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Design of Forward Adaptive Uniform Quantizer for Discrete Input Samples for Laplacian Source

M. Savić, Z. Perić, M. Dinčić

Department of Telecommunications, Faculty of Electronic Engineering, University of Nis,
Aleksandra Medvedeva 14, 18000 Nis, Serbia, phone: +381 18/529225, e-mail: zoran.peric@elfak.ni.ac.rs

Introduction

Quantization is a very important part in almost all modern telecommunication systems, and it is usually done in two steps: firstly quantization with high number of levels is done with the aim of the analog-to-digital conversion and after that the second quantization with smaller number of levels is done with the aim of compression. Let's denote the first N_0-level quantizer with Q_0 and the second N-level quantizer with Q, $N < N_0$. Input of the first quantizer Q_0 consists of continual input samples which can take any real value from the interval $(-\infty,+\infty)$. But, input of the second quantizer Q consists of discrete input samples which can take only N_0 discrete values, equal to the output levels of the first quantizer Q_0. Therefore, input samples of the second quantizer Q are bounded by amplitude. Quantizers with continual input samples are largely analyzed in literature (e.g. [1] and the paper [2] – where design of nonuniform quantizers for exponential source for very small bitrates was done).

Since the most used types of signals (speech, audio, video) are not stationary (i.e. signal variance varies with time), adaptation should be applied to achieve good signal quality in the wide range of input variances. Adaptation can be forward and backward. The forward adaptation was analyzed in [1, 3, 4, 5]. It is known [6] that the forward adaptation gives for about 1 dB higher *SQNR* compared to the backward adaptation.

In this paper, the uniform quantizer for discrete input samples is analyzed, for Laplacian source. Uniform quantizer is considered since it is the most used type of quantizers.

Firstly, the fixed uniform quantizer is analyzed, and it is shown that performances of the quantizer for discrete and for continual input samples are different. For continual input samples, the overload distortion exists which causes the fall of *SQNR* (signal-to-quantization noise ratio) for high values of the variance. For discrete input samples (which are bounded by amplitude) there is no overload distortion and *SQNR* continues to increase for high variances.

After that, the forward adaptation is done for the uniform quantizer with discrete input samples. It is shown in this paper that adaptation gives good performances only if $\sigma < \sigma_0$ (σ^2 is the input variance and σ_0^2 is the referent variance). If $\sigma > \sigma_0$, adaptation gives worse performances compared to the fixed quantizer, and therefore adaptation should not be done in this case. This is the main difference compared to the forward adaptive quantizer for continual input samples, where adaptation gives good performances in both cases: when $\sigma < \sigma_0$ and when $\sigma > \sigma_0$. There is another effect for forward adaptive quantizer for discrete samples: *SQNR* significantly increases for very small variance σ^2, which is not the case for continual input samples. We can conclude that design and performances of the forward adaptive quantizer for discrete and for continual input samples are very different. The difference in performances is very high when $N_0 / N < 16$, but for $N_0 / N \geq 16$ this difference becomes small.

Performanse of fixed uniform quantizer for continual and discrete input signal

A. Uniform quantizer with continual input samples

Uniform quantizer Q will be considered, with the following parameters: N-number of levels, $x_{\max}$ - maximal amplitude and $\Delta = 2x_{\max} / N$ -quantization stepsize. Thresholds are defined as $x_i = -x_{\max} + i\Delta$, $i = 0,..., N$ and representation levels as $y_i = -x_{\max} + (i - 1/2)\Delta$, $i = 1,..., N$. Samples of the signal, which come on the input of the quantizer, are continual, i.e. they can take any real value from the interval $(-\infty, \infty)$. In this paper we consider zero mean input signal with Laplacian distribution, with variance σ^2, whose pdf (probability density function) is

$$p(x) = \frac{1}{\sqrt{2}\sigma} e^{-\frac{\sqrt{2}|x|}{\sigma}}. \qquad (1)$$

During quantization, an error is made which can be measured with distortion. The total distortion D is equal to the sum of the granular D_g and the overload D_{ov} distortion, i.e. $D = D_g + D_{ov}$. Granular distortion is defined as $D_g = \frac{\Delta^2}{12}\int_{-x_{\max}}^{x_{\max}} p(x)dx$ and overload distortion as $D_{ov} = 2\int_{x_{\max}}^{\infty} (x - y_N)^2 p(x)dx$. For $p(x)$ given with (1), these expressions become:

$$D_g = \frac{k^2 t^2 \sigma^2}{3N^2}\left(1 - \exp\left(-\sqrt{2}kt\right)\right), \quad (2)$$

$$D_o = \exp\left(-\sqrt{2}kt\right)\sigma^2\left(1 + \sqrt{2}\frac{kt}{N} + \left(\frac{kt}{N}\right)^2\right), \quad (3)$$

where $t = \sigma_0 / \sigma$ and $k = x_{\max} / \sigma_0$. σ_0^2 denotes a referent variance. Design of the quantizer is done for the referent variance. By minimizing distortion D, i.e. solving the equation $\frac{\partial D(t=1)}{\partial k} = 0$, the optimal value of the factor k, denoted with k_{opt} is found. On the basis of k_{opt}, maximal amplitude and minimal distortion are calculated. Values of the parameter k_{opt} for some values of levels' number N are given in Table 1.

Table 1. Values of parametar kopt for some values of the number of representation levels N

N	16	32	64	128	256	512
k_{opt}	3.71	4.49	5.3	6.15	7.02	7.91

Quality of the quantized signal is usually defined with signal-to-quantization noise ratio *SQNR*

$$SQNR[\text{dB}] = 10\log_{10}(\sigma^2 / D), \quad (4)$$

where σ^2 is a variance of the input signal, and D is total distortion. *SQNR* of the uniform quantizers for continual input samples is shown in Fig 1.

B. Discrete input

In this section we will analyze performances of the N-level uniform quantizer Q, whose input consists of samples previously quantized with the N_0-level uniform quantizer Q_0. So, input samples of the quantizer Q can take N_0 discrete values, which are equal to the output levels of the quantizer Q_0, denoted with $X = \{x_1, ..., x_{N_0}\}$. This situation is very common in practice – quantization with high number of levels is done first to achieve the analog-to-digital conversion, and after that quantization with smaller number of levels is done to achieve compression. Quantizers Q and Q_0 have the same maximal amplitude $x_{\max}$. The stepsize of the quantizer Q_0 is $\Delta_0 = 2x_{\max} / N_0$. Probabilities of the discrete levels from the set X are $P(x_i) = p(x_i)\Delta_0 = (1/(\sqrt{2}\sigma))\exp\left(-\sqrt{2}|x_i|/\sigma\right)\Delta_0$, $i = 2, ..., N_0 - 1$ and $P(x_1) = P(x_{N_0}) = (1/2)\exp(-\sqrt{2}x_{\max}/\sigma)$.

Output levels of the quantizer Q are denoted with $y_j, j = 1, ..., N$. It is valid that $N_0 = N \cdot L$, where L is an integer. This means that L discrete input levels $X_j = \{x_{j1}, ..., x_{jL}\} \in X$ are mapped to one output level $y_j, j = 1, ..., N$. Since discrete samples are amplitude limited with $x_{\max}$, quantizer Q will have only the granular distortion given with

$$\sum_{j=1}^{N}\sum_{k=1}^{L}(x_{jk} - y_j)^2 P(x_{jk}). \quad (5)$$

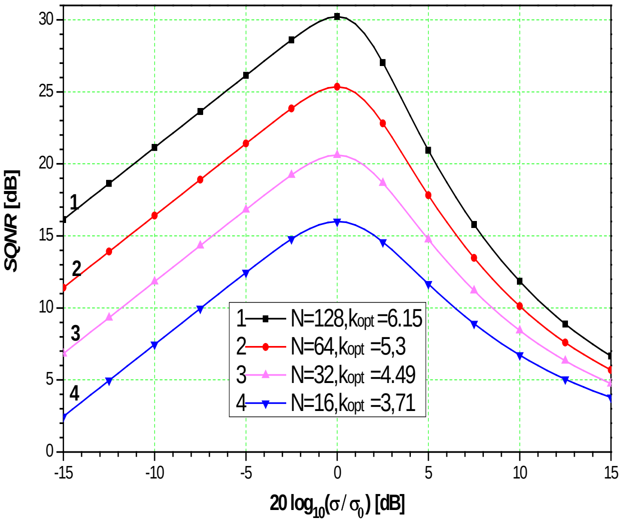

Fig. 1. *SQNR* of the uniform quantizers for continual input samples for different number of levels N

