Multiparameter Electronic Tools Quality Level Linear Transformation Models

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Introduction

Multiparameter mechatronics product defect level nonlinear transformation models in continuous quality control, evaluating separate parameter and the whole product probability characteristics also first and second type errors, when products are classified, described in [1–5]. Electronic tool [ET] quality level probabilistic models, expressed by separate parameters probabilistic characteristics, when defect levels by separate parameters are characterized using beta densities.

Electronic tool quality level directly transformed probabilistic characteristics models from separate parameters probabilistic characteristics transformations and controlled parameters nomenclature variation are described in [6]. When ET are repaired immediately after control operation and returned for repeated control with localized repair operation [1] for fixed second type classification errors by different parameters. For analysis needs we will use defect ET probabilities by i-th parameter θ_i and good product probabilities $\eta_i = 1 - \theta_i$ characteristic models [6] repeatedly, when $\theta_i \sim \mathrm{Be}(b_i, a_i)$, $\eta_i \sim \mathrm{Be}(b_i, a_i)$ – beta laws with parameters a_i, b_i:

averages:

$$\mathrm{E}\theta_i = \mu_i = \frac{a_i}{a_i + b_i}, \quad \mathrm{E}\eta_i = 1 - \mu_i = \overline{\mu}_i, \qquad (1)$$

dispersions:

$$\mathrm{V}\theta_i = \sigma_i^2 = \frac{\mu_i \overline{\mu}_i}{a_i + b_i + 1} = \mathrm{V}\eta_i, \qquad (2)$$

densities:

$$g_i(\theta_i) = B^{-1}(a_i, b_i)\theta_i^{a_i - 1}(1 - \theta_i)^{b_i - 1}, \qquad (3)$$

$$\varphi_i(\eta_i) = g_i(1 - \eta_i) \qquad (4)$$

when

$$B(a_i, b_i) = \frac{\Gamma(a_i)\Gamma(b_i)}{\Gamma(a_i + b_i)} - \text{ beta function,}$$

$\Gamma(z)$ – gama function.

For all l – electronic tool parameter

$$\eta = \prod_{i=1}^{\ell}\eta_i, \ \theta = 1 - \eta, E_n = \overline{\mu} = \prod_{i=1}^{\ell}\overline{\mu}_i, \mathrm{E}\theta = \mu = 1 - \overline{\mu}. (5)$$

Dispersion by two parameters ($i = 1, 2$)

$$\mathrm{V}\theta_{12} = \sigma_{12}^2 = \sigma_1^2\overline{\mu}_2^2 + \sigma_2^2\overline{\mu}_1^2 + \sigma_1^2\sigma_2^2. \qquad (6)$$

Common linear transformation models

Analyzing defect level θ_i linear transformation to defect level τ_i (Fig. 1). Single-stage control K with localized repair operation R is characterized by second type error probability β_{Ri} – in operation R [7–11].
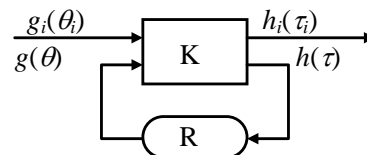

Fig. 1. Single-stage continuous control schematic

If β_{Ri}=const. exists and ED repeats ("spins") through these operations, until all are recognized as good (first type errors probability α_i=0), then both operations K and R are characterized in generalized probability β_{0i} [2]

$$\beta_{0i} = \frac{\beta_i}{1 - \beta_{R_i}(1 - \beta_i)}, \quad i = 1 - \ell. \qquad (7)$$

If control system is made of k serial stages, then for all system by i-th parameter we get (when every stage is characterized $\beta_{0i}(j)$)

$$\beta_{0i} = \prod_{j=1}^{k}\beta_{0i}(j), \quad j = 1 - k. \qquad (8)$$

After control K defected ED probability τ_i and good ED probability $\zeta_i = 1 - \tau_i$ by i-th parameter are equal [16]

$$\tau_i \beta_{0i} \theta_i \,, \zeta_i = \overline{\beta}_i + \beta_{0i} \eta_i \,, \overline{\beta}_i = 1 - \beta_{0i} \,. \qquad (9)$$

Averages and dispersions are:

$$\begin{cases} E\tau_i = \mu_{\tau_i} = \beta_{0i}\mu_i \,, \ E\zeta_i = \overline{\mu}_{\tau_i} = 1 - \mu_{\tau_i} \,, \\ V\tau_i = V\zeta_i = \sigma_{\tau_i}^2 = \beta_{\tau_i}^2 \sigma_i^2 \,. \end{cases} \qquad (10)$$

