ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2010. No. 8(104)
ELEKTRONIKA IR ELEKTROTECHNIKA

T 180 TELECOMMUNICATIONS ENGINEERING
TELEKOMUNIKACIJŲ INŽINERIJA

Application of Least Square Method with Variable Parameters for GPS Accuracy Improvement

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Introduction

Adaptive information processing methods are widely used in navigation techniques. Most often they are use for tow or more sensor information processing. GPS information is most often complex treated with inertial sensor information using adaptive filtering techniques [1, 2]. If navigation system has only one source of information adaptive filtering can be used [3]. Widely known are following algorithms: Least Mean Square (LMS) algorithm, Recursive Least Squares (RLS) algorithm and Kalman Filtering (KF) algorithm [3]. We research Least Square Method (LSM) for one source information filtering with sliding window. Length of window changes in filtering process and depends on the evaluation results. LSM algorithms for information processing in window are used from [4]. This work describes results of filter modeling and optimization and use of optimal sliding window filter for GPS information processing.

Modeling constant length sliding window filtering for one coordinate of mobile object

Trajectory of mobile object movement is supposed linear with radical change in one point. True coordinate is modeling in i point or in time t_i using expression (1). All modeling values are relative, velocity is a relative coordinate change in the time step between two points

$$xt_i = x_0 + V_x\, i, \qquad (1)$$

where x_0 is coordinate in starting point, but V_x is velocity of x coordinate change.

Maximal object movement dynamic is if velocity changes from positive to negative or contrary. In Fig. 1 is shown true coordinate time function if linear movement and radical change of 180^0 are used for mobile object trajectory modeling. True trajectory measurement errors e_i are modeling as normal process with mean value M_e equal zero and root square value - σ_e. In Fig. 2 are shown error modeling results in case if velocity V_x=0 and σ_e=2. Measured trajectory $xm_i = xt_i + e_i$.

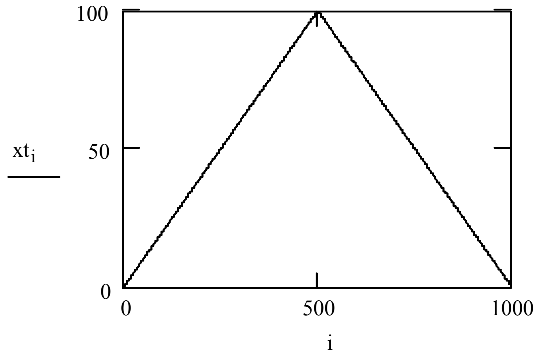

Fig. 1. True coordinate changing model if x_0=0,V_x=0.2 and changes to V_x= -0.2

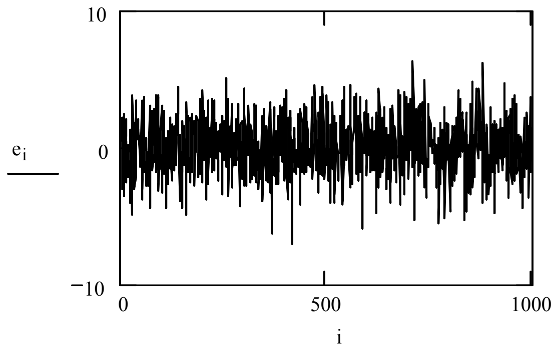

Fig. 2. Measurement error modeling results M_e=0, σ_e=2

Measured coordinate xm_i is estimated using LSM method with sliding window. First results of estimated coordinate xe were obtain with constant length window – w. For information filtering in window following expressions (2) are used [4]. In every times are calculate two estimated values using information inside window: xe_0 and V_{xe}. Coordinate x estimation modeling results are shown in Fig. 3, for coordinate changing model descript in

Fig. 1, error model descript in Fig. 2 and window's length w=200. Estimation error (ee) is shown in Fig. 4 and:

$$xe_0 = \frac{\left[\sum_{i=1}^{w}(i)^2\right]\cdot\left[\sum_{i=1}^{w}(x_i)\right]-\left[\sum_{i=1}^{w}(i)\right]\cdot\left[\sum_{i=1}^{w}i\cdot(x_i)\right]}{\Delta}, \qquad (2)$$

$$V_{xe} = \frac{w\cdot\left[\sum_{i=1}^{w}i\cdot(x_i)\right]-\left[\sum_{i=1}^{w}(x_i)\right]\cdot\left[\sum_{i=1}^{w}(x_i)\right]}{\Delta}, \qquad (3)$$

where Δ is calculate using expression:

$$\Delta = w\cdot\left[\sum_{i=1}^{w}(i)^2\right]-\left[\sum_{i=1}^{w}(i)\right]^2. \qquad (4)$$

