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Design of Quantizers with Huffman Coding for Laplacian Source

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Introduction

Quantization is the main step in the process of analog to digital conversion. Quantizers are included in all modern telecommunication systems. With a good choice of quantizer, we can achieve increase of the signal quality and decrease of the bit-rate. VLC (variable length code) can be used for coding of output levels of quantizer. One of the most popular VLC is Huffman code. This is the optimal code since it gives the shorter average length of codewords, compared to other VLC [1].

In this paper we analyze four types of quantizers for low and moderate bit-rates, for Laplacian source. Firstly, exact analyze of the uniform quantizer is done. Then, nonuniform quantizer is designed using simplified Lloyd-Max algorithm. Construction of the nonuniform quantizers with very small number of levels for exponential source was done in [2]. In [3] the analysis of the uniform quantizer for discretized uniform signal was done. For small bit–rates it is better to use Lloyd-Max algorithm instead of companding technique [4]. After that, we introduce two new types of quantizers: hybrid quantizers type 1 and type 2. Hybrid quantizer type 1 consists of the uniform quantizer in the inner part and of the nonuniform Lloyd-Max quantizer in the outer part. It is combination of the hybrid quantizer proposed in [5] (which consists of the uniform and companding quantizers) and the hybrid quantizer proposed in [6] (which consists of the companding and Lloyd-Max quantizer). Hybrid quantizer type 2 is a modification of the uniform quantizer. It is designed with the aim to keep the same Huffman bit-rate as for the uniform quantizer, but to increase *SQNR* (signal-to-quantization noise ratio). Its decision thresholds are the same as for the uniform quantizer, but its representational levels are determined as for the nonuniform quantizer.

Comparison of these four quantizers is done. Our aim is to determine which quantizer should be used in which situation. Comparison is done for two cases. In the first case VLC is not used and than nonuniform quantizer and hybrid quantizer type 1 are the best. In the second case, VLC Huffman code is used. It is shown that hybrid quantizer type 2 is the best in this case. Therefore, we can conclude that hybrid quantizers proposed in this paper give excellent performances.

Huffman code

Since construction of Huffman code is well known in literature [1], it will not be described here. Output levels of a quantizer can be considered as symbols of a discrete source and they can be coded with Huffman code. For *N*-level quantizer, decision thresholds are denoted with x_i, $i = 0,..., N$ and representation (i.e. output) levels with y_i, $i = 1,..., N$. P_i and d_i denotes probability and the length of Huffman codeword for the output level y_i. Huffman bit-rate *R* and entropy *H* are calculated as:

$$R = \sum_{i=1}^{N} P_i d_i \text{ , [bits/sample],} \quad (1)$$

$$H = \sum_{i=1}^{N} P_i \log_2(1/P_i) \text{ , [bits/sample].} \quad (2)$$

In this paper we will use input signal with zero mean, unit variance Laplacian distribution, whose pdf (probability density function) is given with

$$p(x) = (1/\sqrt{2})\, e^{-\sqrt{2}|x|}, \quad (3)$$

where $p(x)$ is even function, quantizer is symmetrical and we give expression for probabilities P_i only for positive levels

$$P_i = \int_{x_{i-1}}^{x_i} p(x)dx = \frac{1}{2}\left(e^{-\sqrt{2}x_{i-1}} - e^{-\sqrt{2}x_i}\right). \quad (4)$$

Design of quantizer

We will describe design of four types of quantizers: uniform, nonuniform and two types of hybrid quantizer.

Uniform quantizer. Let us consider the uniform quantizer with parameters: *N* – number of levels, $x_{\max}$ – maximal amplitude, $\Delta = 2x_{\max}/N$ – quantization stepsize. The thresholds are defined as $x_i = -x_{\max} + i\Delta$, $i = 0,..., N$ and representation levels with $y_i = -x_{\max} + (i - 1/2)\Delta$, $i = 1,..., N$. The total distortion D is equal to the sum of the granular D_g and the overload D_{ov} distortion, i.e. $D = D_g + D_{ov}$. Granular distortion is defined as $D_g = \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} (x - y_i)^2 p(x)dx$ and overload distortion as $D_{ov} = 2\int_{x_{\max}}^{\infty} (x - y_N)^2 p(x)dx.$ For $p(x)$ given with (3), it is obtained that:

$$D_g = 2e^{-\frac{x_{\max}}{\sqrt{2}}} \mathrm{Sinh}\left(\frac{x_{\max}}{\sqrt{2}}\right)\left(1 + \frac{x_{\max}^2}{N^2} - \frac{\sqrt{2}x_{\max}}{N} \mathrm{Coth}\left(\frac{\sqrt{2}x_{\max}}{N}\right)\right), \quad (5)$$

$$D_{ov} = e^{-\sqrt{2}x_{\max}} \frac{N^2 + \sqrt{2}Nx_{\max} + x_{\max}^2}{N^2}. \quad (6)$$

Approximation $D_g = \frac{\Delta^2}{12}\int_{-x_{\max}}^{x_{\max}} p(x)dx == \frac{\Delta^2}{12}\left(1 - e^{-\sqrt{2}x_{\max}}\right)$ can be used for a large *N*. But, for a small and a moderate *N* (this case is considered in this paper), exact expression (5) should be used. Optimal value of $x_{\max}$ is obtained by minimization of the total distortion D, i.e. by solving the equation $\partial D / \partial x_{\max} = 0$. Optimal value of $x_{\max}$ depends on *N*. Signal-to-quantization noise ratio (*SQNR*) is defined as

$$SQNR[\mathrm{dB}] = 10\log_{10}(1/D). \quad (7)$$

Nonuniform quantizer. Now, the nonuniform quantizer with *N* levels will be considered. In [7], one variant of Lloyd-Max algorithm with effective initialization was analyzed. In this paper we propose another, simplified version of the Lloyd-Max algorithm.

Thresholds of the nonuniform quantizer are denoted with $-\infty = x_0 < x_1 < ... < x_{N/2} = 0 < ... < x_N = +\infty$ and representation levels with $y_1 < y_2 < ... < y_N$, $x_{i-1} < y_i < x_i$. Since $p(x)$ is even function, quantizer is symmetrical, i.e. $x_{N/2-i} = -x_{N/2+i}$, $y_{N/2-i+1} = -y_{N/2+i}$, $i = 1,..., N/2$. Therefore, only the positive thresholds and representation levels will be considered. We define the following parameters:

$$\begin{cases} \delta_i = x_{N-i} - y_{N-i}, \\ \Delta_i = x_{N-i} - x_{N-i-1}. \end{cases} \quad (8)$$

where for $i = 1,..., N/2-1$. It holds that

$$\delta_i = \Delta_{i-1} - \delta_{i-1}. \quad (9)$$

For the optimal quantizer the following two rules (according to Lloyd-Max algorithm) should be fulfilled

Centroid rule:

$$y_i = \frac{\int_{x_{i-1}}^{x_i} xp(x)dx}{\int_{x_{i-1}}^{x_i} p(x)dx},. \quad (10)$$

Nearest neighbor rule

$$x_i = (y_i + y_{i+1})/2. \quad (11)$$

For Laplacian source expression (10) becomes

$$y_{N-i} = x_{N-i-1} + \frac{1}{\sqrt{2}} + \frac{\Delta_i}{1 - e^{\sqrt{2}\Delta_i}}, \ i = 1,..., N/2-1, \quad (12)$$

$$y_N = x_{N-1} + 1/\sqrt{2}. \quad (13)$$

Applying (11) for $i = N-1$ and using (8) and (13), it is obtained that $\delta_1 = 1/\sqrt{2}$. Using (8), we obtain $y_{N-i} = x_{N-i} - \delta_i$. Also, y_{N-i} is defined with (12). Equaling these two expressions for y_{N-i}, the following equation is obtained

$$\Delta_i\left(1 - \frac{1}{1 - e^{\sqrt{2}\Delta_i}}\right) = \delta_i + \frac{1}{\sqrt{2}}, \ i = 1,..., N/2-1, \quad (14)$$

