Outage Probability of SSC Receivers in Correlated Rayleigh Fading Channel with Multiple Cochannel Interferers

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Introduction

In the cellular mobile terrestrial and satellite communication systems, increasing of spectral efficiency is one of the main design goals. In such systems, signals from two or more channels and different locations operate at the same frequency interfere due to frequency reuse. This will result in cochannel interference as a general distortion in the performance of wireless communication [1, 2]. So, it is important to analyse how the interference affects well-accepted criteria of performance of wireless systems, such as outage probability, in order to implement practical system which satisfies predetermined minimum performance levels [3].

In addition to interference, fading is also a major obstacle in a mobile wireless environment, by introducing random amplitude and phase distortion to the transmitted signal [2]. An efficient technique to diminish the bad influences of fading and co channel interference is space diversity reception (analyses are presented in [4–5]). The principle of space diversity reception is to provide the receiver with multiple faded replicas of the same information-bearing signal. The goal is to upgrade transmission reliability without increasing transmission power and bandwidth, as well as to increase channel capacity.

Among the well-known diversity schemes, switch-and-stay combining (SSC) is one of the simplest to implement since the receiver processes the information from a single branch only. In fading environments as cellular systems, where the level of the cochannel interference is sufficiently high compared to thermal noise, SSC receiver adheres to one branch as long as the signal-to-interference ratio (SIR) is greater than a specific threshold. Once the SIR drops below this threshold, the receiver switches to the other branch no matter whether its SIR-ratio is greater or less than the threshold.

Most of the recently published papers assume independent fading between the diversity branches and between the cochannel interferers in Rayleigh fading channel. However, independent fading assumes antenna elements to be placed sufficiently apart, which is not always realized in practice due to insufficient antenna spacing when diversity is applied in small terminals. So the correlation between the diversity branches should be taken in consideration [6].

In this paper, we consider correlated SIR-based SSC diversity system with Rayleigh fading channel in the presence of an arbitrary number of multiple co-channel interferences. For proposed system model, through the short mathematical analysis, closed form expressions for probability density function (pdf) and cumulative distribution function (cdf) of the output SIR are derived. Furthermore, effects of the number of multiple interferers and correlation coefficient of desired signal as well as correlation coefficient of interferences to the system performances are shown. To the best author's knowledge, no similar results for the proposed system model have been reported in the literature.

System model

We consider wireless communication system with dual-branch SSC receiver operating over the Rayleigh fading channels (Fig. 1).

The envelope of desired signal at the *i*-th input branch (*i*=1,2) has Rayleigh pdf [7]

$$p_{R_i}(R_i) = 2\left(\frac{1}{\Omega_d}\right)R_i \times e^{-\frac{R_i^2}{\Omega_d}}, \qquad (1)$$

where $\Omega_d = \mathrm{E}(R_i^2)$ is the average desired signal power at both branches.

The pdf of *M* interferers is also Rayleigh

$$p_{r_i}(r_i) = 2\left(\frac{1}{\Omega_{ck}}\right)^M \frac{r_i^{2M-1}}{\Gamma(M)} \times e^{-\frac{r_i^2}{\Omega_{ck}}}, \qquad (2)$$

where Ω_{ck} is the average power of the single co-channel interference ($\Omega_c = M\Omega_{ck}, k = 1,2,\ldots,M$ is the total average interference signal, $\Omega_c = E\left(r_i^2\right)$).
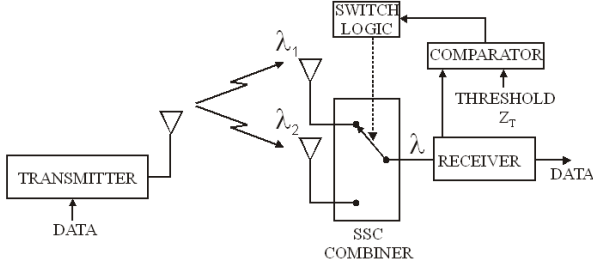

Fig. 1. Block scheme of the system

The pdf and cdf of instantaneous SIR, defined as $\lambda = \dfrac{R^2}{r^2}$, can be evaluated [8], respectively:

$$p_\lambda(\lambda) = \frac{1}{2\sqrt{\lambda}} \int_0^\infty r p_R\left(r\sqrt{\lambda}\right) p_r(r) dr = \frac{S^M M}{(\lambda + S)^{M+1}}, \quad (3)$$

$$F_\lambda(\lambda) = \int_0^\lambda p_\lambda(u) du = M \times B\left(\frac{\lambda}{\lambda + S}, 1, M\right), \quad (4)$$

where $S = \dfrac{\Omega_d}{\Omega_{ck}}$ is the average SIR at both input branches and $B(z,a,b)$ being the incomplete Beta function [9, eq. 8.391].

