

## ECG Research Using Elements of Matrix Analysis and Phase Planes

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### Introduction

Phase portraits are an invaluable tool in studying dynamical systems [1]. They consist of a plot of typical trajectories in the state space. This reveals information such as whether an attractor, a repeller or limit cycle is present for the chosen parameter value. It is important in classifying the behaviour of systems by specifying when two different phase portraits represent the same qualitative dynamic behavior.

Repeated moderate physical loads lead to adaptation of the organism, not only increasing its resistance to large physical loads but also providing a broad-spectrum protection, for example, against hypobaric hypoxia, toxins, and ionizing radiation [2].

Statistical or analytical methods can be used for analysis of two synchronous time series. Previous studies have shown that statistical methods are not suitable for biosignals [3], therefore, analytical methods should be considered

Previous studies [4, 5] have shown that discriminant better reveals the relation between two ECG signals than other characteristics. Therefore, the main aim of this paper is to search the ways to purify the information.

In this study the coherence of two ECG parameters (RR and JT intervals) are investigated during physical load. The phase plane represents homogeneity of repolarisation processes in myocardium of increased heart work during physical load.

### Theoretical Background

Previous studies made by scientist from all over the world have showed that concatenation has been the object of interest for years. The coherence between two different human body systems or different system parts was explored by many scientists: motor synergy [6], graphical investigation of coherence [7], complexity of biosignals [8,

9]. This study investigates the coherence between two different ECG parameters RR and JT.

Two synchronous time series  $X = (x_0, x_1, x_2, \dots)$  and  $Y = (y_0, y_1, y_2, \dots)$ , where  $x_n$  and  $y_n$  are real-valued terms, represent ECG measurement data. Mathematical methods based on matrix analysis are applied when the assumption that  $X$  and  $Y$  are determined is made [10]. Using  $X$  and  $Y$  the matrix time series are constructed:

$$M_n := \begin{pmatrix} x_n & x_{n+1} - y_{n+1} \\ x_{n-1} - y_{n-1} & y_n \end{pmatrix}. \quad (1)$$

Each  $n$ th matrix  $M$  from (1) can be expressed as follows [11]:

$$M^j := \begin{cases} \lambda_1^j I_1 + \lambda_2^j I_2, & \text{if dsk } M \neq 0, \\ \lambda^j E + (j-1)\lambda^j N, & \text{if dsk } M = 0, \end{cases} \quad (2)$$

where  $\lambda_1, \lambda_2$  are eigenvalues of matrix  $M$ ,  $I_1, I_2$  are idempotents,  $N$  is nulpotent. Therefore, several characteristics time series are proposed: discriminants (3), discriminant coefficients (4), idempotent coefficients (5):

$$\text{dsk } M_n = (x_n - y_n)^2 + 4(x_{n-1} - y_{n-1})(x_{n+1} - y_{n+1}), \quad (3)$$

$$\text{dskCoeff } M_n = \frac{(x_n - y_n + \alpha - \beta)^2 + 4(x_{n-1} - y_{n-1} + \alpha - \beta)(x_{n+1} - y_{n+1} + \alpha - \beta)}{\sqrt{((x_n + \alpha)^2 + 4(x_{n-1} + \alpha)(x_{n+1} + \alpha))(y_n + \beta)^2 + 4(y_{n-1} + \beta)(y_{n+1} + \beta)}}, \quad (4)$$

$$\text{IdeCoeff } M_n = \frac{(x_n - y_n) + \sqrt{(x_n - y_n)^2 + 4(x_{n-1} - y_{n-1})(x_{n+1} - y_{n+1})}}{2\sqrt{(x_n - y_n)^2 + 4(x_{n-1} - y_{n-1})(x_{n+1} - y_{n+1})}}, \quad (5)$$

where  $n$  is the number of cardio cycle,  $\alpha$  and  $\beta$  parameters.

## Results

The ECG parameters in 12 leads of continuous monitoring during provocative physical load test (Rouffier test) were investigated. The test was repeated four times: 1<sup>st</sup> day before 1<sup>st</sup> training session, after 1<sup>st</sup> training session, after 2<sup>nd</sup> training session and 2<sup>nd</sup> day before 1<sup>st</sup> training session. The recorded ECG was divided into following stages: before physical load the ECG of sportsman's steady state was recorded (1 min), then the participants of investigation performed Rouffier test (30 squats per 45 s) followed by 2 min of recovery (divided into beginning (1 min) and the end (1 min) of recovery).