which is similar to the one obtained in [2] for the exponential source. Therefore, knowing δ_i, Δ_i can be found by numerical solving of the equation (14). Starting from $\delta_1 = 1/\sqrt{2}$ and solving equation (14) we obtain Δ_1. Then, using (9) we obtain δ_2. Then, solving (14), Δ_2 is obtained. And so on. Generally, in the *i*-th iteration, δ_i is calculated firstly using (9), and then Δ_i is calculated solving equation (14). The last iteration is for $i = N/2-1$. Also, we calculate $\delta_{N/2}$ using (9). The thresholds and the representation levels in the positive part can be calculated using the following expressions:

$$x_{N/2+1} = \delta_{N/2} + \delta_{N/2-1}; \ y_{N/2+1} = \delta_{N/2}, \quad (15)$$

$$x_{N/2+i} = \delta_{N/2} + 2\sum_{j=1}^{i-1}\delta_{N/2-j} + \delta_{N/2-i}, i = 2,..., N/2-1, \quad (16)$$

$$y_{N/2+i} = \delta_{N/2} + 2\sum_{j=1}^{i-1}\delta_{N/2-j} \ \ i = 2,..., N/2. \quad (17)$$

Therefore, we can see that thresholds and representation levels are completely determined with parameters δ_i. For a given number of levels *N*, we take parameters $\delta_1, \ldots, \delta_{N/2}$, and using (15)–(17), we calculate thresholds and representation levels. These parameters do not depend on *N*, e.g. δ_3 is always the same for any *N*. This is very important since these parameters should be calculated only once and than can be used for any *N*.

Distortion *D* of the nonuniform quantizer is defined as

$$D = 2\sum_{i=N/2+1}^{N-1} \int_{x_{i-1}}^{x_i} (x - y_i)^2 p(x)dx + 2\int_{x_{N-1}}^{\infty} (x - y_N)^2 p(x)dx. \quad (18)$$

Hybrid quantizer type 1. Hybrid quantizer type 1 consists of the uniform quantizer (in the inner part) and the nonuniform quantizer (in the outer part). Number of levels of the hybrid quantizer is $N = N_1 + N_2$, where N_1 and N_2 are number of levels of the uniform and the nonuniform quantizers. t_1 denotes the border amplitude between the uniform and the nonuniform quantizers, i.e. in the interval $(-t_1, t_1)$ the uniform quantizer is used and in the interval $(-\infty, -t_1) \cup (t_1, \infty)$ the nonuniform quantizer is used. $\Delta = 2t_1 / N_1$ is the stepsize of the uniform quantizer. The thresholds of the nonuniform quantizers are denoted with x_i^n, $i = 0,..., N_2$ and the representation levels with y_i^n, $i = 1,..., N_2$. It holds that $x_0^n = -\infty$ and $x_{N_2}^n = \infty$. The nonuniform quantizer is designed using simplified Lloyd-Max algorithm, described in subsection B. Since quantizer is symmetrical, we will give expressions only for positive thresholds and representation levels of the nonuniform quantizer:

$$x^n_{N_2/2+1} = t_1 + \delta_{N_2/2} + \delta_{N_2/2-1}, \quad y^n_{N_2/2+1} = t_1 + \delta_{N_2/2}, \tag{19}$$

$$x^n_{N_2/2+i} = t_1 + \delta_{N_2/2} + 2\sum_{j=1}^{i-1} \delta_{N_2/2-j} + \delta_{N_2/2-i}, \quad i = 2,..., N_2/2-1, \tag{20}$$

$$y^n_{N_2/2+i} = t_1 + \delta_{N_2/2} + 2\sum_{j=1}^{i-1} \delta_{N_2/2-j}, \quad i = 2,..., N_2/2. \tag{21}$$

The total distortion D is the sum of the distortion D_u of the uniform and the distortion D_n of the nonuniform quantizer, i.e. $D = D_u + D_n$. Distortion D_u is calculated using (5), where $x_{\max}$ is replaced with t_1 and N with N_1. Distortion D_n is calculated using (18), where N is replaced with N_2. Distortion D is function of t_1. Therefore, minimizing D (i.e. solving equation $\partial D / \partial t_1 = 0$) the optimal value of t_1 is obtained. This type of the hybrid quantizer is designed to achieve maximal *SQNR* for a given number of levels *N*.

