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Design of Quasi-elliptic Filters with Specified Q-factor Parameter

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Introduction

At the present day there are many types of filters for numerous filtering and signal processing tasks. This article reviews filters approximating the amplitude-frequency response (AFR).

Filters with the highest gain slope can be synthesized on the basis of elliptic approximation. The elliptic functions can be used to create the highest selective filters [1]. However, the high gain slope of elliptic filter is achieved at the cost of the higher Q-factor of filter sections, which requires complex calculations and accurate tuning. The scheme of such filter is not only larger and more complicated, but also more sensitive to the values of its parts and environment conditions, such as a temperature.

By reducing Q-factor of filter it is possible to simplify its circuitry. It is obvious, though, that selectivity of the filter with limited Q-factor shall be lower, compared to selectivity of the elliptic filter of same order. However, by using zeros in the transfer function, it is possible to create a filter with higher selectivity, compared to the selectivity of Chebyshev filter of the same order and higher Q-factor.

Estimation of the Q-factor for pole pairs

When synthesizing filters, the value of Q-factor has decisive importance for their selectivity [2]. Taking these values into account is the most relevant when realizing the filters with losses in reactive elements (inductivities and capacities) and during the synthesis of the active RC-filters. High Q-factor value in passive analog schemes requires either the higher quality of the reactive elements (smaller losses) or the increased filter order. In case of the active RC-filters the higher Q-factor of filter section with order 2 can require more active (such as transistors or operational amplifiers) and passive elements. That is why it's so important to develop the algorithms for the optimal filters design with the specified or minimal poles Q-factor.

Since a transfer function (TF) of the elliptic filer with order 2 has complex conjugate poles and purely imaginary zeros, it can be written the following way

$$H_2(p) = \frac{p^2 + b_0}{p^2 + a_1 p + a_0}, \tag{1}$$

where b_i and a_i are the quotients of a numerator and denominator polynomials of complex variable p.

For quadratic reduced polynomial of type

$$p^2 + a_1 p + a_0, \tag{2}$$

where a_1 and a_0 are polynomial quotient. In this case Q-factor can be calculated the following way

$$Q = \frac{\sqrt{a_0}}{a_1}. \tag{3}$$

When considering Q-factor of the elliptic filter TF numerator, it can be shown that the Q-factor tends to infinity, since the zeros are purely imaginary (quotient of p is zero). Therefore, Q-factor of zeros is omitted in the computation equations during design of filter schematic. However, it's obvious, that the scheme of the 2-nd order filter section with fractionally rational TF (1) shall be more complicated than the scheme of the 2-nd order filter section with polynomial TF.

Since the TF denominator of the stable circuit is the Hurwitz polynomial, the roots of this polynomial can be either real negative, or they can form the complex conjugate pairs with negative real part. In both cases, the Q-factor of the pole pair can be calculated by following formula [3]

$$Q = \frac{\sqrt{\alpha^2 + \beta^2}}{2\alpha} = \sqrt{\frac{1}{4} + \frac{\beta^2}{4\alpha^2}}, \tag{4}$$

where α and β are real and imaginary parts of the pole pair, respectively.

The expression (4) makes it obvious that, when the Q-factor of the poles increases, they tend to the imaginary axis of the complex plane, which increases sensitivity of the entire filter. There is another obvious fact – the purely real pole with the multiplicity of 2: $p_i = p_i^* = -\alpha$, has the minimal Q-factor. Synthesis of such filters is described by authors in their article [4]. The same article contains table of TF quotients for some orders.

Filter design problem with specified Q-factor

The filter synthesis problem with specified Q-factor parameter was solved by applying optimization procedures.

During the filter synthesis, designer usually specifies bounds for pass band and for stop band. The AFR is not allowed to exceed these bounds. These requirements can be shown graphically as two passages (Fig. 1).