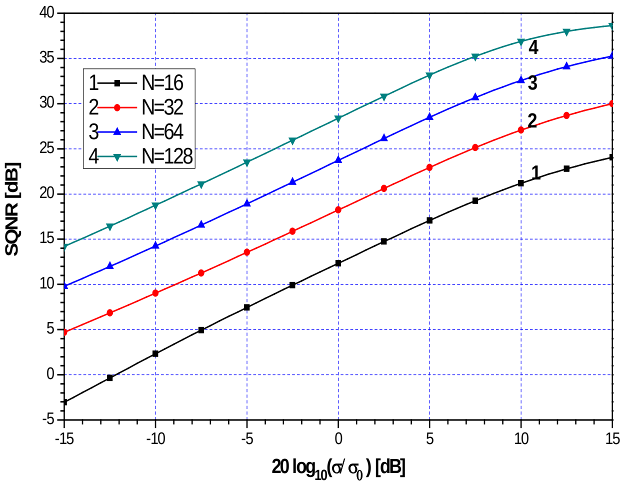

Fig. 2. *SQNR* of the fixed uniform quantizer for discrete input samples with $N_0 = 256$, for different values of N

SQNR for the uniform quantizer Q, for discrete input samples is in Fig. 2 We can see from Fig. 1 and Fig. 2 we can see that *SQNR* curves are very different for continual and for discrete input samples. For continual input samples there is overload distortion and *SQNR* falls for $\sigma > \sigma_0$. For discrete input samples there is no overload distortion, and therefore *SQNR* continue to increase for $\sigma > \sigma_0$. This is one of the motives for analysis and construction of adaptive uniform quantizers for discrete input samples.

Performance of adaptive uniform quantizer for discrete input signal

In this section, the forward adaptation of the uniform quantizer with discrete input samples will be done. Since the forward adaptation is well known in literature [1, 3, 4, 5], it will be described here very shortly. The scheme of the forward adaptive quantizer is shown in Fig. 3. Firstly, there is N_0-level uniform quantizer Q_0 whose input consists of continual samples and whose output consists of discrete samples. These discrete samples are input samples for the adaptive quantizer. The adaptive quantizer consists of a buffer, a gain estimator, a quantizer Q_{gain}, a divider and a fixed quantizer Q_{fixed}. Frame of the discrete input samples is formed in the buffer. This forward adaptation process works on the frame-by-frame basis. Firsly, the variance of the frame σ^2 is estimated and gain $g = \sigma / \sigma_0$ is calculated in the gain estimator. The gain g is quantized with the M – level log-uniform quantizer Q_{fixed} (it is uniform in the logarithmic domain of the gain: $20\log_{10} g$). We obtain the quantized gain which can take M discrete values $\hat{g}_j, j = 1,..., M$. $\hat{g}_j$ is sent to the receiver as an additional information with $\log_2 M$ bits. Samples from the frame in the buffer are divided with $\hat{g}_j$ and quantized with the fixed quantizer Q_{fixed}. Quantizers Q_0 and Q_{fixed} are designed for the same referent variance σ_0^2 and they have the same maximal amplitude $x_{\max}^{\mathrm{f}} = k\sigma_0$. So, discrete input samples of the adaptive quantizer are bounded in amplitude with $x_{\max}^{\mathrm{f}}$. Thresholds of the fixed quantizer Q_{fixed} are denoted with $x_i^{\mathrm{f}}, i = 0,..., N$ and representation levels are denoted with $y_i^{\mathrm{f}}, i = 1,..., N$.

