

## Modeling and Estimation of Temperature Transient Process of Heat Carrier in Heating System

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### Introduction

Efficient work of heating devices is hardly imaginable without their automation. The temperature is the most important measured and controlled parameter in heating systems. The temperature of the heat carrier, which is supplied to the heating system, usually is regulated according to the outside temperature, automatically changing position of the valve, which mixes streams of supplied and returning heat carrier [1]. The transient process of the heat carrier temperature take place when the position of valve is changed. While designing and adjusting the controllers of heating devices one has to know the dynamic parameters of the supplied and returning heat carrier temperature measurement devices. These parameters are mostly determined by the transient process of the heat carrier temperature.

There are some propositions in the literature which suggest to consider the transient process of the temperature of heat carrier, which flows through the conduit, as the first order lag process with deadtime, in the form of delayed exponent [2]. While investigating the control of bio-fuel boiler [3], the non-exponential transient process of heat carrier temperature has been noticed. Non-exponential transient process of heat carrier temperature can also be noticed in some simulation results of these processes delivered in the literature [4]. There is ambiguous data about form of the transient processes of heat carrier temperature in the literature.

The purpose of this research paper is to estimate the form and main parameters of the temperature transient process of heat carrier, flowing through the conduit.

### Methods

The temperature transient process is provoked by instantaneous change of control valve position. The change of heat carrier temperature in the selected distance from the control valve is analyzed. Certain magnitude front of heat carrier temperature in certain time reaches the selected distance of the conduit. The size of this front is calculated

by solving analytically the differential equation describing the front temperature.

For the analysis of the later change of temperature the electro-analogical model of heat carrier flowing through the conduit is designed. The developed electro-analogical model is realized and investigated by electric circuits design and analysis software OrCAD which include analogical and digital simulator PSpice AD.

Relative heat indicators recalculated for conduit of 1 m length are used in modeling and calculating:

$$c_V = \rho_V c_{V1} \pi D_1^2 / 4, \quad (1)$$

here  $c_V$  – relative heat capacity of heat carrier (water), situated in conduit of 1 m length,  $\text{Jm}^{-1}\text{K}^{-1}$ ;  $\rho_V$  – density of heat carrier,  $\text{kgm}^{-3}$ , for 70 °C water  $\rho_V = 977 \text{ kgm}^{-3}$  [5];  $c_{V1}$  – specific heat capacity of heat carrier,  $\text{Jkg}^{-1}\text{K}^{-1}$ , for 70 °C water  $c_{V1} = 4190 \text{ Jkg}^{-1}\text{K}^{-1}$  [5];  $D_1$  – internal diameter of the conduit, m.

$$c_S = \rho_S c_{S1} \pi (D_2^2 - D_1^2) / 4, \quad (2)$$

here  $c_S$  – relative heat capacity of the 1 m conduit wall,  $\text{Jm}^{-1}\text{K}^{-1}$ ;  $\rho_S$  – density of wall material,  $\text{kgm}^{-3}$ , for cooper  $\rho_S = 8930 \text{ kgm}^{-3}$  [5];  $c_{S1}$  – specific heat capacity of wall material,  $\text{Jkg}^{-1}\text{K}^{-1}$ , for cooper  $c_{S1} = 383 \text{ Jkg}^{-1}\text{K}^{-1}$  [5];  $D_2$  – external diameter of conduit, m.

$$\alpha_{VS} = \alpha_{VS1} \pi D_1, \quad (3)$$

here  $\alpha_{VS}$  – relative heat transfer coefficient from heat carrier to conduit wall of 1 m conduit,  $\text{Wm}^{-1}\text{K}^{-1}$ ;  $\alpha_{VS1}$  – heat transfer coefficient from heat carrier to conduit wall,  $\text{Wm}^{-2}\text{K}^{-1}$ .