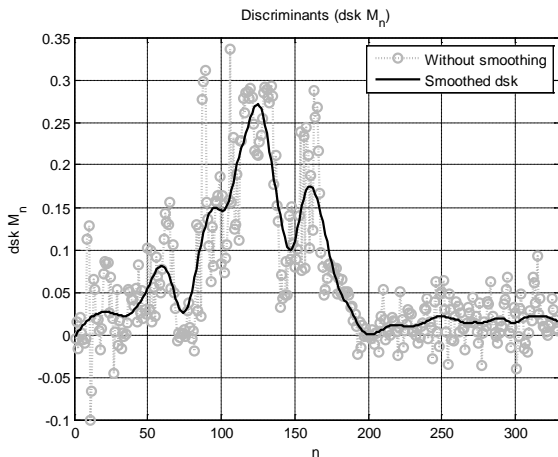
Thus the heart needs to overcome different stages of adaptation during physical load. Therefore intrinsic functional adaptation features and their homogeneity can be investigated using cointegration of two time series taken from ECG monitoring during different states of body (rest, physical workout etc.).

The time series of discriminants ( $dsk M_n$ ), discriminant coefficients ( $dskCoeff M_n$ ) and idempotent coefficients ( $IdeCoeff M_n$ ) are calculated from two time series: duration of RR interval taken from the II standard lead and duration of JT interval of the V standard lead.

In order to reduce noise and correct baseline locally weighted linear regression smoothing (LOESS) method is applied (see Fig. 1.) [12]. The smoothing process is considered local because each smoothed value is determined by neighboring data points defined within the span. The process is weighted because a regression weight function is defined for the data points contained within the span. The weight function used for LOESS is:

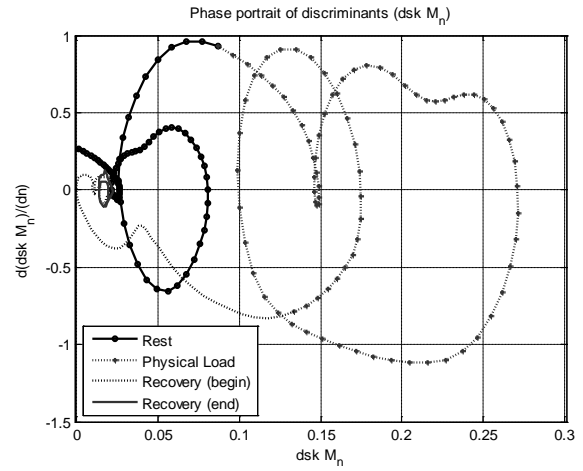
$$w_i = \left( 1 - \left| \frac{k - k_i}{d(k)} \right|^3 \right)^3, \quad (6)$$

where  $k$  is predictor value associated with the response value to be smoothed,  $k_i$  are the nearest neighbors of  $k$  as define by the span, and  $d(k)$  is the distance along the abscissa from  $k$  to the most distant predictor value within the span.

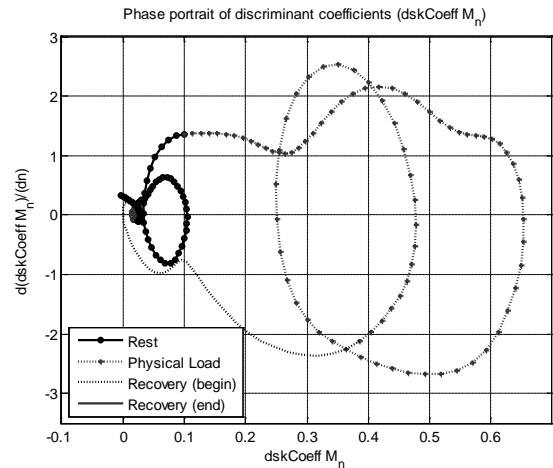


**Fig. 1.** Parameter  $dsk M_n$  before and after smoothing

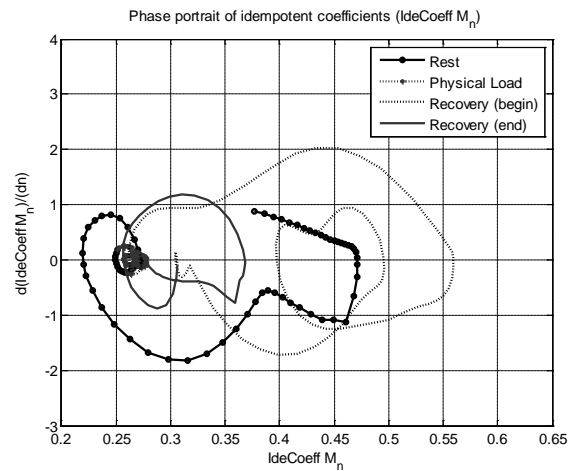
To specify the differences in dynamics of  $dsk M_n$  parameter a phase portrait for each sportsman was drawn. Phase portrait reveals how  $dsk M_n$  change velocity depends on  $dsk M_n$ . Therefore, y-axis represents derivative of  $dsk M_n$  and x-axis parameter  $dsk M_n$  (Fig. 2).