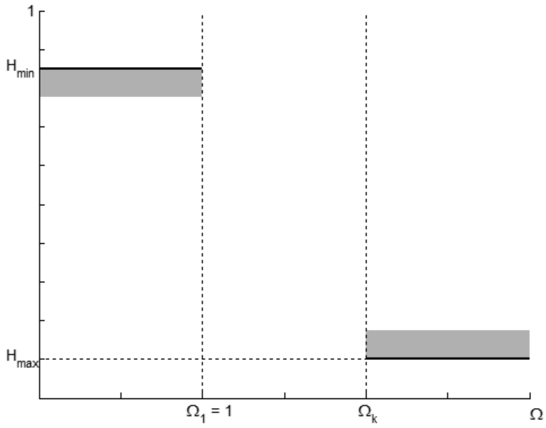

Fig. 1. AFR passages for pass band and stop band

For solution of similar problems, MATLAB package includes different Optimization toolbox procedures, for example, *fminsearch* and *fmincon* [5]. In order to switch from the problem with multiple criteria to the problem with single criteria, we shall use linear convolution method [6]

$$F(\mathbf{x}) = \sum_{i=1}^{L} \alpha_i f_i(\mathbf{x}), \qquad (5)$$

where $f_i(\mathbf{x})$ are penalty functions for controlled parameters and $\alpha_i > 0$ – weight quotients, which can be considered to be the relative indexes of the penalty functions importance; L is the number of the penalty functions. In such case, the multiple criteria optimization problem shall be following

$$F(\mathbf{x}) = \sum_{i=1}^{L} \alpha_i f_i(\mathbf{x}) \rightarrow \min; \quad \sum_{i=1}^{L} \alpha_i = 1. \qquad (6)$$

To solve this problem, we need to compose the penalty function for pass band of the filter. This function can be represented by a sum of the all AFR samples, which exceed the given bound for band pass flatness (Fig. 2). Such sum can be written by the following expression

$$f_1(\mathbf{x}) = \sum_{k=1}^{K} \left[H_{\min} - H(\mathbf{x}, \Omega_k) \right], \quad \text{when } H(\mathbf{x}, \Omega_k) < H_{\min}, \qquad (7)$$

where $\Omega_1 = 0$, $\Omega_K = 1$ (normalized cut-off frequency), $H(\mathbf{x}, \Omega_k)$ is the absolute value of the transfer function for current argument $\mathbf{x}$ values at the frequency Ω_k, $H_{\min}$ is minimal allowed value of the AFR pass band.

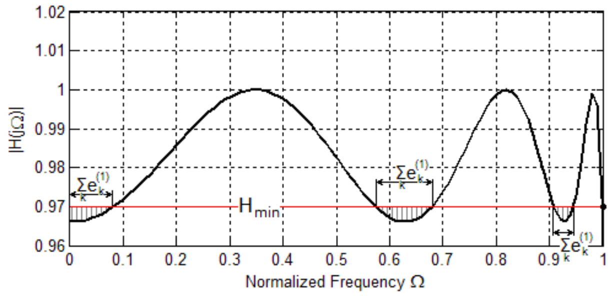

Fig. 2. Forming AFR error in pass band

In similar way, we must also compose the penalty function for stop band. This function can also be represented by a sum of the all AFR samples exceeding given minimal attenuation at the control frequency (Fig. 3).

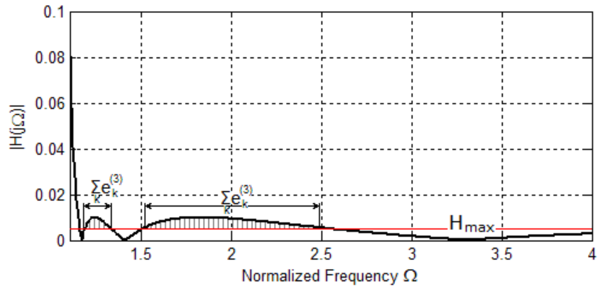

Fig. 3. Forming AFR error in stop band

Such sum can be written by the following expression

$$f_2(\mathbf{x}) = \sum_{m=1}^{M} \left[H_{\max} + H(\mathbf{x}, \Omega_m) \right], \quad \text{when } H(\mathbf{x}, \Omega_m) > H_{\max}, \qquad (8)$$

where Ω_1 is the normalized limiting frequency of the stop band (control frequency); Ω_M is AFR calculation limiting frequency; $H(\mathbf{x},\Omega_m)$ is the absolute value of the transfer function for current argument $\mathbf{x}$ values at the frequency Ω_m; $H_{\max}$ is maximal allowed value of the AFR stop band.

