 9, pp. 460–461, 2009. [Online]. Available: http://dx.doi.org/10.1049/el.2009.0597

[2] D. B. da Costa, S. Aissa, “Cooperative dual-hop relaying systems with beamforming over Nakagami-*m* fading channels”, *IEEE Trans. Wire. Commun*., vol. 8, no. 8, pp. 3950–3954, 2009. [Online]. Available: http://dx.doi.org/10.1109/TWC.2009.081353

[3] S. S. Ikki, M. H. Ahmed, “Performance analysis of dual hop relaying over non-identical Weibull fading channels”, in *Proc. IEEE Vehicular Technology Conf. (VTC)*, vol. 69, Barcelona, Spain, 2009, pp. 1–5.

[4] S. S. Ikki, M. H. Ahmed, “Performance analysis of dual hop relaying communications over generalized Gamma fading channels”, in *Proc. IEEE Global Telecommunications Conf. (GLOBECOM 2007)*, Washington, USA, 2007, pp. 3888–3893.

[5] W. Xu, J. Zhang, P. Zhang “Performance analysis of dual-hop amplify-and-forward relay system in mixed Nakagami-*m* and Rician fading channels”, *Electron. Letts*., vol. 46, no. 17, pp. 1231–1232, 2010. [Online]. Available: http://dx.doi.org/10.1049/el.2010.1555

[6] H. A. Suraweera, R. H. Y. Louie, Y. Li, G. K. Karagiannidis, B. Vucetic, “Two-hop amplify-and-forward transmission in mixed Rayleigh and Rician fading channels”, *IEEE Commun. Lett.*, vol. 13, no. 4, pp. 227–229, 2009. [Online]. Available: http://dx.doi.org/10.1109/LCOMM.2009.081943

[7] H. A. Suraweera, G. K. Karagiannidis, P. J. Smith, “Performance analysis of the dual-hop asymmetric fading channel”, *IEEE Trans. Wire. Commun*., vol. 8, no. 6, pp. 2783–2788, 2009. [Online]. Available: http://dx.doi.org/10.1109/TWC.2009.080420

[8] J. Ouyang, M. Lin, Y. Zhuang, “Performance analysis of cooperative relay networks over asymmetric fading channels”, *Electron. Letts*., vol. 48, no. 21, pp. 1370–1371, 2012. http://dx.doi.org/10.1049/el.2012.1737

[9] N. Kapucu, M. Bilim, I. Develi, “Outage probability analysis of dual-hop decode-and-forward relaying over mixed Rayleigh and generalized Gamma fading channels”, *Wireless Personal Communications,* vol. 71, no. 2, pp. 947–954, 2013. [Online]. Available: http://dx.doi.org/10.1007/s11277-012-0853-6

[10] E. Lee, J. Park, D. Han, G. Yoon, “Performance analysis of the asymmetric dual-hop relay transmission with mixed RF/FSO links”, *IEEE Photon. Tech. Lett.*, vol. 23, no. 21, pp. 1642–1644, 2011. [Online]. Available: http://dx.doi.org/10.1109/LPT.2011.2166063

[11] P. Spalevic, M. Stefanovic, S. Panic, S. Minic, Lj. Spalevic, “Amplify-and-forward relay transmission system over mixed Rayleigh and Hoyt fading channels”, *Elektronika ir Elektrotechnika*, no. 4, pp. 21–25, 2012.

[12] G. Tzeremes, C. G. Christodoulou, “Use of Weibull distribution for describing outdoor multipath fading”, in *Proc. IEEE Antennas, Propagation Soc. Int. Symp.*, vol. 1, San Antonio, TX, 2002, pp. 232–235.

[13] S. S. Ikki, S. Aissa, “Performance analysis of amplify-and-forward relaying over Weibull-fading channels with multiple antennas”, *IET Commun.*, vol. 6, no. 2, pp. 165–171, 2012. [Online]. Available: http://dx.doi.org/10.1049/iet-com.2011.0264

[14] I. S. Gradshteyn, I. M. Ryzhik, *Table of Integrals Series, and Products.* San Diego, CA: Academic, 2000.