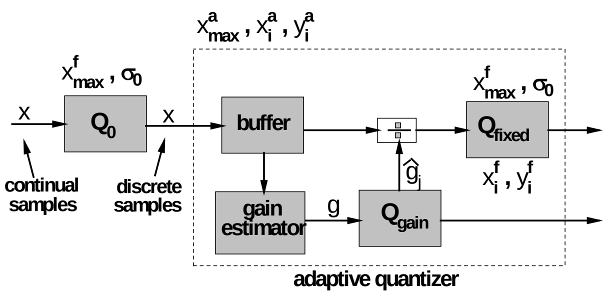

Fig. 3. The scheme of the forward adaptive quantizer for discrete input samples

Parameters of the adaptive quantizer are: $x_{\max}^{\mathrm{a}}$ - maximal amplitude, $x_i^{\mathrm{a}}, i = 0,..., N$ - thresholds and $y_i^{\mathrm{a}}, i = 1,..., N$ - representation levels. These parameters are constant inside one frame, but their values change from one frame to another. Let's suppose that in some frame quantized gain takes value $\hat{g}_j$. Then, in that frame, it is valid that $x_{\max}^{\mathrm{a}} = \hat{g}_j x_{\max}^{\mathrm{f}} = (\sigma/\sigma_0)k\sigma_0 = k\sigma$, $x_i^{\mathrm{a}} = \hat{g}_j x_i^{\mathrm{f}}$, $i = 0,..., N$ and $y_i^{\mathrm{a}} = \hat{g}_j y_i^{\mathrm{f}}, i = 1,..., N$.

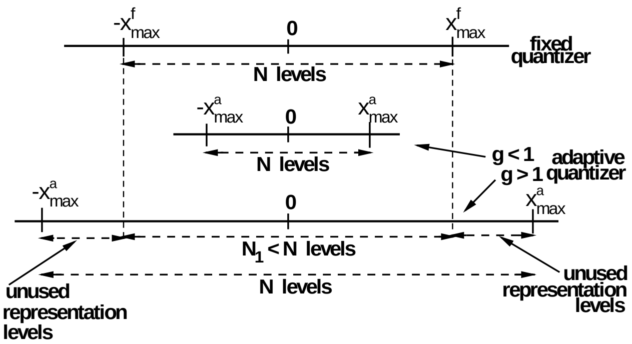

Fig. 4. The range of the fixed and the forward adaptive quantizers

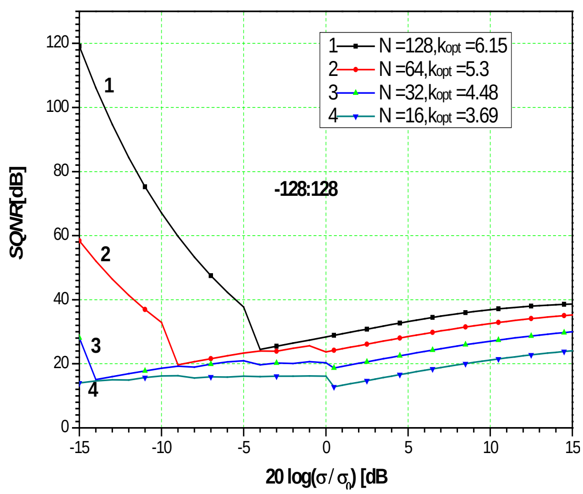

Fig. 5. *SQNR* of adaptive uniform quantizer for discrete input samples with $N_0 = 256$ levels for different number of levels N

Using adaptation, we adjust the amplitude range of the quantizer to the variance of the input signal. In Fig. 4, the range $\mathrm{I_f} = (-x_{\max}^{\mathrm{f}}, x_{\max}^{\mathrm{f}})$ of the fixed quantizer Q_{fixed} is shown firstly. After that, the range $\mathrm{I_a} = (-x_{\max}^{\mathrm{a}}, x_{\max}^{\mathrm{a}})$ of the adaptive quantizer is shown for two cases: when $g < 1$ (i.e. $\sigma < \sigma_0$) $\mathrm{I_a}$ becomes narrower than $\mathrm{I_f}$ and when $g > 1$ (i.e. $\sigma > \sigma_0$) $\mathrm{I_a}$ becomes wider than $\mathrm{I_f}$. The case when $g > 1$ will be deeply analyzed. In this case $x_{\max}^{\mathrm{a}} > x_{\max}^{\mathrm{f}}$, so N representation levels are placed in wider range $(-x_{\max}^{\mathrm{a}}, x_{\max}^{\mathrm{a}})$. But, as it was said earlyer, input samples are bounded in amplitude with $x_{\max}^{\mathrm{f}}$. Input samples cannot take values from the range $(-x_{\max}^{\mathrm{a}}, -x_{\max}^{\mathrm{f}}) \cup (x_{\max}^{\mathrm{f}}, x_{\max}^{\mathrm{a}})$ and therefore representation levels in that range are not used. So, not all N representation levels are used, but only N_1 (N_1 is some number smaller than N). Since smaller number of representation levels is used, higher distortion (i.e. lower *SQNR*) is obtained, compared to the fixed quantizer Q_{fixed}. So, we can conclude that if adaptation is done when $g > 1$, worse performances are obtained. Therefore, adaptation should not be done in this case. We can introduce the following rule: adaptation should be done only when