As in non-linear transformation case, densities $h(\tau)$ are more useful to approximate using simple models for engineering analysis. Referencing single-parameter model, we use generalized beta density $h_\sigma(\tau)$ [6]

$$h_\sigma(\tau) = \frac{B^{-1}(a^*, b^*)}{\beta_0^{a^* + b^* - 1}} \tau^{a^* - 1} (\beta_0 - \tau)^{b^* - 1}, \tau \in (0, \beta_0)\,, \qquad (11)$$

here

$$a^* = \frac{\mu_\tau}{\beta_0}\left[\frac{\mu_\tau(\beta_0 - \mu_\tau)}{\beta_\tau^2} - 1\right], b^* = \left(\frac{\beta_0}{\mu_\tau} - 1\right)a^*$$

and mode

$$\tau_M = \beta_0 \frac{a^* - 1}{a^* + b^* - 2} \text{ with maximum } h_{M\sigma} = h_\sigma(\tau_M).$$

Valid formulas

$$\tau_\tau = \beta_0 \frac{a^*}{a^* + b^*}, \sigma_\tau^2 = \beta_0 = \frac{\mu_\tau \cdot b^*}{(a^* + b^*)(a^* + b^* + 1)}.$$

Multiparameter electronic tool

Multiparameter ET is characterized when $l \geq 3$ and accidental values ζ and τ are described analogically [16], when β_{0i} varies in interval.

Multiparameter ET is characterized when $l \geq 3$, and

$$\zeta \in (\overline{\beta}_0, 1), \tau \in (0, \beta_0), \qquad (12)$$

here
$$\overline{\beta}_0 = \overline{\beta}_1 \overline{\beta}_2, \beta_0 = 1 - \overline{\beta}_0, \overline{\beta}_i = 1 - \beta_{0i}, i = 1,2,3.$$
In this case

$$\overline{\beta}_0 = \prod_{i=1}^3 \overline{\beta}_i, \beta_0 = 1 - \overline{\beta}_0, \overline{\beta}_i = 1 - \beta_{01}, i = 1,2,3\,. \qquad (13)$$

Accidental multiplicative value $\zeta = \zeta_1 \zeta_2 \zeta_3$ available values range in coordinate system $(\zeta_1, \zeta_2, \zeta_3)$ is a 3-dimensional parallelepiped rectangular where ζ gets values $\overline{\beta}_1 \overline{\beta}_2 \overline{\beta}_3, \overline{\beta}_1 \overline{\beta}_2, \overline{\beta}_1 \overline{\beta}_3, \overline{\beta}_1, \overline{\beta}_2, \overline{\beta}_3, 1$ on its tops. When $\overline{\beta}_1 \neq \overline{\beta}_2 \neq \overline{\beta}_3$, these values divide the whole accidental value ζ variety range to 7 partial ranges dependent on transformation coefficient ratios $\beta_{01}/\beta_{02}, \beta_{01}/\beta_{03}, \beta_{02}/\beta_{03}$. In these partial intervals, function $\dot\Phi(\zeta)$ models are different. For more simple analysis we will use partial case $\beta_{01} = \beta_{02} = \beta_{03} = \beta_{0i}$, when ζ varies in interval $(\overline{\beta}_i^3, 1)$ and possible values range is a cube, where $\dot\Phi(\zeta)$ is described with three different models in partial intervals:

$\overline{\beta}_i^3 \leq \zeta \leq \overline{\beta}_i^2, \overline{\beta}_i^2 \leq \zeta \leq \overline{\beta}_i, \zeta \in (\overline{\beta}_i^3, \overline{\beta}_i^2)$. Integration ranges, when $\zeta \in (\overline{\beta}_i^3, \overline{\beta}_i^2)$ and $\zeta \in (\overline{\beta}_i^2, \overline{\beta}_i)$ are shown in Fig. 2. In interval $\overline{\beta}_i < \zeta < 1$ integration range is made for function $1 - \dot\Phi(\zeta)$ and equals to non-linear transformed beta density case. Expressing functions $\dot\Phi(\zeta)$, $\dot\varphi(\zeta)$ models by multidimensional definite integrals (14, 15).