Hybrid quantizer type 2. This hybrid quantizer is obtained by modification of the uniform quantizer described in subsection A. The hybrid quantizer has *N* levels. We firstly design the uniform quantizer with parameters: $N, \Delta, x_{\max}, x_i$ and y_i defined as in subsection A. Huffman bit-rate *R* of the uniform quantizer depends on probabilities P_i, $i = 1,..., N$. We can see from (4) that these probabilities depend on thresholds x_i, $i = 2,..., N-1$. The aim of designing the hybrid quantizer type 2 is: to provide the same the Huffman bit-rate *R* as for the uniform quantizer, but to increase *SQNR*. It will be done in the following way.

Thresholds of the hybrid quantizer will be the same as thresholds for the uniform quantizer, therefore probabilities P_i will stay the same and therefore Huffman bit-rate *R* will stay the same. Actually, we put $x_0 = -\infty$ and $x_N = \infty$, but x_0 and x_N do not have influence on *R*.

Representation levels y_i will be calculated using the centroid rule (12)-(13), which now becomes:

$$y_N = x_{N-1} + 1/\sqrt{2}, \tag{22}$$

$$y_{N-i} = x_{N-i-1} + \frac{1}{\sqrt{2}} + \frac{\Delta}{1 - e^{\sqrt{2}\Delta}}, \quad i = 1,..., N/2-1, \tag{23}$$

$$y_{N/2-i+1} = -y_{N/2+i}, \quad i = 1,..., N/2. \tag{24}$$

y_i is not in the middle of the (x_{i-1}, x_i) interval, but in the centroid of this interval. In that way *SQNR* increases.

Therefore, in the hybrid quantizer type 2, the thresholds are calculated as for the uniform quantizer, but the representation levels are calculated as for the nonuniform quantizer.

Table 1. *SQNR* and Huffman bit-rate *R* for the uniform quantizer and for the hybrid quantizer type 2

N	Uniform	Hybrid type 2,variant 1	Hybrid type 2,variant N/2	*R* [bits/ sample]	R_1 [bits/ sample]
	SQNR [dB]				
4	7.07	7.20	7.53	1.82	2
8	11.44	11.77	12.02	2.48	3
16	15.96	16.39	16.50	3.07	4
24	18.66	19.10	19.17	3.53	5
32	20.60	21.04	21.09	3.82	5
40	22.12	22.56	22.59	4.04	6
48	23.37	23.80	23.83	4.25	6

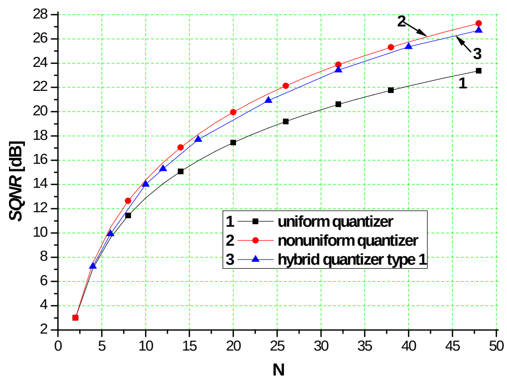

Fig. 1. Dependence of *SQNR* on number of levels *N*

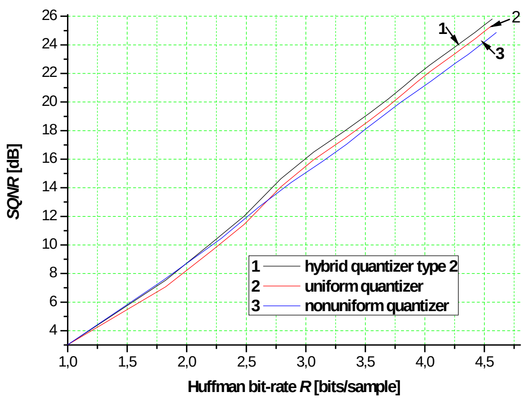

Fig. 2. Dependence of *SQNR* on Huffman bit-rate *R*

The main idea of construction of the hybrid quantizer type 2 was explained above. But, actually, there are $N/2$ variants of this idea. Variant i, $i=1,...,N/2$ is defined in the following way. Output levels $y_1,...,y_i,y_{N-i+1},...,y_N$ are calculated using the centroid rule (22)-(24). All other representation levels $y_{i+1},...,y_{N-i}$ are the same as in the uniform quantizer.