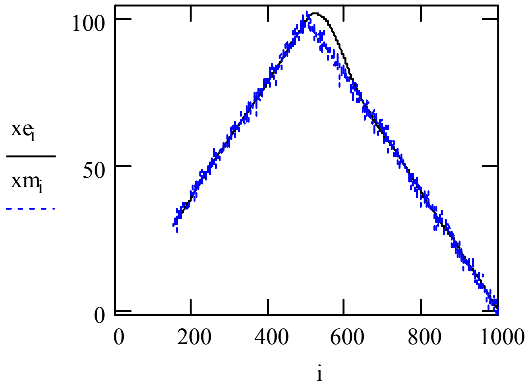

Fig. 3. Estimation results with constant length sliding window w=200 and velocity change in point i=500 from +0.2 to -0.2

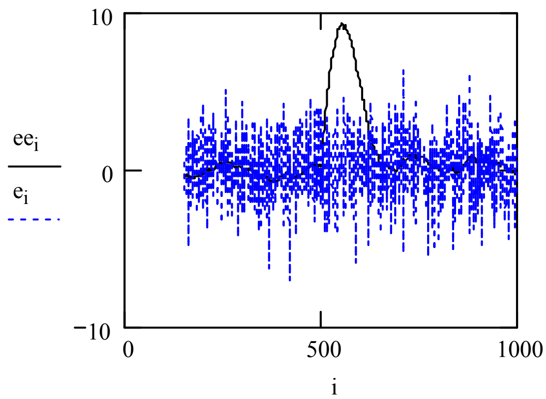

Fig. 4. Estimation error ee_i and measuring error e_i modeling results with constant length sliding window w=200 and velocity change in point i=500 from +0.2 to -0.2

How it is seen from Fig. 3 and Fig. 4 estimation results is good if coordinate change is linear. When velocity changes estimation error has very high value. Error's maximal value is proportional windows length and velocity changing. If velocity is constant filtering efficiency is higher using longer window. In Fig. 5 are shown modeling results of least square estimation error σ_{ee} depending from sliding window length w for two values of velocity and its change from positive to negative value as show in Fig. 1. Results show, that there is same optimal value of windows length w_o for every velocity which gives minimal estimation error. If velocity change is smaller the optimal length of window is longer and minimal σ_{ee} is smaller. If w=0, the estimation error is equal modeling error ($\sigma_{ee}= \sigma_e$).

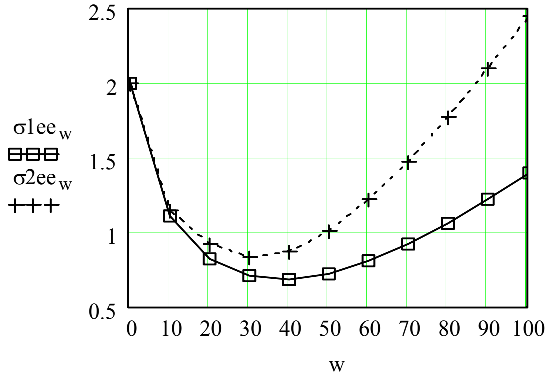

Fig. 5. Least square estimation error σ_{ee} depending from sliding window length w for two values of velocity ($\sigma 1ee_w$ for V_x =0.2, $\sigma 2ee_w$ for V_x =0.4)

If system have sensors measuring velocity, the windows length can be change proportional velocity. This is one of possible methods using adaptive trajectory date filtering for GPS accuracy improvement. For example if GPS system is used in automobile odometer can be used for velocity measuring. What do if GPS system is used autonomous? In this case sliding window's length can be change using detected error value between priory calculated coordinate xp_{i+1}, based on sliding window filtering, and measured coordinate xm_{i+1}.

Modeling variable length sliding window filtering for one coordinate of mobile object

We research different adaptive filtering algorithms with sliding window. Relatively good results were reaching if window is change using expressions (3):

$$\begin{cases} w = w + 1, \text{ if } |xm_{i+1} - xp_{i+1}| \le n1\cdot \sigma_e, \text{ but no more } w_{max}, \\ w = w - 1, \text{ if } n1\cdot \sigma_e < |xm_{i+1} - xp_{i+1}| \le n2\cdot \sigma_e, \\ w = w/2, \text{ if } n2\cdot \sigma_e < |xm_{i+1} - xp_{i+1}| \le n3\cdot \sigma_e, \\ w = w_{min}, \text{ if } n3\cdot \sigma_e < |xm_{i+1} - xp_{i+1}|. \end{cases} \qquad (5)$$

Algorithm choice is based on error estimation with constant length sliding window. First must be choice values of minimal and maximal windows length (w_{min}, w_{max}). Minimal length can be chose from Fig.5, when windows length has week influence on dynamic error and this is when value of w_{min} is about 10. Maximal value depends from date processing system parameters and can be about 400 – 1000 time steps. We use value w_{max}=500. Choice of numbers n1, n2 and n3. Windows length must increase when estimation is with high probability (0.95-0.997) and this is when n1 = (2…3)σ_e for normal measuring error distribution. Length decreasing has three steps: little decreasing – reduce per one unit, reduce per 50% and reduce till minimal length. In the Fig.6 and Fig.7

results of modeling estimation with variable parameter sliding window are shown. True trajectory model is so as show in Fig. 1. Modeling estimation parameters are following w_{min}=10, w_{max}=500, n1=2σ_e, n2=3σ_e and n3=4σ_e.

