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# The Optimal Distribution of Cooling Elements

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### Introduction

The effect of heating or cooling at the junctions of two different conductors exposed to the current was discovered in 1834 by French Jean Peltier. Only last two decades this effect begun to use in the electronics. And now Peltier effect is widely applied in the manufacturing of the mini-refrigerators or for the cooling of the personal computers processors. There is presented one of the fresh application areas of this method – the cooling of the human body surface [1, 2]. The cooling effect depends on distribution of cooling elements on the cooling capacity surface. There is very important subject to choose right thermoelectric cooling element for a specific application, such as physical dimensions, input power limitations or cost.

System analysis considering thermal power and geometry of cooling elements is presented, also the calculation of the basic parameters for the choice of the thermoelectric devices is introduced.

### Parameters required for cooling element selection

There are three basic parameters that need to be known for Peltier device choosing: the two temperatures that define the gradient across the thermoelectric element and the total amount of heat that must be pumped by the Peltier [3].

The temperature difference across the thermoelectric device (actual $\Delta T$) is not the same as the named $\Delta T$ (system $\Delta T$). The difference between these two $\Delta T$ is often ignored, which results wrong function of the cooling system. The magnitude of the difference in $\Delta T$ is most dependent on the type of heat exchangers that are utilized on either the hot or cold sides of the system. Since the heat flux densities on the cold side of the system are considerably lower than those on the hot side, an allowance of about 50% of the hot side figures (assuming similar types of heat exchangers) can be used.

The third parameter that must be identified for the choosing process is the total heat to be pumped by the thermoelectric device. To reduce the temperature of the body, heat must be removed from it faster than heat enters it. There are generally two broad classifications of the heat that must be removed from the element. The first is the real, sensible heat load (called "active"). This is the load that is representative of what wants to be done. This load could be the $I^2R$ load of an electric component or the load of cooling objects. The other kind of load is often referred to as the parasitic load. This is the load due to the fact that the object is cooler than the surrounding environment. This load can be composed of conduction and convection of the surrounding air, leak through insulation, conduction through wires, condensation of water, and in some cases formation of ice. Regardless of the source of these parasitic loads, they must not be ignored. Conductive losses may be minimized by limiting direct contact between the cooled object and external structural segment. All interfaces between system components must be flat, parallel, and clean to minimize thermal resistance. High conductivity thermal interface material is often used to ensure good contact between surfaces.

### Calculation of cooling elements

First of all we need to build a mathematical model of the thermoelectric cooling element. There are several important coefficients of the element that must be evaluated before creating the cooling module: the module's effective Seebeck coefficient ($\alpha_M$, V/°C), electrical resistance ($r_M$, Ω) and thermal conductance ($k_M$, W/°C). These values of $\alpha_M$, $r_M$, and $k_M$ can be expressed mathematically by polynomial equations.

If $\Delta T$>0, the Seebeck coefficient must be evaluated at both temperatures $T_h$ and $T_c$ using the expressions [3, 5]:

$$\alpha_{MTh} = \alpha_1 T_h + \frac{\alpha_2 T_h^{\,2}}{2} + \frac{\alpha_3 T_h^{\,3}}{3} + \frac{\alpha_4 T_h^{\,4}}{4}, \quad (1)$$

$$\alpha_{MTc} = \alpha_1 T_c + \frac{\alpha_2 T_c^{\,2}}{2} + \frac{\alpha_3 T_c^{\,3}}{3} + \frac{\alpha_4 T_c^{\,4}}{4}, \quad (2)$$

$$\alpha_M = \frac{(\alpha_{MTh} - \alpha_{MTc})}{\Delta T}, \quad (3)$$

where: $\alpha_{MTh}$ is the module's Seebeck coefficient at the hot side temperature $T_h$, $\alpha_{MTc}$ is the module's Seebeck coefficient at the cold side temperature $T_c$. Each manufacturer company mostly indicate the specified equation coefficients, they depend on temperature, for example, TE Technology, Inc. of Traverse City, Michigan (USA) produce thermoelectric elements derived from an industry-standard 71-couple, 6-ampere module which is made from bismuth – telluride ($Bi_2Te_3$) and their coefficients are the following [5]: $\alpha_1$ = 1.33450 x $10^{-2}$ V/°C$^2$, $\alpha_2$ = –5.37574 x $10^{-5}$ V/°C$^3$, $\alpha_3$ = 7.42731 x $10^{-7}$ V/°C$^4$, $\alpha_4$ = –1.27141 x $10^{-9}$ V/°C$^5$.