$$I = I(y_{0i}, y_{1i}) = \int_{y_{01}}^{y_{11}} \int_{y_{02}}^{y_{12}} \int_{y03}^{y_{13}} \dot\varphi_1(\zeta_1) \times \dot\varphi_2(\zeta_2) \times \\ \times \dot\varphi_3(\zeta_3) d\zeta_3 d\zeta_2 d\zeta_1, \qquad (14)$$

$$I^* = I^*(y_{0i}, y_{1i}) = \int_{y_{01}}^{y_{11}} \int_{y_{02}}^{y_{12}} \frac{1}{\zeta_1} \dot\varphi_1(\zeta_1) \frac{1}{\zeta_2} \times \\ \times \dot\varphi_2(\zeta_2) \dot\varphi_3\left(\frac{\zeta}{\zeta_1 \zeta_2}\right) d\zeta_2 d\zeta_1, \ i = 1,2. \qquad (15)$$
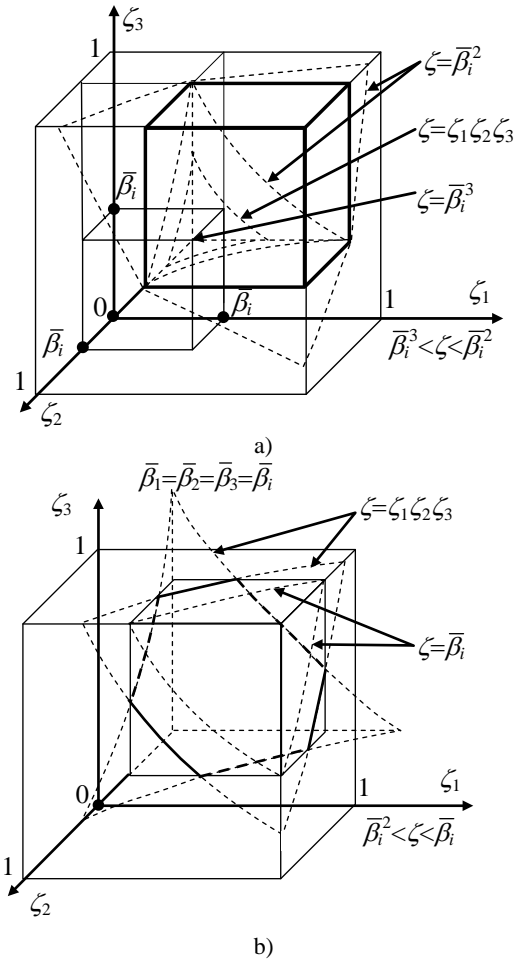

a)

b)

Fig. 2. Functions $\dot\Phi(\zeta)$, $\ell=3$ integration ranges, $i=1,2,3$, $\beta_{01} = \beta_{02} = \beta_{03}$

Mark third parameter integration ranges $y_{03} \leq \zeta \leq y_{13}$ and get

- $\overline{\beta}_i^3 \leq \zeta \leq \overline{\beta}_i^2$: $y_{01} = \overline{\beta}_i$, $y_{11} = \zeta/\overline{\beta}_i^2$; $y_{02} = \overline{\beta}_i$, $y_{12} = \zeta/\overline{\beta}_i \zeta_i$; $y_{03} = \overline{\beta}_i$, $y_{13} = \zeta/\zeta_i \zeta_2$ and $\dot\Phi(\zeta) = I = I_1$, $\dot\varphi(\zeta) = I^* \equiv I_1^*$;

- $\overline{\beta}_i^2 \le \zeta \le \overline{\beta}_i$: $y_{01} = 1$, $y_{11} = \zeta / \overline{\beta}_i^2$;

  $y_{02} = \overline{\beta}_i$, $y_{12} = \zeta / \overline{\beta}_i \zeta_i$; $y_{03} = \overline{\beta}_i$, $y_{13} = \zeta / \zeta_i \zeta_2$

  and $\dot{\Phi}(\zeta) = I - 3I_1$, $\dot{\varphi}(\zeta) = I_1^* - 3I^*$;

- $\overline{\beta}_i \le \zeta \le 1$: $y_{01} = \zeta$, $y_{11} = 1$; $y_{02} = \zeta / \zeta_1$, $y_{12} = 1$;

  $y_{03} = \zeta / \zeta_i \zeta_2$, $y_{13} = 1$

  and $\dot{\Phi}(\zeta) = 1 - I$, $\dot{\varphi}(\zeta) = I^*$.

