ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2010. No. 7(103)
ELEKTRONIKA IR ELEKTROTECHNIKA

ELECTRICAL ENGINEERING
T 190
ELEKTROS INŽINERIJA

# Optimization of Two–Mass Electromechanical System

**B. Karaliūnas**
*Department of Automation, Vilnius Gediminas Technical University,*
*Naugarduko str. 41, LT-03227 Vilnius, Lithuania, phone: +370 274 50 63, e-mail: bronius.karaliunas@el.vgtu.lt*

## Introduction

Electric drives are applied in modern mechatronic systems, their motors are connected to the moving mechanisms by means of various mechanical links. Therefore, there are used toothed, worm – gear or composite speed – reducers, screw, belt and rack bar/link mechanisms as well as flexible shafts and crankshafts or drums with ropes and others. The mentioned above links experience elastic torsion, deformations of bending, tension and springing while transferring the motion of a motor to the mechanism. In majority cases they have specific backlash, without which there could be impossible to transmit mechanical movement at all.

In majority cases the coefficients of stiffness of mechanical links for mechatronic systems are minor. That is why the elasticity of these links and their deformations are considered to be significant as they distort the transferred motion of an electric motor. The phenomenon is typical for mechanical units, the motor of which is usually connected via the elastic elements of high inertia, e.g. a long shaft, a long rope of a hoisting unit, a belt of a long conveyor or a chain. Elasticity is also specific for long kinematic chains, as the sum of deformations of separate elements might be of high intensity.

When analyzing the dynamic processes of the mentioned above type of systems and the forces affecting them, all the inertia masses have to be substituted by two masses, one of which has to be rigidly connected to the shaft of a motor, while the second is to be separated from the shaft by kinematic clearance and by an elastic element. Recently, there appeared scientific publications, investigating the characteristics of two–mass electromechanical system.

The work [1] presents the description of high quality control unit, the electrical part of which is comprised of nonlinear two – mass system. Some issues of optimization of two – mass electromechanical systems are analyzed in article [2]. In article [3] there is analyzed the two – mass system with a finite stiffness shaft and signified by non-rectilinear phenomenon with kinematic looseness. The issues of synthesis in automatic control systems with two – mass mechanical link DC motors are analyzed in article [4]. The software package is used for the algorithm synthesis of control system, it is comprised of the algorithms of simplex search, of control effects formation, of indexes quality and adaptation programs. Article [5] presents the results of dynamic characteristics modeling for two – mass with elasticity system.

The review of literature sources indicates that because of the elastic bond and backlash of two – mass system the second mass is tend to oscillate more then the first. That is why the dynamic characteristics and transitional processes of such type drives are not considered to be optimal.

The objectivity of the work is to submit one presumable method of optimization for two – mass electromechanical system and to receive optimal transitional processes of the system.

## Optimization methods of electromechanical systems

To solve the optimization tasks of electromechanical systems there are widely applied methods of simulation, search optimization, algorithmic synthesis of systems, optimal control. We present one of the probable solutions when the system of algorithmic synthesis is applied.

If we assume that the object of control is expressed by vector differential equation

$$\dot{y} = f(\mathbf{y}, \mathbf{u}), \tag{1}$$

where **y** is $n$ – dimensional vector of state; **u** is $n$ – dimensional control vector.

When vector function (1) is not given or is of a complicated form, it is not possible to solve the task of optimal control by the analytic method. When border conditions are given as well as limitations of certain coordinates of the object, then

$$\left|y^{(s)}\right| \le y_m^{(s)}, \tag{2}$$

there exists the law of optimal control $u^*(t)$, providing the minimum to the functional

$$J = \int_{t_0}^{t_f} f_0(\mathbf{y}, \mathbf{u})\,dt. \tag{3}$$

Then the approximate value of the law of optimal control $u^*(t)$ is possible to be found in the process of search optimization by the following way. In the interval of control $t_0 \le t \le t_f$, when $t_0 = 0$, by means of discrete values of the components of vector **u**, there is introduced $k = mr -$ dimensional vector **x**

$$\mathbf{x} = \{x_1 = u_1[0], x_2 = u_1[1T], ..., x_{k-1} = u_m[(r-2)T], \\ x_k = u_m[(r-1)T]\}, \tag{4}$$

where $T = t_f / r$; $T$ is the quantifying period; $r$ – is quantifying step.