For this specific problem we had to add another penalty function to control the Q-factor of poles. This function can be represented by a sum, which consists of the Q-factor values exceeding given value

$$f_3(\mathbf{x}) = \sum_{i=1}^{n} Q_i, \; Q_i > Q_{\max}, \qquad (9)$$

where Q_i is the Q-factor of the i-th complex conjugate pole pair; n is the number of these pole pairs; $Q_{\max}$ is specified Q-factor limit.

Calculation algorithm for design of the filters with the specified Q-factor parameter

Authors of this article created a function for MATLAB software package. This function calculates quasi-elliptic prototype, which complies the specified AFR and Q-factor requirements

The algorithm (Fig. 4) of this problem solution is given further as a sequence of following actions.

1. Specify prototype requirements:
   - N – the order of filter;
   - N_Z – the number of zero pairs;
   - Q_{max} – maximal allowed Q-factor of pole pairs;
   - a_{max} – AFR band pass flatness, dB;
   - a_{min} – stop band minimal attenuation, dB;
   - Ω_K – control frequency.
2. Calculate analog elliptic prototype. Calculate Chebyshev prototype if number of zero pairs $N_Z = 0$. Create an argument **x** for target function $F(\mathbf{x})$ and minimize it according to (6).
3. Q-factor maximization: if maximal Q-factor doesn't reach specified limit, increase a_{min} and repeat step 2.
4. Minimization of the transition band width: if minimal attenuation at the control frequency exceeds given value, increase control frequency and repeat steps 2 and 3.

At the end of the optimization procedure the program outputs zero and pole values for a prototype, which complies specified requirements.

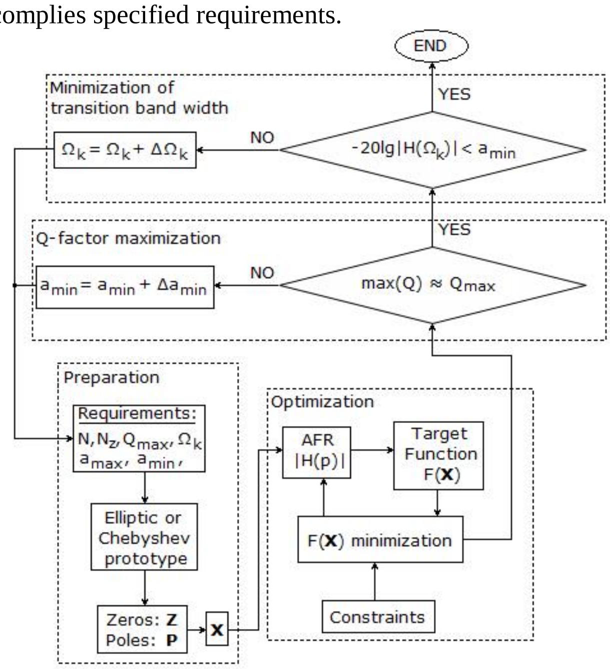

Fig. 4. Blog-diagram of the optimization algorithm

Authors of this article created a program for MATLAB software pack, which calculates and sequentially improves (if this is possible) the quasi-elliptic filter, which complies specified requirements.

Quasi-elliptic prototype example with specified Q-factor of poles

Let's consider a prototype with following requirements: the order is $N = 7$, number of zero pairs is $N_z = 3$. Maximal pole pair Q-factor is limited to a value of $Q_{max} = 5$. Let's also specify band pass flatness $a_{max} = 0.3$ dB and minimal attenuation $a_{min} = 65$ dB starting by control frequency of $\Omega_K = 1.6$.

By using MATLAB program, which realizes described before algorithm (Fig. 4), we calculated values of zeros and poles. These values are available in Table 1.

Table 1. Poles and zeros of the synthesized prototype TF

Poles	Zeros
-0.1165823 ± 1.0866546i -0.3264229 ± 0.6176561i -0.1825970 ± 1.0227336i -0.4103039	±3.0950120i ±1.8793181i ±1.5682279i

With these zeros and poles it is possible to calculate transfer function of the prototype in following way:

$$H(p) = \prod_{i=1}^{N_z}\left(p^2 + |z_i|^2\right)\prod_{i=1}^{N}\frac{1}{p - p_i}, \qquad (10)$$

where z_i and p_i are zeros and poles of the transfer function, respectively, N_Z is the number of zero pairs, N is the order of prototype.