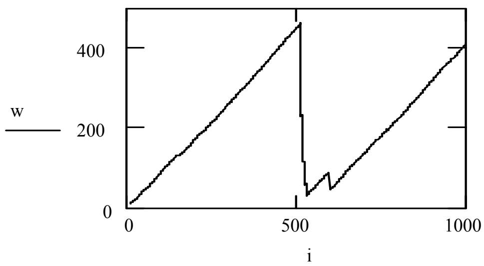

Fig. 6. Sliding window length change w_{min}=10, w_{max}=500, n1=2σ_e, n2=3σ_e and n3=4σ_e

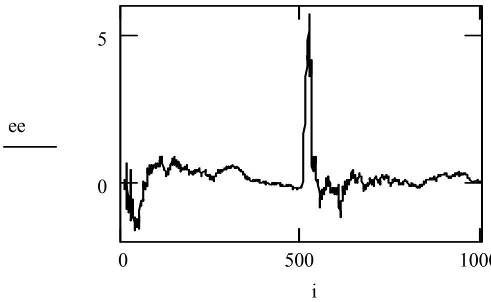

Fig. 7. Estimation error for variable parameter trajectory determination w_{min}=10, w_{max}=500, n1=2σ_e, n2=3σ_e and n3=4σ_e

Fig. 6. shows that in the moment of velocity change length of sliding window decrease: first for 50% and then till w_{min}. When movement is stabilized windows length increments and reaches maximal value. Fig. 7 shows estimation errors depending of time or i. In the moment of velocity change there is error, but only short time compared with error, which take place in case of parameter determination using constant length window (Fig. 4). Maximal value of error also decreases, compared Fig. 4 and Fig. 7.

In Fig. 8 are shown modeling results of least square estimation error σ_{ee} depending from velocity if it change from positive to negative value as show in Fig. 1. For velocities higher relative value 0.1 estimations error is constant, for small velocity values error is decreasing. If compare Fig. 5 and Fig. 8 can see, that estimation error is smaller in the case of variable window length estimation and do not increase for high dynamic objects how it take place using constant length window.

Influence of parameter n1 is very small. Modeling results show that changing n1 in diapason 1.5 - 2.5, σ_{ee} changes from 0.68 till 0.69 for modeling parameters σ_e=2 and V_x=0.2. Modeling results show that value of windows change in point n2 is optimal in diapason 0.3 – 0.6. We chose 0.5 or 50% change. Value of n3 also has small influence on estimation error if window length is change till w_{min} in this point. For example changing n3 from 3.5 till 5 if V_x=0.2 do not affect value of σ_{ee}=constant.

Modeling results using variable sliding window estimation, when for window change expressions (3) are used, estimation error decrease about 10 time if object movement parameters are constant and decrease about 3 times relative measuring error if parameters change very rapidly.

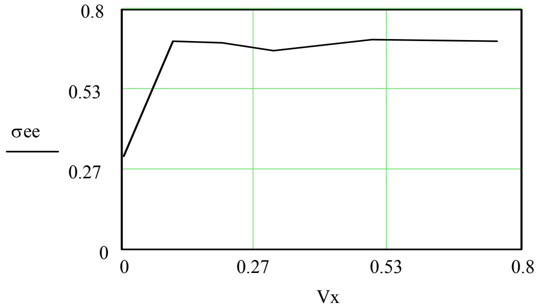

Fig. 8. Least square estimation error σ_{ee} with variable sliding window depending from velocity

GPS data filtering with variable length sliding window

Next step is to use the same method for some fragment of GPS data. There are 4 turns in this data fragment. Fig. 9 shows measured data (xm) and estimated data (xe) with fixed sliding window length w=20.