The heat transfer coefficient from heat carrier to conduit wall in case of turbulent flow is calculated [5]

$$\alpha_{VS1} = 0,023 \text{Re}^{0,8} \text{Pr}^{0,33} \lambda / D_1, \quad (4)$$

here  $\lambda$  – thermal conductivity of heat carrier,  $\text{Wm}^{-1}\text{K}^{-1}$ , for 70 °C water  $\lambda = 0,66 \text{ Wm}^{-1}\text{K}^{-1}$  [5]; Re – Reynolds criterion [5]:

$$\text{Re} = \rho_V D_1 v / \mu, \quad (5)$$

here  $v$  – velocity of heat carrier flow,  $\text{ms}^{-1}$ ;  $\mu$  – dynamic viscosity of heat carrier,  $\text{Nsm}^{-2}$ , for  $70^\circ\text{C}$  water  $\mu = 412 \cdot 10^{-6} \text{ Nsm}^{-2}$  [5];  $\text{Pr}$  – Prandtl number, for  $70^\circ\text{C}$  water  $\text{Pr} = 2,6$  [5].

$$\alpha_{SA} = \alpha_{SA1} \pi D_2, \quad (6)$$

here  $\alpha_{SA}$  – relative heat transfer coefficient from conduit wall to surrounding of 1 m conduit,  $\text{Wm}^{-1}\text{K}^{-1}$ ;  $\alpha_{SA1}$  – heat transfer coefficient from conduit wall to surrounding,  $\text{Wm}^{-2}\text{K}^{-1}$ .

Real temperatures of heat carrier  $\Theta_V$  and surrounding  $\Theta_A$  are evaluated when defining coefficient  $\alpha_{SA1}$ . In still air, when  $\Theta_V = 70^\circ\text{C}$  and  $\Theta_A = 20^\circ\text{C}$ , for non insulated conduit of heating system  $\alpha_{SA1} \approx 12 \text{ Wm}^{-2}\text{K}^{-1}$  [6].

Assumptions that are widely used in the automatic control theory and also facilitate thermal calculations are applied in modeling and calculating:

- 1) the change of the temperature of heat carrier (water) at the output of the control valve (input of the analysed link) is unitary step mode;
- 2) the initial conditions are null, i.e. the temperatures of the heat carrier, the conduit wall and surrounding are the same before the jump of temperature; this assumption is not applied when selecting concrete digital values of heat transfer coefficient;
- 3) the velocity of heat carrier flow in the conduit is constant;
- 4) the temperature of the conduit wall is the same throughout the wall thickness;
- 5) there is no conduction and convection in the axial (flow) direction of heat carrier;
- 6) the temperature of the surrounding of the conduit is constant;
- 7) thermal parameters of heat carrier and conduit (specific heat capacity, heat transfer coefficient) are stable during the analysed transient process.

### Analytical calculations

The selected distance  $l$  is reached by the front of heat carrier temperature in time  $t_0 = l/v$ . The time  $t_0$  is delay of variation of temperature of the heat carrier and the conduit wall. During the time  $t_0$  the front of heat carrier temperature spreads through the conduit, wall temperature of which is equal to zero (null initial conditions). The cooling of the front of heat carrier is described with equation

$$c_V \frac{d\Theta_V}{dt} = -\alpha_{VS} \Theta_V. \quad (7)$$

When (7) is solved under the initial conditions  $\Theta_V = 1$ , when  $t = 0$ , we obtain

$$\Theta_V = e^{-(\alpha_{VS}t)/c_V}. \quad (8)$$

The front of heat carrier temperature  $\Theta_V(l)$  which reaches the distance  $l$

$$\Theta_V(l) = e^{-(\alpha_{VS}l)/(vc_V)}. \quad (9)$$

The steady state temperature of the heat carrier is found when the equation of the power balance in the conduit of length  $dx$  is solved

$$-vc_V d\Theta_V = \frac{\alpha_{VS} \alpha_{SA}}{\alpha_{VS} + \alpha_{SA}} \Theta_V dx. \quad (10)$$