$$\sigma < \sigma_0 \Leftrightarrow k\sigma < x_{\max}^{\mathrm{f}} . \quad (6)$$

If condition (6) is not fulfilled, the fixed quantizer Q_{fixed} should be used. This is the main difference between adaptive quantizers for continual and for discrete input samples, because for continual input samples adaptation give good results in both cases: $g < 1$ and $g > 1$.

Dependence of *SQNR* on the input variance, for the forward adaptive quantizer with discrete input samples is shown on Fig. 5. From Fig. 5 we can see an interesting effect: *SQNR* increases for small σ, i.e. small *g*. Now, we will explain this effect considering one example with parameters $N_0 = 256$, $N = 32$, $L = 8$, and $g = 1/4$. For the fixed quantizer, we choose 32 output levels from the set of 256 input levels, i.e. matching between input and output levels is 12.5%. For the adaptive quantizer it is valid that $x_{max}^{a} = (1/4)x_{max}^{f}$. Therefore, inside the range of the adaptive quantizer $(-x_{max}^{a}, x_{max}^{a})$ there are $N_0/4 = 64$ input levels and among them we choose $N = 32$ output levels. So, for the adaptive quantizer, matching between input and output levels is 50%. If some input level is equal to some output level, distortion for that input level is zero. Therefore, if percentage of matching between input and output levels increases then distortion decreases and then *SQNR* increases. It is clear from the above example that matching percentage increases when *g* (i.e. σ) decreases.

We can see from Fig. 5 that *SQNR* curve 4 for $N = 16$ (i.e. when $N_0/N = 16$) is similar to *SQNR* curves for the forward adaptive quantizer with continual input samples, described in [5]. We can conclude that behavior of the adaptive uniform quantizer for discrete input samples is similar to behavior of the forward adaptive uniform quantizer for continual input samples when $N_0/N \geq 16$, but these behaviors are very different when $N_0/N < 16$.

Conclusions

The uniform quantizer for discrete input samples was analyzed in this paper. It is very common in practice and therefore, its analysis is very significant. It was shown that *SQNR* dependence on the input variance for discrete input samples differs compared to the case for continual input samples, both for the fixed and for the adaptive quantizers. For the fixed quantizer, there is no fall of *SQNR* for $\sigma > \sigma_0$ since there is no overload distortion. For the adaptive quantizer, *SQNR* significantly increases for very small σ, since the matching between input and output levels increases. Also, there is one significant difference in the construction of the forward adaptive quantizer for discrete input samples: adaptation should be done only when $\sigma < \sigma_0$.

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M. Savić, Z. Perić, M. Dinčić. Design of Forward Adaptive Uniform Quantizer for Discrete Input Samples for Laplacian Source // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 9(105). – P. 73–76.

In this paper, the problem of construction of the fixed and the forward adaptive uniform quantizer for input samples with discrete amplitude is analyzed, for Laplacian source. This quantizer is very common in practice; therefore its analysis is very significant. It is shown that design and performances of the quantiazer for discrete input samples are very different, compared to quantizer for continual input samples. Ill. 5, bibl. 7, tabl. 1 (in English; abstracts in English and Lithuanian).

M. Savić, Z. Perić, M. Dinčić. Laplaso šaltiniui skirto priešakinio adaptyviojo kvantoriaus projektavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 9(105). – P. 73–76.

Aprašyta priešakinio adaptyviojo kvantoriaus, skirto Laplaso šaltiniams, projektavimo problemos. Tokio kvantoriaus analizė yra labai svarbi dėl paplitimo praktikoje. Nustatyta, kad kvantoriaus našumas diskretinių ir nuosekliųjų įėjimo signalų atžvilgiu yra labai skirtingas. Il. 5, bibl. 7, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).