The electrical resistance of a thermoelectric module can be expressed as a function of temperature:

$$r_{MTh} = r_1 T_h + \frac{r_2 T_h^2}{2} + \frac{r_3 T_h^3}{3} + \frac{r_4 T_h^4}{4}, \tag{4}$$

$$r_{MTc} = r_1 T_c + \frac{r_2 T_c^2}{2} + \frac{r_3 T_c^3}{3} + \frac{r_4 T_c^4}{4}, \tag{5}$$

$$r_M = \frac{(r_{MTh} - r_{MTc})}{\Delta T}, \tag{6}$$

where $r_M$ is the module's resistance in ohms, $r_{MTh}$ is the module's resistance at the hot side temperature $T_h$, $r_{MTc}$ is the module's resistance at the cold side temperature $T_c$ .

Coefficients for a 71-cpl, 6-amp module: $r_1$ = 2.08317 Ω/°C , $r_2$ = –1.98763 x $10^{-2}$ Ω/°C$^2$, $r_3$ = 8.53832 x $10^{-5}$ Ω/°C$^3$, $r_4$ = –9.03143 x $10^{-8}$ Ω/°C$^4$.

And the thermal conductance of a thermoelectric module can be expressed as a function of temperature:

$$k_{MTh} = k_1 T_h + \frac{k_2 T_h^2}{2} + \frac{k_3 T_h^3}{3} + \frac{k_4 T_h^4}{4}, \tag{7}$$

$$k_{MTc} = k_1 T_c + \frac{k_2 T_c^2}{2} + \frac{k_3 T_c^3}{3} + \frac{k_4 T_c^4}{4}, \tag{8}$$

$$k_M = \frac{(k_{MTh} - k_{MTc})}{\Delta T}, \tag{9}$$

where $k_M$ is the module's thermal conductance, $k_{MTh}$ is the thermal conductance at the hot side temperature $T_h$, $k_{MTc}$ is the thermal conductance at the cold side temperature $T_c$.
Coefficients for a 71-cpl, 6-amp module: $k_1$ = 4.76218 x $10^{-1}$ W/°C$^2$, $k_2$ = –3.89821 x $10^{-6}$ W/°C$^3$, $k_3$ = –8.64864 x $10^{-6}$ W/°C$^4$, $k_4$ = 2.20869 x $10^{-8}$ W/°C$^5$.

The $\alpha_M$, $r_M$, and $k_M$ parameters shown are calculated for a 71-couple, 6-ampere thermoelectric module. If a new or different module configuration is to be modelled, it is necessary to apply a conversion factor to each of these parameters, as follows:

$$\alpha_{new} = \alpha_M \cdot \frac{n_{new}}{71}, \tag{10}$$

$$r_{new} = r_M \cdot \frac{6}{I_{new}} \cdot \frac{r_{new}}{71}, \tag{11}$$

$$k_{new} = k_M \cdot \frac{I_{new}}{6} \cdot \frac{n_{new}}{71}, \tag{12}$$

where $\alpha_{new}$ is the Seebeck coefficient for the new module, and accordingly $r_{new}$ is the electrical resistance, $k_{new}$ is the thermal conductance, $n_{new}$ is the number of couples, $I_{new}$ is the optimum or maximum current of the new module.