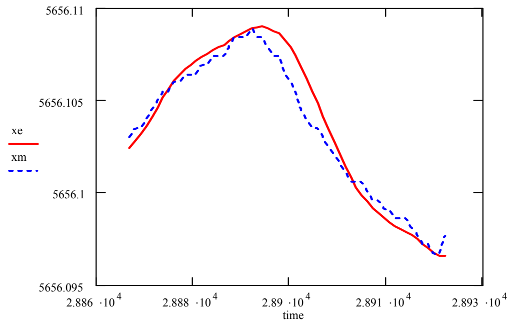

Fig. 9. Estimation results with constant length sliding window w=20

Filtering gives quite significant error in turns where we need to drop sliding window length quickly. Estimation results using variable sliding window, when for window change, expressions (3) are used, are shown in Fig.10. Sliding window change: w_{min}=5 and w_{max}=50. As shown in the Fig. 10 result of date filtering is much better.

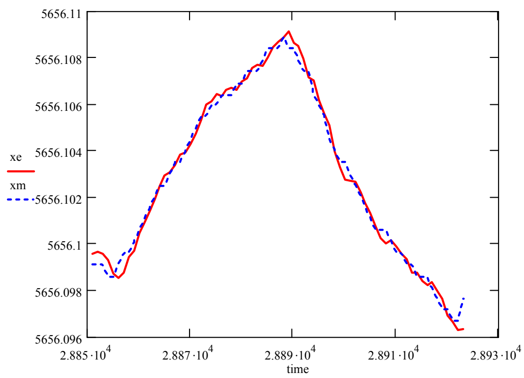

Fig. 10. Estimation results with window length sliding w_{min}=5, w_{max}=50, n1=2σ_e, n2=3σ_e and n3=4σ_e

Conclusions

This work describes results of filter modeling and optimization and use of optimal sliding window filter for GPS information processing**.** Results of modeling mobile object trajectory detection, using constant length sliding window filtering, show that for every velocity is optimal length of window. Modeling results using variable sliding window estimation show that estimation error decrease about 10 time if object movement parameters are constant and decrease about 3 times relative measuring error if parameters change very rapidly. Expressions for windows parameter change are developed. Variable parameter estimation is used for GPS date filtering and shows that modeling results is equal real date filtering.

Acknowledgement

This work has been supported by the European Social Fund within the project „Support for the implementation of doctoral studies at Riga Technical University".

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Received 2010 01 31

V. Belinska, A. Kluga, J. Kluga. Application of Least Square Method with Variable Parameters for GPS Accuracy Improvement // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 8(104). – P. 109–112.

In this work research results of Least Square Method for information filtering with sliding window are presented. Length of window changes in filtering process and depends on the evaluation results between measuring and expected coordinate. Modeling results of filtering with variable parameter window are comparing with constant length sliding window filtering. Algorithms for sliding window change are developed and research. Modeling results using variable sliding window coordinate estimation show that estimation error decrease about 10 time if object movement parameters are constant and decrease about 3 times relative measuring error if parameters change very rapidly. Results of sliding filter modeling and filter parameter optimization and use of optimal sliding window filter for GPS receiver information processing are described. Ill. 10, bibl. 4 (in English; abstracts in English, Russian and Lithuanian).

В. Белинска, А. Клуга, Я. Клуга. Применение среднеквадратического метода с меняющимися параметрами для увеличения точности GPS // Электроника и электротехника. – Каунас: Технология, 2010. – № 8(104). – С. 109–112.

В данной работе приводятся результаты исследования среднеквадратического метода со скользящим окном для фильтрации информации. Длина окна изменяется в процессе фильтрации и зависит от результатов сравнения измеренной и прогнозируемой координаты. Результаты моделирования процесса фильтрации со скользящим окном с изменяемыми параметрами сравниваются с результатами фильтрации со скользящим окном постоянной длины. Разработаны и исследованы алгоритмы изменения длины скользящего окна. Результаты моделирования оценки координат методом скользящего окна показали, что ошибка оценивания уменьшается примерно 10 раз, если параметры движения объекта постоянны, и примерно 3 раза относительно погрешности измеряемого параметра при быстрых эволюциях объекта. Приводятся результаты моделирования и оптимизации фильтра со скользящим окном и результаты применения оптимального фильтра для обработки информации GPS приемника. Ил. 10, библ. 4 (на английском языке; рефераты на английском, русском и литовском яз.).

V. Belinska, A. Kluga, J. Kluga. GPS imtuvo tikslumo didinimas taikant kintamų parametrų vidutinį kvadratinį metodą // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 8(104). – P. 109–112.

Aprašomi slankiosios įrangos informacijos filtravimo rezultatai. Lango ilgio įvertinimo kriterijai gauti modeliuojant filtracijos procesą. Įrodyta, kad naudojant slenkiųjų langų filtraciją, paklaida sumažėja 10 kartų, kai objekto judesys yra pastovus, ir 3 kartus, kai objekto judesys greitai keičiasi. Rezultatai panaudoti kuriant naujos kartos imtuvus ir modeliuojant bei optimizuojant slankiųjų langų filtrų parametrus. Il. 10, bibl. 4 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).