**Fig. 2.** Phase portrait of coherence dynamic between RR and JT intervals in subject A after the 1<sup>st</sup> training session ( $dsk M_n$ )



**Fig. 3.** Phase portrait of coherence dynamic between RR and JT intervals in subject A after the 1<sup>st</sup> training session ( $dskCoeff M_n$ )



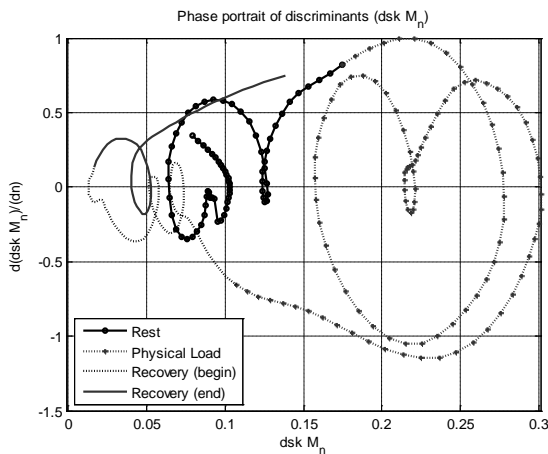
**Fig. 4.** Phase portrait of coherence dynamic between RR and JT intervals in subject A after the 1<sup>st</sup> training session ( $IdeCoeff M_n$ )

Two sportsmen with different duration of training experience were compared. Fig. 2, 3 and 4 represent subject A, who has less than 5 years of training experience and Fig. 5, 6 and 7 represent subject B, who has more than 5 years training experience.

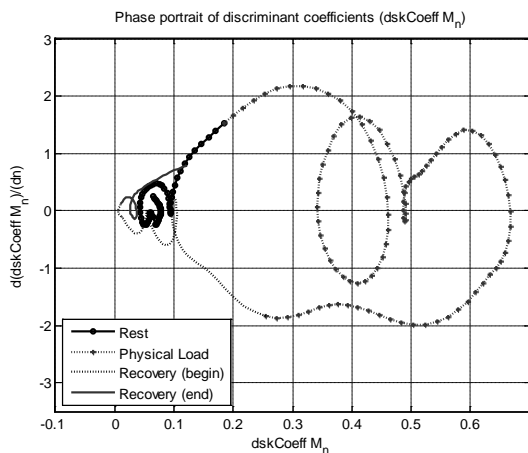
The dynamics of RR and JT intervals ratio shown in the following figures represents the interaction of the processes between regulatory and supplying systems depending on the performed physical load.

The obtained results showed that oscillations occur at the onset of investigated electrocardiogram signals. Depending on the stage of the testing the attractors form the coherence dynamics of the investigated parameters, thereafter bifurcation emerge and the dynamics moves and approaches new stable state.

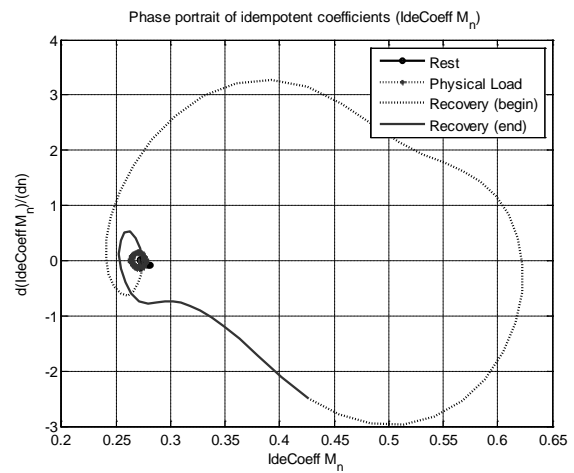
Parameters  $dsk M_n$  and  $dskCoeff M_n$  reveal very similar information (Fig. 3, Fig. 4) but in different scale. Parameter  $dskCoeff M_n$  is more sensitive than  $dsk M_n$ : discriminant coefficients grow faster than discriminants. Meanwhile, the parameter  $IdeCoeff M_n$  gives the opposite result comparing with previous ones. While  $dsk M_n$  and  $dskCoeff M_n$  grow during physical load, the parameter  $IdeCoeff M_n$  acts on the contrary. The same situation is seen for subject B.