In partial case, when $\beta_{03} = 1$ and $\beta_{01} = \beta_{02} = \beta_{0i}$, $i=1,2$, we have to analyze integration ranges, because in common models with equal β_{0i} ($i=1,2,3$) different parameters levels together. Lets analyze $\dot{\Phi}(\zeta)$ model design geometrical interpretation, when $\overline{\beta}_3 = 0$.

$\zeta = \zeta_1 \zeta_2 \zeta_3$ possible values range – dimensional parallelepiped rectangular bounded with six planes: $\zeta_1 = \overline{\beta}_1 = \overline{\beta}_i$, $\zeta_2 = \overline{\beta}_2 = \overline{\beta}_i$, $\zeta_3 = 0$ ir $\zeta_1 = \zeta_2 = \zeta_3 = 1$. Everywhere in $0 < \zeta < 1$ function $\dot{\Phi}(\zeta)$ value equals to figure slice of volume, which is between plane $\zeta_3 = 0$ and hyperbolical surface $\zeta = \zeta_1 \zeta_2 \zeta_3$ (Fig. 3). To avoid indefinite "ln 0" in function $\dot{\Phi}(\zeta)$ model, which emerges when the lower range is $\zeta_3 = 0$, we find the left volume on the figure: from the surface $\zeta = \zeta_1 \zeta_2 \zeta_3$ to the plane $\zeta_3 = 1$. This equals to the value of the function $1 - \dot{\Phi}(\zeta)$.
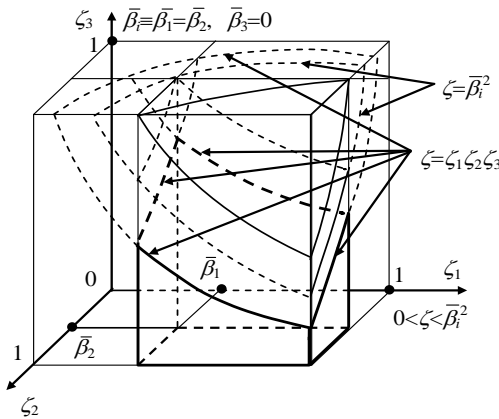

Fig. 3. Functions $\dot{\Phi}(\zeta)$, $\ell = 3$ integration ranges, $\beta_{03} = 1$, $\beta_{01} = \beta_{02}$

Functions $1 - \dot{\Phi}(\zeta)$ value in interval $\overline{\beta}_i < \zeta < 1$ equals to volume bounded with surface $\zeta = \zeta_1 \zeta_2 \zeta_3$ and three planes $\zeta_1 = \zeta_2 = \zeta_3 = 1$. The bounded volume in interval $\overline{\beta}_i^2 < \zeta < \overline{\beta}_i$ is bigger than the $1 - \dot{\Phi}(\zeta)$ value, because it use more space than the surface $\zeta = \zeta_1 \zeta_2 \zeta_3$ severs from parallelepiped rectangular. In this case, two equal (by volume) figures has to be subtracted from the whole volume:

- the first, bounded with surface $\zeta = \zeta_1 \zeta_2 \zeta_3$ planes $\zeta_1 = \overline{\beta}_1$, $\zeta_2 = \zeta_3 = 1$;
- the second, bounded with surface $\zeta = \zeta_1 \zeta_2 \zeta_3$ and planes $\zeta_2 = \overline{\beta}_2$, $\zeta_1 = \zeta_3 = 1$.

It is obvious that it is possible to subtract both volumes separately or double volume of the one figure by integration. After analogical elimination of two additional volumes in interval $0 < \zeta < \overline{\beta}_i^2$ as in interval $\overline{\beta}_i^2 < \zeta < \overline{\beta}_i$,

we get double subtract of the figure volume, bounded with surface $\zeta = \zeta_1 \zeta_2 \zeta_3$ and planes $\zeta_2 = \overline{\beta}_2$, $\zeta_2 = \overline{\beta}_2$, $\zeta_3 = 1$. Therefore we add a single figure volume to this subtract. Case $\overline{\beta}_2 = \overline{\beta}_3 = 0$ will not be analyzed.