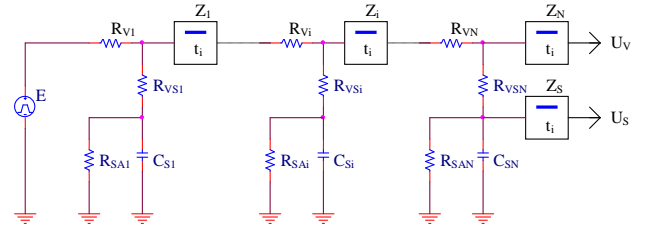
Steady state temperature  $\Theta_{VN}(l)$  of heat carrier in distance  $l$

$$\Theta_{VN}(l) = e^{-\frac{\alpha_{VS} \alpha_{SA} l}{(\alpha_{VS} + \alpha_{SA})vc_V}}. \quad (11)$$

For conduit without heat losses to surrounding  $\Theta_{VN}(l) = 1$ .

### Electro-analogical model

When creating the model of the conduit, through which heat carrier flows, length  $l$  is divided to  $N$  equal segments. Every segment is modeled as the electric circuit with concentrated parameters. Electric capacities are analogous to heat capacities and electric resistances are analogous to thermal resistances. Scheme of electro-analogical model of temperature conditions of conduit and heat carrier flowing in it is in Fig. 1.



**Fig. 1.** Electro analogical model of temperatures of conduit and heat carrier fluid flowing inside it:  $E$  – source of unitary jump voltage;  $Z_i$  – delay line modeling heat carrier flow from one segment to another;  $U_V$ ,  $U_S$  – electrical voltages modeling temperatures of heat carrier and conduit wall respectively; explication of other notations are presented in the text

The parameters of electro-analogical model are calculated according to expressions listed below

$$R_{Vi} = K_R / (vc_V), \quad (12)$$

here  $R_{Vi}$  – electrical resistance, which models the resistance of heat transfer in flowing heat carrier,  $\text{M}\Omega$ ;  $K_R$  – coefficient of dimensions conversion,  $K_R = 1 \text{ M}\Omega\text{WK}^{-1}$ .

$$R_{VSi} = (NK_R) / (\alpha_{VS}l), \quad (13)$$

here  $R_{VSi}$  – electrical resistance, which models resistance of heat transfer from the heat carrier to conduit wall,  $\text{M}\Omega$ ;  $N$  – number of segments to which the analysed conduit length  $l$  is divided.

$$R_{SAi} = (NK_R) / (\alpha_{SA}l), \quad (14)$$

here  $R_{SAi}$  – electrical resistance, which models resistance of heat transfer from the conduit wall to surrounding,  $\text{M}\Omega$ .

$$C_{Si} = c_S l K_C / N, \quad (15)$$

here  $C_{Si}$  – electrical capacity, which models the heat capacity of the conduit wall,  $\mu\text{F}$ ;  $K_C$  – coefficient of dimensions conversion,  $K_C = 1 \mu\text{FKJ}^{-1}$ .

$$t_i = l / (Nv), \quad (16)$$

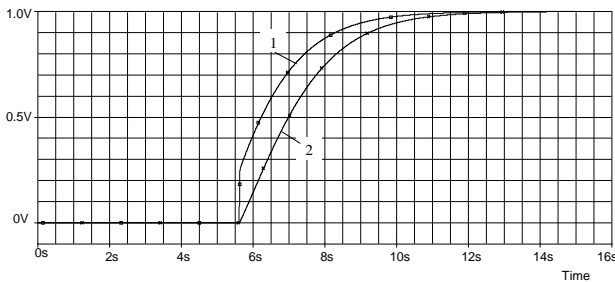
here  $t_i$  – time delay, which models flow of heat carrier from one segment to another, s.