In article [4], there is submitted the formation of function $u(t)$ applying the components $x_i$ of vector **x**. As vector **x** comprises the law of control $u(x,t)$ in interval $t_0 \le t \le t_f$, the task of optimal control is possible to be written in the form of search optimization. That is why it is required to determine vector $x^*$ ensuring the minimum of the functional

$$J(x) = J[\mathbf{y}, \mathbf{u}(\mathbf{x}, t)], \quad 0 \le t \le t_f \tag{5}$$

by observing limitations (2) and (6):

$$g_j[\mathbf{y}, \mathbf{u}(\mathbf{x}, t)] \ge 0, \quad j = 1, ..., q, \tag{6}$$

where $J$, $g_j$ are indexes of quality of control, e. g, time of regulation is $t_r$, dynamic deviation is $\sigma$, control error is $\Delta y$ and etc.

This method could be implemented when solving the tasks of optimization of electromechanical systems, which couldn’t be solved by means of classical methods of optimization. Hereafter is analyzed the electromechanical system, comprised of DC motor, controlled by changing the voltage of an armature, and of two – mass mechanical link.

## Structural diagram of two – mass electromechanical system optimization

Article [4] presents the derived differential equations for the system and the function of transfer. As the result of that, there was compiled Matlab/Simulink model for optimized electromechanical systems and diagrams of transitional processes.

The structural diagram of optimization that is presented in Fig.1, is compiled in the following way. PID regulator is attached to the previously compiled Matlab/Simulink model as well as signal limitation units are introduced. The initial values of $K_p$, $K_i$ and $K_d$ coefficients of regulator amplification are written into these units. There are also proposed parameters of the ideal desirable curve of the transitional process. This characteristics is presented in Fig. 2.

The following initial data are placed into the Matlab/Simulink model to be able to receive the findings of modeling:

- Resistance of armature circuit of a motor is $R_a$ = 0,.28 Ω;
- Electromechanical time constant is $T_e$ = 0,01 s;
- Coefficient of internal feedback is cϕ=0,7 V·s;
- Coefficient of feedback velocity is $k_\omega$= 0,5 V·s/rad;
- Stiffness of elastic bond is $c_{12}$ = 3 N·m/rad;
- Inertia moment of the first mass is $J_1$= 0,1 kg·m$^2$;
- Inertia moment of the second mass is $J_2$= 0,2 kg·m$^2$.

The process of optimization is considered finalized when the characteristics of transitional process corresponding the prescribed parameters are determined. Then the real values of coefficients $K_p$, $K_i$ and $K_d$ of the regulator for the optimized system appear in the window of executed iterations. These values of the coefficients are applied afterwards while executing parametric synthesis of the real system.

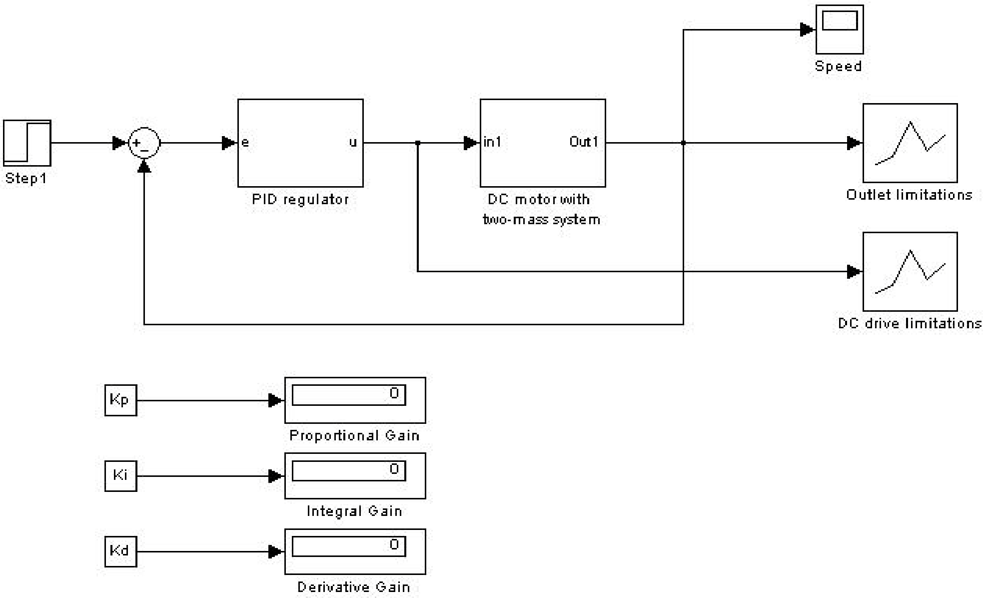
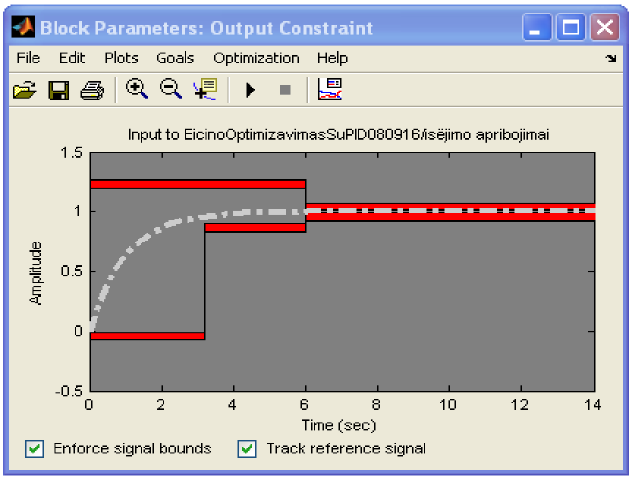