For comparison, the plot contains also curve of the Chebyshev prototype of the 7-th order with AFR pass band flattening $a_{max} = 0.3$ dB.

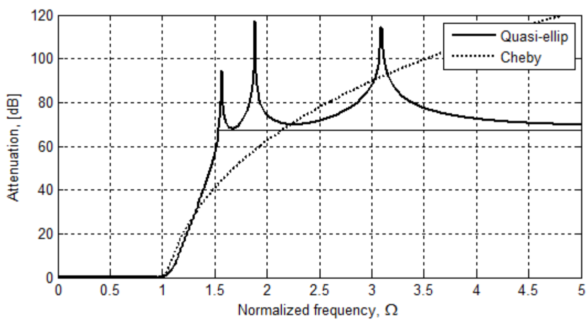

Fig. 5. Comparative curves of the attenuation for quasi-elliptic and Chebyshev prototypes

Even though maximal Q-factor of Chebyshev prototype poles exceeds the specified value (max(Q) = 7.68), the synthesized quasi-elliptic prototype provides attenuation of 65 dB at the lower control frequency. Note that Chebyshev prototype of given order can't satisfy given requirements at all.

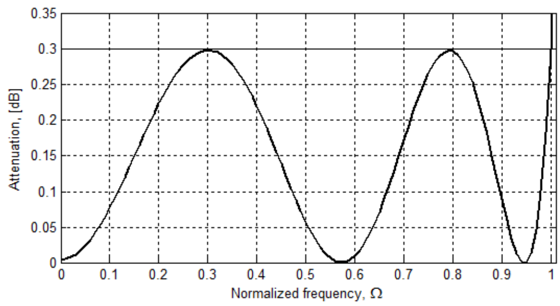

Fig. 6. Pass band of the synthesized quasi-elliptic prototype

Also note that behaviour of the transfer function in the passband is equiripple (Fig. 6), and behaviour in the stop band is also sufficiently close to equiripple (Fig. 5).

Usually such filters realized as a cascade of the 2-nd order filter sections with a transfer function (1). Therefore, such synthesis approach described in this article simplifies realization of a filter, especially realization of the filter sections with high Q-factor values.

It is obvious, that selectivity of the synthesized prototypes is significantly lower, than selectivity of elliptic prototypes. However elliptic filters are much harder to realize, since all their filter sections have fractionally-rational transfer functions. Chebyshev filters, on the other hand are much simpler, but their transfer functions doesn't have zeros, which can significantly increase selectivity.

Conclusions

This article describes the algorithm for quasi-elliptic filters synthesis with specified Q-factor parameter. Such approach allows simplifying of the filter schematics, compared to ones of the elliptic filters. At the same time, selectivity of the quasi-elliptic filters is often higher than selectivity of the Chebyshev filters of the same order (Fig. 5) with Q-factor of poles greater than specified, in some cases much greater.

In the example prototype it has been shown, that AFR of the synthesized prototype can be equiripple in both frequency bands.

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S. Sharkovsky, E. Grab. Design of Quasi-elliptic Filters with Specified Q-factor Parameter // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 9(105). – P. 77–80.

This article describes filters with specified Q-factor parameter of poles. This reduces complexity of circuitry, helps to pick elements with reasonable parameters and makes it easier to adjust circuit. There is also demonstrated algorithm for calculations of the transfer function, which uses optimisation functions of MATLAB software package. This article contains graphs of the amplitude-frequency response curves for synthesized filters. Ill. 6, bibl. 6, tabl. 1 (in English; abstracts in English and Lithuanian).

S. Sharkovsky, E. Grab. Pusiau elipsinio filtro su nustatytais Q faktoriaus parametrais projektavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 9(105). – P. 77–80.

Filtrai su nustatytais Q faktoriaus polių parametrais supaprastina schemas. Pateiktas perdavimo funkcijos apskaičiavimo algoritmas, kuris taiko MATLAB programų paketo optimizavimo funkcijas. Pateiktos susintetintų filtrų dažninės amplitudės charakteristikos. Il. 6, bibl. 6, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).