In this case, when $\overline{\beta}_3 = 0$ ($i=3$ not controlled), we get:

- $\overline{\beta}_i \le \zeta \le 1$: $y_{01} = \zeta$, $y_{11} = 1$; $y_{02} = \zeta / \zeta_1$, $y_{12} = 1$;

  $y_{03} = \zeta / \zeta_i \zeta_2$, $y_{13} = 1$ ir $\dot{\Phi}(\zeta) = 1 - I$, $I \equiv I_3$;

  $\dot{\Phi}(\zeta) = 1 - I$, $I \equiv I_3$; $\dot{\varphi}(\zeta) = I^* \equiv I_3^*$; $i = 1,2$;

- $\overline{\beta}_i^2 \le \zeta \le \overline{\beta}_i$: $y_{01} = 1$, $y_{11} = \overline{\beta}_i$;

  $y_{02} = \zeta / \zeta_1$, $y_{12} = 1$; $y_{03} = \zeta / \zeta_i \zeta_2$, $y_{13} = 1$ ir

  $\dot{\Phi}(\zeta) = 1 - (I_3 - 2I)$, $I \equiv I_4$;

  $\dot{\varphi}(\zeta) = I_3^* - 2I^*$, $I^* \equiv I_4^*$; $i = 1,2$;

- $0 \le \zeta \le \overline{\beta}_i^2$: $y_{01} = \zeta / \overline{\beta}_i$, $y_{11} = \overline{\beta}_i$;

  $y_{02} = \zeta / \zeta_1$, $y_{12} = \overline{\beta}_i$; $y_{03} = \zeta / \zeta_i \zeta_2$, $y_{13} = 1$ ir

  $\dot{\Phi}(\zeta) = 1 - (I_3 - 2I_4 + I)$, ; $\dot{\varphi}(\zeta) = I_3^* - 2I_4^* + I$;

  $i = 1,2$.

For three-parameter ET analysis we use the most simple $g_i(\theta_i)$ variant with $a_i = b_i = 1$, $i = 1,2,3$. Then

$$I^* = \frac{1}{\beta_{0i}^3} \int_{y_{01}}^{y_{11}} \int_{y_{02}}^{y_{12}} \frac{d\zeta_2 d\zeta_1}{\zeta_1 \zeta_2} = \frac{1}{\beta_{0i}^3} \int_{y_{01}}^{y_{11}} \frac{1}{\zeta_1} \ln \frac{y_{12}}{y_{02}} d\zeta_1.$$

When $\beta_{0i} < 1$ (all $\ell = 3$ parameters are checked), we get:

$\mu_{\tau_1} = 1/2$, $\sigma_i^2 = 1/12$, $\mu = 7/8 = 0{,}875$, $\sigma^2 = 0{,}0214$,

$g(\theta) = \frac{1}{2} \ln^2(1 - \theta)$ and

$$h(\tau) = \frac{1}{2\beta_{0i}^3} \begin{cases} \ln^2(1 - \tau), \ \tau \in (0, \beta_{0i}), \\ \ln^2 \frac{1 - \tau}{\overline{\beta}_i} - 3\ln^2 \frac{1 - \tau}{\overline{\beta}_i^2}, \ \tau \in (\beta_{0i}, 1 - \overline{\beta}_i^2), \\ \ln^2[(1 - \tau) / \overline{\beta}_i^3], \tau \in (1 - \overline{\beta}_i^2, \beta_0), \beta_0 = 1 - \overline{\beta}_i^3. \end{cases}$$

When $\beta_{03} = 1$, o $\beta_{01} = \beta_{02} = \beta_{0i} < 1$ we get:

$$h(\tau / \overline{\beta}_3 = 0) = \frac{1}{2\beta_{0i}^2} \begin{cases} \ln^2(1 - \tau), \ \tau \in (0, \beta_{0i}), \\ \ln^2(1 - \tau) - 2\ln^2[\overline{\beta}_i(1 - \tau)^{-1}], \tau \in (\beta_{0i}, 1 - \overline{\beta}_i^2), \\ 2\ln^2 \overline{\beta}_i, \tau \in (1 - \overline{\beta}_i^2, 1), i = 1,2. \end{cases}$$

For transformed densities $h(\tau)$ approximation $h_\sigma(\tau)$ models (11), (12) in case of $\ell = 2$ are valid, when $\ell = 3$ models mathematical characteristics are used and $h_i(\tau_i)$ – see (5).

Mathematical realizations ($\ell = 3$), when $a_i = b_i = 1$.