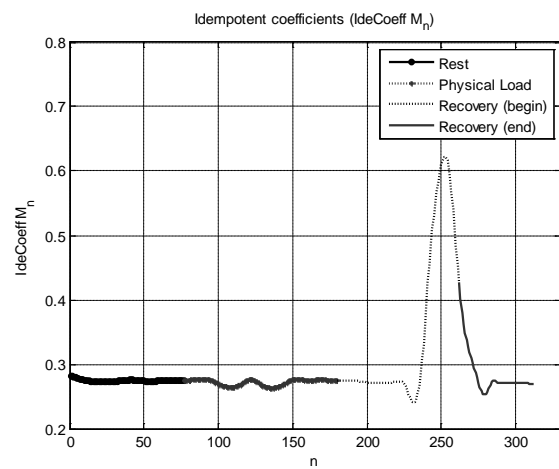
**Fig. 5.** Phase portrait of coherence dynamic between RR and JT intervals in subject B after the 1<sup>st</sup> training session ( $dsk M_n$ )



**Fig. 6.** Phase portrait of coherence dynamic between RR and JT intervals in subject B after the 1<sup>st</sup> training session ( $dskCoeff M_n$ )



**Fig. 7.** Phase portrait of coherence dynamic between RR and JT intervals in subject B after the 1<sup>st</sup> training session ( $IdeCoeff M_n$ )



**Fig. 8.** Parameter  $IdeCoeff M_n$  for subject B after the 1<sup>st</sup> training session

Even though, Fig. 7 differs from Fig. 4 and is not as informative but it does not deny the fact that idempotent coefficient acts opposite to discriminants. Fig. 7 has a different view because a problem related to the situation when discriminant is close to zero (and idempotent coefficient grows very fast) has not been solved yet. This situation is confirmed by linear graph (Fig. 8).

Comparing A and B person we can conclude that amplitude of discriminant and its derivative is much higher in person whom has longer duration of training experience (person B). Residual fatigue features has influence on the decrease of amplitude of these changes observed after training sessions.

## Conclusions

Phase plane of electrocardiogram parameters discriminants revealed that coherence between regulatory and supplying systems in human body on different stages of adaptation to load have dynamic character. The increase of fluctuation and new stable state reflect different metabolic rate depending on the change of the body state.

The limits of such fluctuations for healthy persons as well as for patients are still not known. But the results of

this study and future works might be useful for diagnostic purposes in patients or for the evaluation of the effect of physical training for athletes etc.

### Acknowledgement

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### References

1. **Jordan, D. W., Smith, P.** Nonlinear Ordinary Differential Equations (fourth ed.). – Oxford University Press, 2007. – 589 p.
2. **Malyshev I. Yu., Prodlus P. A., Meerson F. Z.** Effect of adaptation to moderate physical loads on the increased resistance of the isolated heart to ischemia and reperfusion // Bulletin of Experimental Biology and Medicine. – New York: Springer, 1995. – No. 1. – Vol 119. – P. 20–22.
3. **Cunha J. P. S., de Oliveira P. G.** A new and fast nonlinear method for association analysis of biosignals // Biomedical Engineering IEEE Transactions. – 2006. – Vol. 47. – No. 6. – P. 757–763.
4. **Venskaityte E., Poderys J., Balagué N, Bikulciene L.** Assessment of Dynamics of Inter-Parameter Concatenation Exercise Tests // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 6(94). – P. 89–92
5. **Smidtaite R., Navickas Z., Vainoras A, Bikulciene L, Poskaitis V.** Evaluation of Coherence of T-Wave in Different Leads // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 5(93). – P. 113–116.
6. **Latash M. L., Scholz J. P., and Schöner G.** Toward a New Theory of Motor Synergies // Motor Control. – 2007. – No. 11. – P. 276–308.
7. **Balagué N., Hristovski R.** Modelling fatigue and task failure as non linear processes // Abstracts Collection Computer Science in Sport – Mission and Methods. Dagstuhl Seminar Proceedings. – 2008.
8. **Sliupaite A., Navickas Z., Vainoras A.** Evaluation of Complexity of ECG Parameters Using Sample Entropy and Hankel Matrix // Electronics and Electrical Engineering. – Kaunas : Technologija, 2009. – No. 4(92). – P. 107–110.
9. **Kersulyte G., Navickas Z., Vainoras A., Gargasas L.** Calculation of Electric and Haemodynamic Processes in the Heart // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 3(91). – P. 43–48.
10. **Berskiene K., Lukosevicius A., Jarusevicius G., Jurkonis V., Navickas Z., Vainoras A., Daunoraviciene A.** Analysis of Dynamical Interrelation of Electrocardiogram Parameters // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 7(95). – P. 95–98.
11. **Bikulčienė L., Navickas Z., Vainoras A., Poderys J., Ruseckas R.** Matrix Analysis of Human Physiologic Data // Proceedings of International Conference on Information Technology Interfaces. – University of Zagreb. – 2009. – P. 41–46.
12. **Cleveland W. S., Devlin S. J.** Locally Weighted Regression: An Approach to Regression Analysis by Local Fitting // Journal of the American Statistical Association. – 1988. – Vol. 83. – P. 596–610.