When we get thermoelectric parameters we can calculate single-stage thermoelectric module [3, 5].
The temperature difference $\Delta T$ across the module in °C

$$\Delta T = T_h - T_c . \tag{13}$$

Heat removed by the module $Q_c$ in watts

$$Q_c = (\alpha_M \cdot T_c \cdot I) - (0.5 \cdot I^2 \cdot r_M) - (k_M \cdot \Delta T). \tag{14}$$

The input voltage $V_{in}$ to the module in volts

$$V_{in} = (\alpha_M \cdot \Delta T) + (I \cdot r_M). \tag{15}$$

The electrical input power $P_{in}$ to the module in watts

$$P_{in} = V_{in} \cdot I . \tag{16}$$

The heat wasted by the module $Q_h$ in watts

$$Q_h = P_{in} + Q_c . \tag{17}$$

The coefficient of performance $\eta$ as a cooler

$$\eta = \frac{Q_c}{P_{in}} . \tag{18}$$

**Thermoelectric temperature distribution in Peltier element**

The basic equation of the heat flow in the thermoelectric analysis

$$\rho c \frac{\partial T}{\partial t} + \nabla \cdot \boldsymbol{q} = \dot{q} . \tag{19}$$

We use the electric current density continuity equation, too

$$\nabla \left( \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \right) = 0 . \tag{20}$$

These two equations have thermoelectric relation that can be written:

$$\boldsymbol{q} = [\Pi] \cdot \boldsymbol{J} - [k] \cdot \nabla T, \tag{21}$$

$$\boldsymbol{J} = [\sigma] \cdot (\boldsymbol{E} - [\alpha] \cdot \nabla T), \tag{22}$$

where $\boldsymbol{q}$ is the heat flux density vector, $\boldsymbol{J}$ is the electric current density vector, $\boldsymbol{E}$ is the electric field intensity vector, $[\Pi]$ is the Peltier coefficient matrix, $[k]$ is the thermal conductivity matrix, $[\sigma]$ is the electrical conductivity matrix, $[\alpha]$ is the Seebeck coefficient matrix [5]. The electric flux density vector $\boldsymbol{D}$ has the following relation to the electric field intensity vector:

$$\boldsymbol{D} = [\varepsilon] \cdot \boldsymbol{E}, \tag{23}$$

where $[\varepsilon]$ is the dielectric permittivity matrix.

Substituting equations (21)-(23) to equations (19)-(20) and considering that $\boldsymbol{E} = -\nabla\varphi$, we obtain the thermoelectric equations system [6]:

$$\rho c \frac{\partial T}{\partial t} + \nabla \cdot ([\Pi] \cdot \boldsymbol{J}) - \nabla \cdot ([k] \cdot \nabla T) = \dot{q}, \tag{24}$$

$$\nabla\left( [\varepsilon] \cdot \nabla \frac{\partial \varphi}{\partial t} \right) + \nabla([\sigma] \cdot [\alpha] \cdot \nabla T) + \nabla \cdot ([\sigma] \cdot \nabla \varphi) = 0. \tag{25}$$

**Distribution dependence on sizes of the Peltier elements**

The thermoelectric elements are produced all over the world. Very strong positions have American companies (TE Technology, Melcor, Watronix and Marlow, for example) who have been in the market for many years and have maintained leading positions, also – Russian and Ukrainian. Most of them are young and started only a decade ago but are based on a high thermoelectric technology level and scientific basis. Chinese companies are also relatively young. They have demonstrated a fast expansion in the thermoelectric market and dominate in low-cost types. Production of Peltier modules is also developing rapidly in Japan and Europe. Peltier devices are available in great variety of sizes, shapes, operating currents, operating voltages and ranges of heat pumping capacity.

The thermoelectric elements are of various sizes, the smallest can be from 2 x 2 mm and height from 2 mm (for example, USA Watronix Inc.) and the biggest one are till 62 x 62 mm and respectively height till 5.6 mm (for example, USA Melcor Inc.).