$\beta_{0i} = 3/8$, $i = 1,2,3$; $\beta_0 = 0{,}756$

$\mu_{\tau_i} = 3/16$, $\sigma_{\tau_i}^2 = 3/256$, $\mu_\tau = 0{,}4636$, $\sigma_\tau^2 = 0{,}0156$,

$a^* = 4{,}715$, $b^* = 2{,}974$;

$$h(\tau) = 9{,}4815 \begin{cases} \ln^2(1 - \tau), \tau \in (0, 3/8), \\ \ln^2 4{,}096(1 - \tau) - 3\ln^2 2{,}56(1 - \tau), \tau \in (3/8, 39/64), \\ \ln^2 4{,}096(1 - \tau), \ \tau \in (39/64, 0{,}756); \end{cases}$$

$\tau_M = 0{,}506$, $h_M = 3{,}7142$;

Table 1. $\ell=3$, density values, when $a_i=b_i=1$

	1 var.: $\beta_{0i}=3/8$, $i=1,2,3$									
τ	0,05	0,15	0,25	0,375	0,438	0,5	0,609	0,7	0,75	0,756
$h(\tau)$	0,025	0,250	0,785	2,095	2,823	3,139	2,095	0,403	0,005	0
$h_\sigma(\tau)$	0,004	0,185	0,866	2,230	2,776	2,962	2,056	0,515	0,008	0

$$h_\sigma(\tau) = 572{,}93\tau^{3,715}(0{,}756 - \tau)^{1,974}, \quad \tau \in (0, 0{,}756);$$
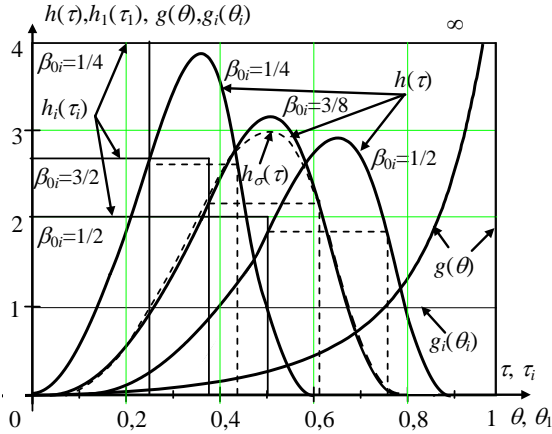$$\tau_{M\sigma} = 0{,}494, \; h_{M\sigma} = 2{,}965.$$

Fig. 4. Three parameters, $i=1,2,3$. Densities $a_i=b_i=1$, $\beta_{01}=\beta_{02}=\beta_{03}$
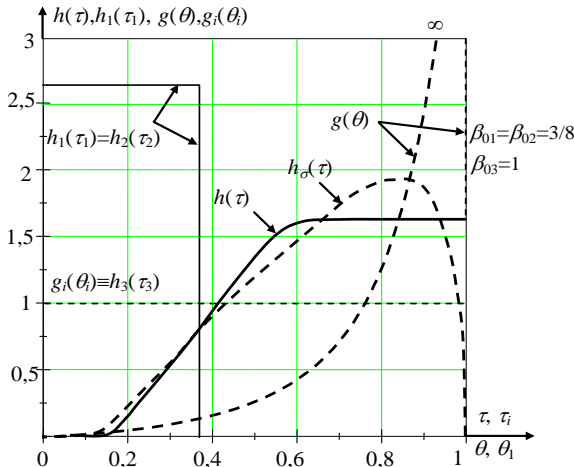

Fig. 5. Three parameters, $i=1,2,3$. Densities $a_i=b_i=1$, $\beta_{03}=1$

Density values are shown in table 1 and densities are graphically shown in Fig. 4, 5. Additionally in Fig. 8 $h(\tau)$ dynamics is shown when $\beta_{0i}(i=1,2,3)$ value varies: when $\beta_{0i}=1/4<3/8$ with $\tau_M=0{,}351$, $h_M=3{,}973$, $\beta_0=0{,}5781$ and $\beta_{0i}=1/2<3/8$ with $\tau_M=0{,}647$, $h_M=2{,}883$, $\beta_0=0{,}875$.

Multiparameter product, $i=1-\ell$.

For engineering analysis, we use $h(\tau)$ approximation for density $h_\sigma(\tau)$ by (11), when accidental values θ_i and θ

mathematical characteristics are calculated by formulas used in "Initial models" and accidental values τ_i and τ – by (3) – (7) expressions (density $g(\theta)$ models – by [9–11]).

When $\beta_{0j}=1$, $j\in(1-i)$, then in τ_j place characteristics, we use $\theta_j \equiv \tau_j$ characteristics.

If $\beta_{0i}<1$, $i=1-\ell$, then $\tau\in(0-\beta_0)$ and

$$\beta_0 = 1 - \prod_{i=1}^{\ell}(1 - \beta_{0i}), \qquad (14)$$

and when $\beta_{0j}=1$, then $\tau\in(0,1)$ and approximation density $h_\sigma(\tau)$ becomes beta density.