**Fig. 1.** Structural diagram of two – mass electromechanical system of optimization



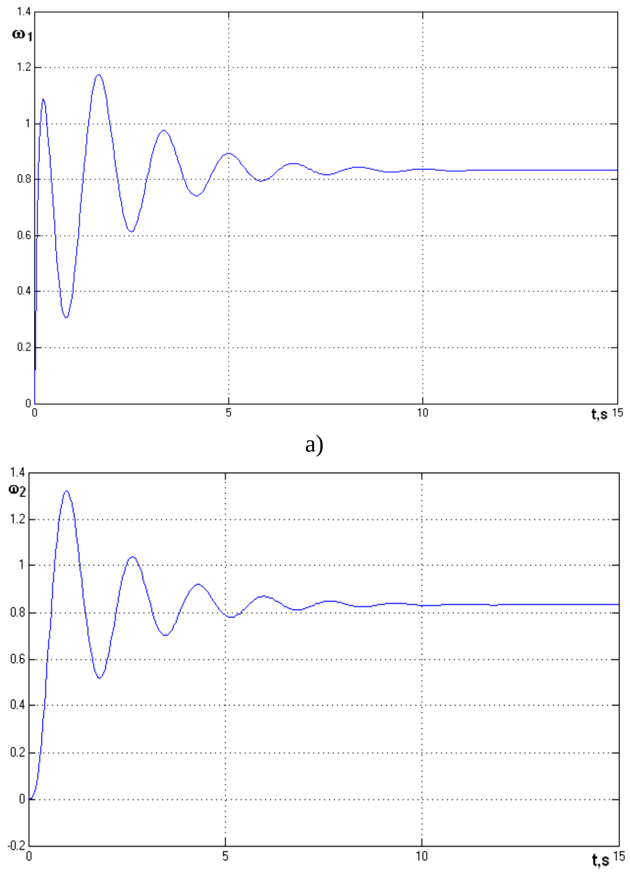
☑ Enforce signal bounds ☑ Track reference signal

**Fig. 2.** Ideal curve of transitional process expected to be received

## Results of optimization

Firstly, there were investigated transitional processes of non – optimized two – mass system by changing various parameters of the motor. Fig. 3 exhibits the transitional processes of velocity of non – optimized system when the motor operates without the load.