Variant 1 is the simplest, because only two terminal levels are calculated as centroids. Variant 2 is more complex but it gives higher *SQNR*. Variant $N/2$ is the most complex, but it gives the highest *SQNR*. All these variants give the same Huffman bit-rate *R* which is equal to Huffman bit-rate of the uniform quantizer. In Table 1 *SQNR* is presented for the uniform and for two variants of the hybrid quantizer type 2 (variant 1 and variant $N/2$). Also, Huffman bit-rate *R*, which is the same for these quantizers for given *N*, is presented. For the purpose of the comparison, the bit-rate without the Huffman coding $R_1=\lceil \log_2 N \rceil$ ($\lceil x \rceil$ is the nearest integer higher than *x*) is also presented in Table 1. It is obvious that the bit-rate decreases significantly when the Huffman coding is used. We can see that for higher *N* ($N > 16$), *SQNR* for variant 1 is very close to *SQNR* for variant $N/2$, but variant 1 is much simpler. So, for higher *N*, variant 1 should be used.

Comparison between quantizers

In this section, comparison of the previously described quantizers will be done. Two cases is considered.

Case 1) VLC (variable-length code) is not used, i.e. output levels of the quantizer are coded using the codewords with the same length. In this case, the aim of design process is to achieve as high *SQNR* as possible, for given number of levels *N*. In Fig. 1 dependence of *SQNR* on number of levels *N* is presented. We can see that the nonuniform quantizer gives the highest *SQNR*, much higher than in the case of the uniform quantizer. Hybrid quantizer type 1 gives *SQNR* very close to the one achieved with the nonuniform quantizer, but hybrid quantizer is simpler than the nonuniform quantizer since it has smaller number of nonuniform levels. Therefore, when VLC is not used, the best solutions are the nonuniform quantizer and the hybrid quantizer type 1. The hybrid quantizer type 2 is not designed for this case.

Case 2) VLC Huffman code is used. Now, the aim of design process is to simultaneously maximize *SQNR* and minimize the bit-rate *R*, i.e. to achieve as high as possible *SQNR* for some bit-rate *R*. In Fig. 2 dependence of *SQNR* on Huffman bit-rate *R* is presented. We can see that the hybrid quantizer type 2 gives the highest *SQNR* for all *R*. *SQNR* of the nonuniform quantizer is almost equal to *SQNR* of the hybrid type 2 for very small bit-rates ($R < 2$ bits/sample), but for $R > 2.6$ bits/sample, it is the smallest. *SQNR* of the uniform quantizer is the smallest for low bit-rates ($R < 2.6$ bits/sample), but for higher bit-rates it is higher than *SQNR* of the nonuniform quantizer. So, we can conclude that the hybrid quantizer type 2 is the best solution when VLC is used. The hybrid quantizer type 1 is not designed for this case.

We can conclude that hybrid quantizers proposed in this paper have very good performances in situations for which they are designed.

Conclusions

In this paper four types of quantizers were analyzed. Among them, two new types of hybrid quantizer were proposed. Our aim was to find out which quantizer is suitable to be used in different considered situations. Therefore, comparison of these quantizers was done in two cases: when VLC (variable length code) was not used and when VLC Huffman code was used. It was shown that when VLC was not used, the nonuniform and the hybrid type 1 quantizers achieved the best performances (hybrid type 1 is simpler than nonuniform). When Huffman VLC was used, the hybrid quantizer type 2 was the best. Therefore, it was shown that hybrid quantizers proposed in this paper could achieve excellent performances.

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Two new types of hybrid quantizers are proposed for low and moderate bit-rates. They are compared with the uniform and the nonuniform quantizers in two cases: when VLC (variable length code) is not used and when Huffman VLC is used. It is shown that our hybrid quantizers have excellent performances. Ill. 2, bibl. 7, tabl. 1 (in English; abstracts in English and Lithuanian).

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Pasiūlytos dvi mišrių kvantorių rūšys esant mažai ir vidutinei bitų spartai. Palyginimas atliktas dviem atvejais su vienodais ir skirtingais kvantoriais, kai kintamo ilgio Huffmano kodas nebuvo naudojamas ir kai jis buvo naudojamas. Nustatyta, kad mišrieji kvantoriai yra labai našūs. Il. 2, bibl. 7, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).

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