Modeling examples

Initial values: $\ell=4$; $a_i=1$, $b_i=\{1,2,3,4\}$; $\beta_{0i} = \{1/2,\; 1/3, 1/4, 1/5\}$, $i=1-4$.

Results: $g_i(\theta_i) = b_i(1-\theta_i)^{b_i-1}$; $\mu_i = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$;
$\sigma_i^2 = \left\{\frac{1}{12}, \frac{1}{18}, \frac{3}{80}, \frac{21}{75}\right\}$; $\mu=0{,}8$, $\sigma^2 = \frac{2}{75}$; $g(\theta) = 4\theta^3$ /see [9]- [11]/
$\mu_{\tau_i} = \left\{\frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}\right\}$; $\sigma_{\tau_i}^2 = \left\{\frac{1}{48}, \frac{1}{189}, \frac{3}{1280}, \frac{2}{1875}\right\}$; $\mu_\tau=0{,}4$,
$\sigma_\tau^2 = 0{,}01764$; $a^* = b^* = 4{,}035$; $\beta_0 = 0{,}8$;
$h_\sigma(\tau) = 147{,}64[\tau(0{,}8 - \tau)]^{3,035}$, $\tau \in (0, 0{,}8)$;
$\tau_{M\sigma} = 0{,}4$, $h_{M\sigma} = 2{,}747$.

1.1 var.: $\beta_{03} = \beta_{04} = 1$; $\beta_{01} = 1/2, \beta_{02} = 1/3$; $\tau \in (0,1)$
$\mu_{\tau_i} = \left\{\frac{1}{4}, \frac{1}{9}, \frac{1}{4}, \frac{1}{5}\right\}$; $\sigma_{\tau_i}^2 = \left\{\frac{1}{48}, \frac{1}{189}, \frac{3}{80}, \frac{2}{75}\right\}$;
$\mu_\tau = 0{,}6$, $\sigma_\tau^2 = 0{,}02578$; $a^* = 4{,}986 \approx 5$, $b^* = 3{,}324$;
$h_\sigma(\tau) = 143{,}5\tau^4(1-\tau)^{2,324}$; $\tau_{M\sigma} = 0{,}634$, $h_{M\sigma} = 2{,}242$.

1.2. var.: $\beta_{01} = \beta_{02} = 1$; $\beta_{03} = 1/4, \beta_{04} = 1/5$; $\tau \in (0,1)$
$\mu_{\tau_i} = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{16}, \frac{1}{25}\right\}$; $\sigma_{\tau_i}^2 = \left\{\frac{1}{12}, \frac{1}{18}, \frac{3}{1280}, \frac{2}{1875}\right\}$;
$\mu_\tau = 0{,}7$, $\sigma_\tau^2 = 0{,}04552$; $a^* = 2{,}53$, $b^* = 1{,}084$;
$h_\sigma(\tau) = 2{,}903\tau^{1,53}(1-\tau)^{0,084}$; $\tau_{M\sigma} = 0{,}948$, $h_{M\sigma} = 2{,}087$

Density values are in table 2, densities – Fig. 6 /$h_i(\tau_i)$ models – by (5).

Table 2. $\ell=4$, density values

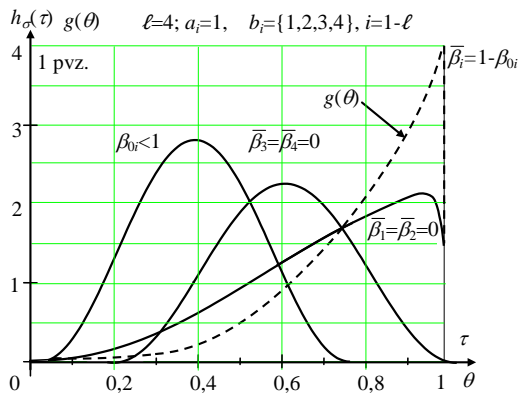|		$a_i=1, b_i=\{1,2,3,4\}$, $i=1-\ell$; $\beta_{0i}=\{1/2, 1/3, 1/4, 1/5\}<1$								
$\beta_{0i}<1$	τ	0,05	0,1	0,15	0,2	0,3	0,4	0,5	0,6	0,7
	$h_\sigma(\tau)$	0,034	0,224	0,611	1,147	2,258	2,747	2,258	1,147	0,224
$\beta_{03}=\beta_{04}=1$	τ	0,1	0,2	0,3	0,4	0,5	0,7	0,8	0,9	0,99
	$h_\sigma(\tau)$	0,011	0,137	0,507	1,121	1,791	2,099	1,396	0,447	0,003
$\beta_{01}=\beta_{02}=1$	$h_\sigma(\tau)$	0,085	0,243	0,447	0,685	0,948	1,520	1,803	2,036	1,942