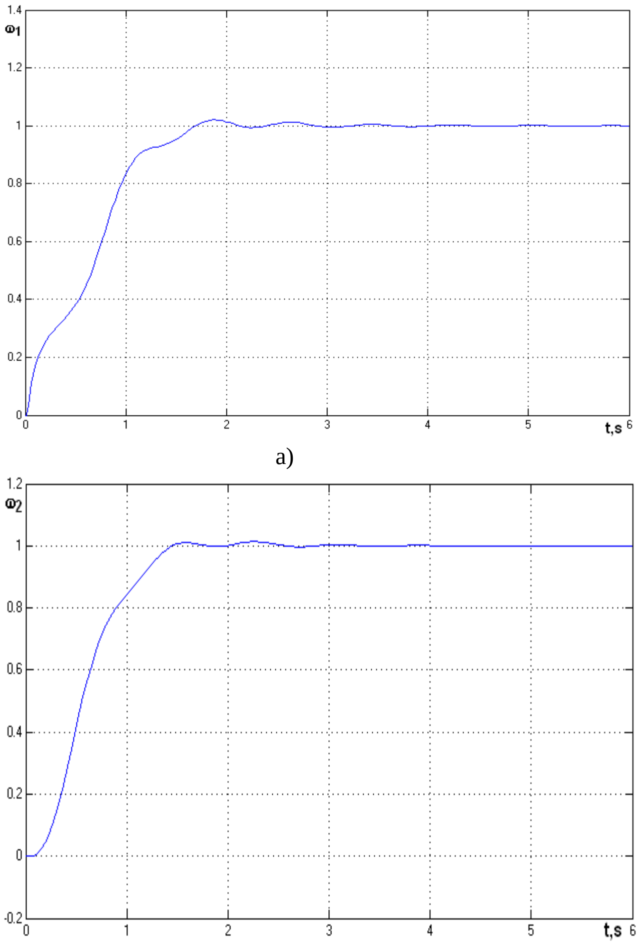
During the process of optimization there were changed the inertia moments $J_1$ and $J_2$ of both masses, stiffness $c_{12}$ of elastic bond of the masses, torque $T_{st1}$ of the load of the motor shaft, torque $T_{st2}$ of resistance.



b)

**Fig. 3.** Transitional processes of velocity of non – optimized system: a – of the first mass; b – the second mass

Fig. 4. presents transitional processes of velocity of the optimized system. The transitional process of the motor current is presented in Fig. 5 and in Fig. 6 – of the motor torque.



b)

**Fig. 4.** Velocity transitional processes of the optimized system: a – the first mass; b – the second mass

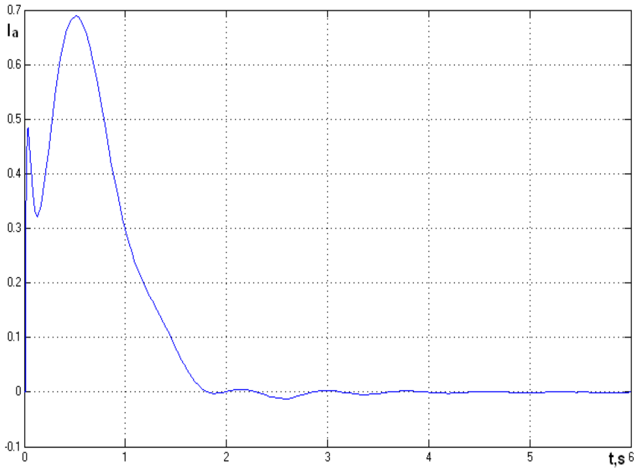


**Fig. 5.** Transitional process of the armature current when starting the unloaded motor

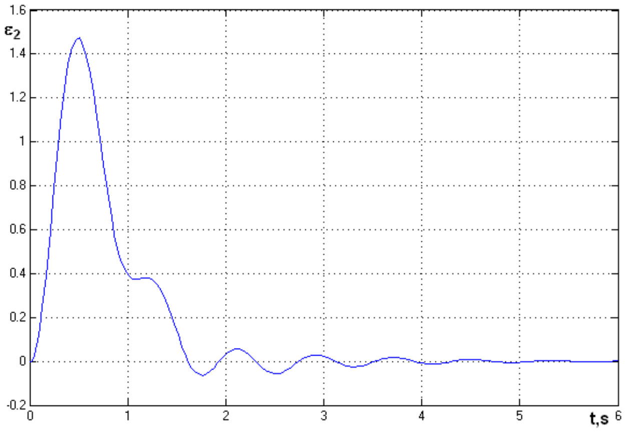


**Fig. 6.** Transitional process of the DC motor torque after optimization

## Conclusions

1. The results of the analysis indicate that the current and torque oscillations of the shaft of the motor are caused by the elasticity of the mechanical link via the internal feedback of velocity, which in their turn influences the velocity of the shaft of the motor. The transitional processes of velocity, acceleration, current and torque for the mentioned system are not optimal.
2. The presented structural diagram of the electromechanical system of optimization is comprised of Matlab/Simulink model for DC motors with two – mass mechanical link, PID regulator and units of signal limitations.
3. The derived results of optimization indicate that when the optimal values of the coefficients of amplification for PID regulator are found, then it is possible significantly reduce oscillations of drive velocity, acceleration, current and torque as well as derive optimal dynamic characteristics.
4. The optimal values of coefficients of amplification derived afterwards were applied in carrying out the parametric synthesis of the real system and in works of regulator adjustment.