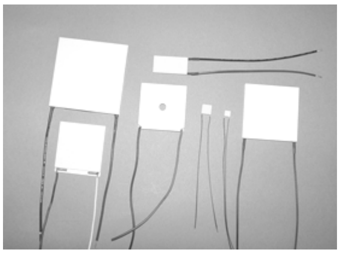

**Fig. 1.** The thermoelectric device of various sizes

The single stage thermoelectric elements are mostly usable, its basic temperature difference Δ*T* is around 70°C. Also there is the possibility to use the two (Δ*T*=90°C) or three stages (Δ*T*=110°C) Peltier element. Our creative system required temperature difference does not exceed 60°C, so the number of stages more than one is unnecessary. And more than one stage Peltier device is not comfortable for integration into human wearing clothes for cooling system.

Analysis of the energy balance in the system human – clothing – environment is presented in the reference [4]. By using received results of the human thermal balance, we can choose the effective thermoelectric device.

The thermoelectric module consists of two or more elements of semiconductor material that are connected electrically in series and thermally in parallel. The effectiveness of the Peltier elements directly depends on the number of the thermocouples – it can be 7, 31, 63, 71 or 127.

There is presented preliminary calculation of the choosing the thermoelectric devices. It was picked, for example, for elimination of the heat power around 300 W. Thermoelectric devices were picked from manufacturer TE Technology, Inc. of Traverse City, Michigan (USA) [5].

First of all, we choose the smallest buyable thermoelectric device, it sizes are those: width – 8x8 mm, height – 4.3 mm. So the area of the Peltier surface is 64 mm$^2$. Thermoelectric parameters are: $Q_{\mathrm{Max}}$=1.4W, $I_{\mathrm{Max}}$=2.3A, $V_{\mathrm{Max}}$=1V. This maximum values are achieved when there is no difference in temperature (Δ*T*=0) on the modules two surfaces, if there requires cooling, the heat pumping capacity will be less. The element consists of 7 thermocouples. Approximately evaluating if need to eliminate 300 W of the heat, we need 214 those elements, and of course this is if we ignore thermal waste.

The choice of the bigger thermoelectric element: 71 thermocouples, sizes: width – 30x30 mm, height – 4.2 mm, the area of the Peltier is 900 mm$^2$. Thermoelectric parameters are: $Q_{\mathrm{Max}}$=30.6 W, $I_{\mathrm{Max}}$ =5.1 A, $V_{\mathrm{Max}}$=9.9 V. Therefore for 300 W needs 10 those elements.

And the biggest element – 127 thermocouples, sizes are those: width – 48x48 mm, height – 4.8 mm, the area is 2304 mm$^2$. Thermoelectric parameters are: $Q_{\mathrm{Max}}$=83.4 W, $I_{\mathrm{Max}}$=7.6 A, $V_{\mathrm{Max}}$=18.1 V. For 300 W needs 4 those elements.

Regarding to the mounting into the clothes is more comfortable to use many small thermoelectric modules, than the few but large modules. The large one can create discomfort and limit the motion of the human. The small elements can be more various distributed then the big one.

Further we prefer the particular calculation evaluating the equations described in the beginning of this paper and applying the datasheets of the Peltier element. There is picked the thermoelectric element TE-127-1.0-0.8, which consist of 127 thermocouples and its size is width – 30x30 mm, height – 3.1 mm, it is the smallest and thinnest element of those number of thermocouples. Its optimal thermal – electrical parameters are $I_{\mathrm{Max}}$=5.8 A, $V_{\mathrm{Max}}$=17.4 V, $Q_{\mathrm{Max}}$= 61.4 W. From equations (1) – (12) there was calculated effective Seebeck coefficient $\alpha_{\mathrm{M}}$, electrical resistance $r_{\mathrm{M}}$ and thermal conductance $k_{\mathrm{M}}$ at various temperature difference and the results are depicted in the table 1.