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**R. Smidtaite, Z. Navickas, E. Venskaityte. ECG Research Using Elements of Matrix Analysis and Phase Planes // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 7(103). – P. 83–86.**

Electrocardiogram (ECG) is a diagnostic tool that measures activity of the heart. The dynamics of coherence between two different ECG parameters is investigated. Two time series, representing RR and JT parameters, are cointegrated into one time series and then matrix analysis is applied. While studying dynamical systems phase portraits are an invaluable tool. Therefore, to reveal similarities and differences of several characteristics phase portraits are drawn. In this study data of over twenty sportsmen are investigated. The view of phase portraits depends on physical load and training. The coherence of two parameters is to be extended to coherence of three parameters in further studies. Ill. 8, bibl. 12 (in English; summaries in English, Russian and Lithuanian).

**P. Шмидтайте, З. Навицкас, Я. Вянкайтите. Исследование электрокардиограмм с помощью матричного анализа и фазовых плоскостей // Электроника и электротехника. – Каунас: Технология, 2010. – № 7(103). – С. 83–86.**

Как правило, для оценки состояния человеческого сердца используется электрокардиограмма (ЭКГ). С помощью данных ЭКГ можно исследовать динамику взаимодействия между разными параметрами сердца. Связь между двумя параметрами оценивается при использовании матричного анализа. При этом из данных ЭКГ строится специальный ряд второго порядка матрицы, после чего полученный ряд обрабатывается математическими методами. Для более полного обозрения полученных результатов целесообразно использовать фазовое пространство. В этом пространстве эволюция исследуемых параметров отражается в виде геометрических кривых, которые наглядно демонстрируют разные нюансы деятельности сердца. Были исследованы данные параметров РР и ООП примерно для двадцати спортсменов. Было замечено, что разнообразность фазовых рисунков обусловило как физическая нагрузка, так и имеющийся спортивный опыт у спортсмена. В дальнейшем планируется исследование особенности взаимосвязи между тремя параметрами. Ил. 8, библи. 12 (на английском языке; рефераты на английском, русском и литовском яз.).

**R. Šmidtaité, Z. Navickas, E. Venskaityte. Elektrokardiogramų tyrimas naudojant matricinę analizę ir fazines plokštumas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 7(103). – P. 83–86.**

Širdies būklei nustatyti dažniausiai naudojama elektrokardiograma (ECG). Pasinaudojant ECG duomenimis stebima skirtingų parametrų tarpusavio ryšio dinamika. Dviejų parametrų ryšys vertinamas tam tikru būdu sudarant antros eilės matricių eilutę ir šiai eilutei taikant matricių analizę. Dinaminėms sistemoms tirti patogiau naudoti fazinę erdvę. Fazinė erdvė patogiai dinaminei sistemai kokybiškai apibūdinti. Šioje erdvėje sistemos evoliucija pateikiama geometriniu pavidalu. Siekiant atskleisti ir palyginti skirtingų charakteristikų panašumus ir skirtumus gauti rezultatai vaizduojami fazinėje erdvėje. Buvo tirti daugiau nei dvidešimties sportininkų RR ir JT parametrų duomenys. Pastebėta, jog skirtingus fazinius vaizdus lėmė tiek fizinis krūvis, tiek sportininko patirtis. Tolesniuose darbuose planuojama tirti ne tik dviejų, bet ir trijų parametrų sąsajas. Il. 8, bibl. 12 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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