## References

1. **Beineke S., Wertz H., Schutte F., Grotstallen H., Frohleke N.** Indentification of Nonlinear Two – Mass Systems for Self – Commisioning Speed Control of Electrical Drives // Proc. of 24th Annual Conference of the IEEE. – Germany, 1998. – Vol. 4. - No. 31. – P. 2251–2256.
2. **Susamu Y., Takuya K., Shigeyuki S., Hironubu H.** Modeling for Two – Mass System Using Two Objective Optimization Genetic Algorithm // Proc. of the Annual Conference of Systems, Control and Information Engineers. – Japan, 1998. – Vol. 42. – P. 241–242.
3. **Smilgevičius A., Rinkevičienė R., Savickienė Z.** Operation of Electric Motor with Elastic Load // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 6(70). – P. 15–18.
4. **Dambrauskas A., Karaliūnas B., Šulskis D.** Synthesis of the Changeable Structure Control Systems with Coordinates Limitations // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 4(84). – P. 31–34.
5. **Juraitis S., Rinkevičienė R., Kriaučiūnas J.** Modeling of Two – Mass System with Elasticity // Proc. of the 4th International Conference on Electrical and Control Technologies, ECT' 2009. – Kaunas: Technologija, 2009. – P. 227–230.

Received 2010 02 04

**B. Karaliūnas. Optimization of Two–Mass Electromechanical System // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 7(103). – P. 43–46.**

The issues of two – mass electromechanical optimization are analyzed. The probable methods of optimization and measures were discussed. The structural diagram of system optimization is presented, it is comprised of DC motor with two – mass mechanical link Matlab/Simulink model, PID regulator and units of signal limitations. The optimization process is finalized, when the characteristics of transitional process corresponding the proposed parameters are derived. The results obtained indicate that after determining the optimal values of the amplifying coefficients for the regulator during the optimization it is possible significantly reduce the oscillations of drive velocity, acceleration, current and torque and obtain optimal dynamic characteristics. Ill. 6, bibl. 5 (in English; abstracts in English, Russian and Lithuanian).

**B. Karaliūnas. Оптимизация двухмассовой электромеханической системы // Электроника и электротехника. – Каунас: Технология, 2010. – № 7(103). – С. 43–46.**

Рассматриваются вопросы оптимизации двухмассовой электромеханической системы, обсуждаются возможные способы и средства оптимизации. Представлена структурная схема оптимизации, которая состоит из Matlab/Simulink модели двигателя постоянного тока, PID регулятора и блоков ограничения сигналов. Полученные результаты показывают, что после нахождения оптимальных значений коэффициентов регулятора, можно значительно уменьшить пульсации скорости, ускорения, тока и момента двигателя и получить оптимальные динамические характеристики. Ил. 6, библ. 5 (на английском языке; рефераты на английском, русском и литовском яз.).

**B. Karaliūnas. Dvimasės elektromechaninės sistemos optimizavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 7(103). – P. 43–46.**

Nagrinėjami dvimasės elektromechaninės sistemos optimizavimo klausimai, aptariami galimi optimizavimo būdai ir priemonės. Pateikiama sistemos optimizavimo struktūrinė schema, kurią sudaro DC variklio su dvimase mechanine grandimi Matlab/Simulink modelis, PID reguliatorius ir signalų apribojimo blokai. Optimizacijos procesas baigiasi, kai randama nustatytus užduotus parametrus atitinkanti pereinamojo proceso charakteristika. Gauti rezultatai rodo, kad optimizavimo metu suradus reguliatoriaus stiprinimo koeficientų optimalias vertes, galima gerokai sumažinti pavaros greičio, pagreičio, srovės bei momento švytavimus ir gauti optimalias dinamines charakteristikas. Il. 6, bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.