Fig. 6. Densities when, $\beta_{0i}=\{1/2,1/3,1/4,1/5\}<1$; $\beta_{03}=\beta_{04}=1$; $\beta_{03}=\beta_{04}=1$

Conclusions

1. It is advisable to use beta densities for different parameters defect level characterization in control system modeling, when linear transformation, also as in nonlinear transformation scheme, is used.
2. Linearly transformed defect levels densities by different parameters are simpler than in case of linear transformation, but for all ET, especially when parameters are increasing, it becomes more complicated because of integral intervals disjunction to separate intervals where density models become different.
3. If one of the parameters is not checked in the control system, approximated density $h_\sigma(\tau)$ from approximated beta density becomes beta density, because τ variation interval becomes $0 \leq \tau \leq 1$, when non-checked parameters increase, density $h_\sigma(\tau)$ and also $h(\tau)$ diverges into initial density $g(\theta)$.
4. It is offered to use this modeling technique for engineering projection of control systems with localized repair places.

References

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D. Eidukas, R. Kalnius. Multiparameter Electronic Tools Quality Level Linear Transformation Models // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 9(105). – P. 23–27.

Continuous control main probability characteristic modeling methods for multiparameter electronics tools has been made, when separate independent device parameters defect level probabilistic distributions are "a priori" known. Defected devices flow in control operation is targeted to localized repair operation in this stage, then devices with second type errors goes again into control and "rotates" until all devices are accepted as good. Second type classification errors probabilities (defect device is accepted as good) in control and repair operations are described by one generalized error model, which is used in linear defect level transformation by different parameters. For all device defect level transformation, defect level transformation by different parameters is used. For all defect level transformation, defect levels densities by different parameters combination, is used referencing by transformation model. It is offered to use approximated models instead of exact whole device defect level probabilistic density by different parameters, described by beta law density, because it is the whole model and the exact defect level density is expressed by many different models in every partial interval of integration. This method simplifies modeling procedure, without decreasing engineering analysis accuracy. Ill. 6, bibl. 11, tabl. 2 (in English; abstracts in English and Lithuanian).

D. Eidukas, R. Kalnius. Daugiaparametrių gaminių kokybės lygio tiesinės transformacijos modeliai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 9(105). – P. 23–27.

Sudaryta daugiaparametrių mechatroninių gaminių ištisinės kontrolės pagrindinių tikimybių charakteristikų modeliavimo metodika, kai atskirų nepriklausomų gaminio parametrų defektingumo lygių tikimybiniai skirstiniai yra aprioriškai žinomi. Išbrauktų gaminių srautas kontrolės operacijoje yra nukreipiamas į šio etapo lokalizuotą remonto operaciją, po kurios gaminiai su antros rūšies klaida vėl patenka į kontrolę ir pakartotinai „sukasi" tol, kol kontrolėje visi gaminiai pripažįstami gerais. Antros rūšies klasifikavimo klaidų tikimybės (defektingas gaminys pripažįstamas geru) kontrolės ir remonto operacijose aprašomas vienu apibendrintos klaidos modeliu, kuris taikomas tiesinei defektingumo lygių transformacijai pagal atskirus parametrus. Viso gaminio defektingumo lygio transformacijai taikomas defektingumo lygių tankių pagal atskirus parametrus sujungimas, remiantis transformacijos modeliu. Pasiūlyta vietoj tikslių viso gaminio defektingumo lygio tikimybių tankio transformuotų modelių taikyti aproksimuotus modelius, aprašomus apibendrinto beta dėsnio tankiu, nes tai vientisas modelis, o tikslus defektingumo lygio tankis išreiškiamas daugeliu skirtingų modelių kiekviename integravimo rėžių daliniame intervale. Tai gerokai supaprastina modeliavimo procedūrą, bet nesumažina inžinerinės analizės rezultatų tikslumo. Il. 6, bibl. 11, lent. 2 (anglų kalba; santraukos anglų ir lietuvių k.).