In purpose to obtain electric model scheme of „useful“ parameters, scaled coefficients of dimensions conversion are applied reducing the capacities and increasing resistances  $10^6$  times, because large heat capacities and small heat transfer resistances are typical to the thermal processes, which proceed in heating systems. Such modification of dimensions into micro-farads and mega-ohms does not change static and dynamic form of voltages (voltage models the temperature) in the analysed electric scheme of the model.

### Modeling results and discussion

The delay node is modeled by applying Laplace transformation, which describes operation of this element. Copper conduit which has 20 mm inner diameter and 1 mm wall thickness and the heat carrier water flowing in this conduit are selected for the modeling. Relative heat capacity of such conduit wall  $c_S = 230 \text{ Jm}^{-1}\text{K}^{-1}$ , relative heat capacity of heat carrier  $c_V = 1285 \text{ Jm}^{-1}\text{K}^{-1}$ , relative heat transfer coefficient from the heat carrier to conduit wall  $\alpha_{VS} = 360v^{0.8} \text{ Wm}^{-1}\text{K}^{-1}$ . The length of modeled conduit is divided into 20 segments ( $N = 20$ ). Thermally insulated conduit is modeled ( $\alpha_{SA} = 0$  and  $R_{SAi} = \infty$ ).

Modeling results in the case when distance  $l = 5 \text{ m}$  and velocity of heat carrier  $v = 0.90 \text{ ms}^{-1}$ , directly obtained by using software PSpice AD, are presented in Fig. 2.

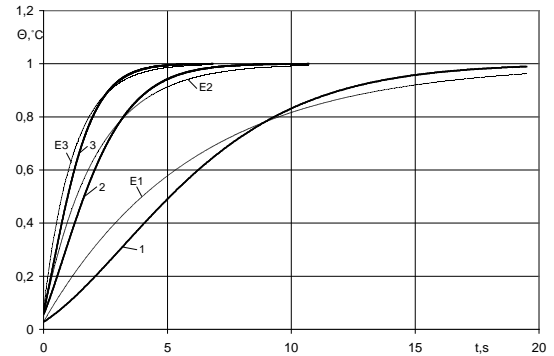


**Fig. 2.** Result window of OrCAD PSpice AD simulation: 1 – temperature of heat carrier; 2 – temperature of conduit wall

In order to facilitate the analysis of the results of modeling, data of transient processes was translated from PSpice AD to Excel and normalized by withdraw delay  $t_0 = l/v$ . The example of modeled and normalized response curves of heat carrier temperature is shown in Fig. 3. Modeled diagrams are approximated with exponents, evaluating initial temperature jumps.

From the delivered example one can see that the non-exponentiality of temperature changes from the initial jump to the steady state increases when the velocity of the flow of heat carrier decreases. When more simulations are

performed it is defined that non-exponentiality of the temperature change increases if the conduit is longer, the relative heat transfer coefficient from the heat carrier to conduit wall is greater, the velocity of heat carrier flow is less and the relative heat capacity of heat carrier is less.



**Fig. 3.** Normalized response curves of heat carrier temperature, when distance  $l = 10 \text{ m}$  and velocity of heat carrier  $v = 0.3 \text{ ms}^{-1}$  (1 graph),  $v = 0.9 \text{ ms}^{-1}$  (2) and  $v = 1.5 \text{ ms}^{-1}$  (3); E1, E2, E3 – exponents approximating graphs respectively

The developed model can be successfully applied for the simulation of the transient processes of the temperature of heat carrier and conduit wall in devices, which demands evaluation of heat transfer into other medium or surrounding, e.g. heat exchangers.

One can see from the modeling results that the variation of heat carrier temperature, at precise evaluation, is non-exponential, but is close to it. While solving many practical tasks of automatic control of heating devices, the change of heat carrier temperature can be considered as a first order lag (exponential) process. For definition of the time constant of this process, the empirical equation supported by simulation results can be suggested

$$T = \frac{lc_S}{vc_V(1 - \Theta_V(l))}. \quad (17)$$

The time constant of heat carrier temperature change, calculated according to (17) equation, differs from modeling results approximating exponent time constant in various practical cases by several percents and does not exceed 10%, when front temperature  $\Theta_V(l) > 0.03$ . The transient process of heat carrier temperature should be modeled if more precise evaluation is needed.