**Table 1.** The effective coefficients of the thermoelectric module

| Δ *T* | $\alpha_{\mathrm{M}}$ | $r_{\mathrm{M}}$ | $k_{\mathrm{M}}$ |
|---|---|---|---|
| °C | V/°C | Ω | W/°C |
| 10 | 0.053 | 4.2724 | 0.2957 |
| 20 | 0.0534 | 4.5156 | 0.2912 |
| 30 | 0.0538 | 4.7742 | 0.2871 |
| 40 | 0.0541 | 5.2179 | 0.2744 |
| 50 | 0.0543 | 5.4334 | 0.2757 |

Using the Peltier element datasheets was received the values of thermoelectric parameters depended on various temperature differences, depicted in the table 2. Input voltage $V_{in}$ to the module was chose constant 9.8 V. Also in this table was calculated the necessary number of the devices for elimination the heat of 300 W from the body surface.

**Table 2.** The results of the calculation of Peltier devices

| Δ *T* | $Q_c$ | $Q_h$ | *I* | *η* | *Quantity* |
|---|---|---|---|---|---|
| °C | W | W | A | | Unit |
| 10 | 35 | 68 | 3.4 | 1 | 8 |
| 20 | 28 | 60 | 3.3 | 0.9 | 11 |
| 30 | 22 | 52 | 3.2 | 0.7 | 13 |
| 40 | 14 | 45 | 3 | 0.4 | 21 |
| 50 | 8 | 35 | 2.95 | 0.3 | 37 |

As see, in the table 2 at $\Delta T$=10°C, is needed the 8 thermoelectric modules, at $\Delta T$= 20°C – 11, at $\Delta T$= 30°C – 13, $\Delta T$= 40°C – 21 and $\Delta T$= 50°C – 37. Considering those results we can propose that element TE-127-1.0-0.8 we can integrate into human cooling system and we can reach the temperature difference from 10°C till 50°C.

The averaged human body surface area is 2 $m^2$ and the human trunk takes 35.94% of all body surfaces [6]. So, the Peltier elements can be allocated on the human back, this part of the surface is most comfortable for the integration of the cooling system, and the most sensitive for the cold and heat.

### The choice of the Peltier devices considering the price

Price of the one thermoelectric module depends on the number of the thermocouples and herewith on the size of the elements. It balances from 12$ to 50$ for one element. Accordingly to creation the all cooling system the more expensive becomes the system made neither from the small size element nor from the large size even fifteen fold.

### Conclusions

The cooling effect depends on distribution of cooling elements on the cooling capacity surface. The basic factors for choice of the thermoelectric modules are physical, such as size, width, number of stages and the thermoelectric parameters, such as temperature difference, removed heat from the element, voltage, current and main coefficients.

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The efficiency of the cooling system depends on the choice of the cooling elements. There is important to consider to thermoelectric parameters, the size, distribution and the price of the elements. There is analyzed the process of the calculation of Peltier elements, presented the most important parameters. In this paper there is analyzed the selection of the thermoelectric elements considering to its size and possibility to integrate it into human wearing clothes. There is totalized how many and what kind of elements are required for the elimination of the heat excess of 300 W from human body and where can be integrated such elements for the best efficiency and comfort. Ill. 1, bibl. 6, tabl. 2 (in English; abstracts in English and Lithuanian).

**R. Račkienė, J. A. Virbalis, S. Bartkevičius, R. Lukočius. Optimalus aušinimo elementų išdėstymas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 9(105). – P. 53–56.**

Aušinimo sistemos efektyvumas priklauso nuo aušinimo elementų parinkimo. Svarbu atsižvelgti į termoelektrinius parametrus, elementų dydį, išdėstymą ir kainą. Išnagrinėta Peltier elementų skaičiavimo eiga, pateikti svarbiausi parametrai. Išanalizuotas termoelektrinių elementų parinkimas atsižvelgiant į dydį, galimybę montuoti į žmogaus dėvimus drabužius. Apskaičiuota, kiek ir kokių elementų užtektų 300 W šilumos pertekliui pašalinti iš žmogaus organizmo ir kur tokius elementus veiksmingiausia ir patogiausia montuoti. Il. 1, bibl. 6, lent. 2 (anglų kalba; santraukos anglų ir lietuvių k.).