### Conclusions

The transient process of heating system heat carrier temperature in the selected distance from the control valve can be considered as tripartite process: the delay, the initial jump of temperature and the temperature change from the initial jump to the steady state. Initial jump and steady temperature of heat carrier are calculated analytically (there are respective equations delivered in the article). The change of heat carrier temperature from the initial jump to steady, at precise evaluation, is not the first order lag (exponential) process.

Electro-analogical model, described in this article,

can be used for modeling the transient processes of heat carrier and conduit wall temperatures. Model is realized using electric scheme design and analysis software OrCAD which contains simulator PSpice AD.

While solving many practical tasks of automatic control of heating devices the change of heat carrier temperature from the initial front to steady can be considered as a first order lag (exponential) process. The time constant of this process can be calculated according to the empirical formula, which is composed from the modeling results and delivered in this article.

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**L. Brazdeikis. Modeling and Estimation of Temperature Transient Process of Heat Carrier in Heating System // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 7(103). – P. 21–24.**

Transient temperature process of the heat carrier flowing in a conduit is investigated at a certain selected distance from the heat carrier temperature unit jump point. An electro-analogical model of heat carrier temperature is designed. The model is realized and analysed by electrical circuits simulator PSpice included in the software OrCAD. It was defined that the transient change of heat carrier temperature is not the first order lag process when assessing it precisely. Expressions achieved by theoretic calculations are presented to calculate heat carrier front temperature and steady fluid temperature. Expression summarizing modelling results according to which it is possible to estimate the time constant of transient process of heat carrier fluid temperature is presented too. Ill. 3, bibl. 6 (in English; abstracts in English, Russian and Lithuanian).

**Л. Браздейкис. Моделирование и оценка переходных процессов температуры теплоносителя в системах отопления // Электроника и электротехника. – Каунас: Технология, 2010. – № 7(103). – С. 21–24.**

Исследуется переходной процесс температуры теплоносителя в определенном расстоянии от точки входного скачка температуры. Составлена электроаналоговая модель температуры теплоносителя. Модель реализована при помощи симулятора PSpice, входящего в состав программного пакета OrCAD. Установлено, что переходное изменение температуры теплоносителя при точной его оценке не является аperiodическим процессом. Представлены аналитические выражения для расчета начального скачка и установившейся температуры теплоносителя. Для оценки постоянной времени изменения температуры теплоносителя предложено эмпирическое выражение, основанное на результатах моделирования. Ил. 3, библи. 6 (на английском языке; рефераты на английском, русском и литовском яз.).

**L. Brazdeikis. Šildymo sistemos šilumnešio temperatūros pereinamojo proceso modeliavimas ir įvertinimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 7(103). – P. 21–24.**

Tiriamas šilumnešio, tekančio vamzdžiu, temperatūros pereinamasis procesas tam tikru pasirenkamu atstumu nuo šilumnešio temperatūros vienetinio šuolio taško. Sudarytas šilumnešio temperatūros elektrinis analoginis modelis. Modelis sukuriamas ir tiriamas elektroninių schemų imitatoriumi PSpice, įeinančiu į programų paketą OrCAD. Nustatyta, kad šilumnešio temperatūros pereinamasis kitimas, tiksliai jį vertinant, nėra pirmojo laipsnio inercinis kitimas. Pateikiamos teoriniai skaičiavimai gautos išraiškos šilumnešio fronto temperatūrai ir nusistovėjusiai šilumnešio temperatūrai apskaičiuoti bei modeliavimo rezultatus apibendrinanti išraiška, pagal kurią galima įvertinti šilumnešio temperatūros pereinamojo kitimo laiko pastoviąją. Il. 3, bibl. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).