When diversity system is applied on small terminals with multiple antennas, correlation arises between branches. The following analysis is based on the results available in literature for the Nakagami-m constant correlation model [10].

Due to insufficient antennas spacing, both desired and interfering signal envelopes experience correlated Rayleigh fading with joint pdfs, respectively

$$p(R_1, R_2) = \left(1 - \sqrt{\rho_d}\right) \sum_{k_1,k_2=0}^\infty \frac{2^2 \Gamma(k_1+k_2+1) \rho_d^{\frac{k_1+k_2}{2}}}{\Gamma(k_1+1)\Gamma(k_2+1)k_1!k_2!} \times$$

$$\times \left(\frac{1}{1+\sqrt{\rho_d}}\right)^{k_1+k_2+1} \left(\frac{1}{\Omega_d\left(1-\sqrt{\rho_d}\right)}\right)^{k_1+k_2+2} \times$$

$$\times R_1^{2k_1+1} R_2^{2k_2+1} e^{-\frac{R_1^2 + R_2^2}{\Omega_d\left(1-\sqrt{\rho_d}\right)}}, \quad (5)$$

$$p(r_1, r_2) = \frac{\left(1-\sqrt{\rho_c}\right)^M}{\Gamma(M)} \sum_{l_1,l_2=0}^\infty \frac{2^2 \Gamma(l_1+l_2+1) \rho_c^{\frac{l_1+l_2}{2}}}{\Gamma(l_1+1)\Gamma(l_2+1)l_1!l_2!} \times$$

$$\times r_1^{2M+2l_1-1} r_2^{2M+2l_2-1} e^{-\frac{M\left(r_1^2+r_2^2\right)}{\Omega_c\left(1-\sqrt{\rho_c}\right)}} \times$$

$$\times \left(\frac{1}{1+\sqrt{\rho_c}}\right)^{l_1+l_2+M} \left(\frac{M}{\Omega_c\left(1-\sqrt{\rho_c}\right)}\right)^{l_1+l_2+2M}. \quad (6)$$

So, the joint pdf of instantaneous SIRs, denoted by $\lambda_1 = \dfrac{R_1^2}{r_1^2}$ and $\lambda_2 = \dfrac{R_2^2}{r_2^2}$, at two input branches can be found by using [8]

$$p_{\lambda_1\lambda_2}(\lambda_1, \lambda_2) = \frac{1}{4\sqrt{\lambda_1,\lambda_2}} \int_0^\infty\int_0^\infty p_{R_1R_2}\left(r_1\sqrt{\lambda_1}, r_2\sqrt{\lambda_2}\right) \times$$

$$\times p_{r_1 r_2}(r_1, r_2) r_1 r_2 dr_1 dr_2 \quad .(7)$$

Substituting (5) and (6) in (7), we get

$$p_{\lambda_1\lambda_2}(\lambda_1, \lambda_2) = \sum_{k_1,k_2,l_1,l_2=0}^\infty A \times \left(S \frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}}\right)^{2M+l_1+l_2} \times$$

$$\times \sum_{i=1}^2 \frac{\lambda_i^{k_i}}{\left(\lambda_i\left(1-\sqrt{\rho_c}\right) + S\left(1-\sqrt{\rho_d}\right)\right)^{M+k_i+l_i+1}}. \quad (8)$$

when

$$A = \left(1-\sqrt{\rho_d}\right)\left(1-\sqrt{\rho_c}\right)^M \frac{\Gamma(k_1+k_2+1)\Gamma(l_1+l_2+M)}{\Gamma(M)\Gamma(k_1+1)\Gamma(k_2+1)} \times$$

$$\times \frac{\Gamma(M+k_1+l_1+1)\Gamma(M+k_2+l_2+1)}{\Gamma(l_1+M)\Gamma(l_2+M)k_1!k_2!l_1!l_2!} \rho_d^{\frac{k_1+k_2}{2}} \rho_c^{\frac{l_1+l_2}{2}} \times$$

$$\times \left(\frac{1}{1+\sqrt{\rho_d}}\right)^{k_1+k_2+1} \left(\frac{1}{1+\sqrt{\rho_c}}\right)^{l_1+l_2+M}. \quad (9)$$

SSC receiver

The SSC diversity technique is the less complex combining technique compared to the other space diversity combining methods. Because of its low complexity implementation requirements, SSC diversity systems are frequently used. The SSC combiner processes one branch unless the instantaneous envelope of that branch falls below the threshold z_T when the combiner switches the treated branch with the other one. So, the cdf of instantaneous SIR can be defined as

$$F_{SSC}(\lambda) = \Pr\left(z_T \le \lambda_1 \le \lambda\right) + \Pr\left(\lambda_2 < z_T \wedge \lambda_1 \le \lambda\right). \quad (10)$$

The pdf of instantaneous SIR at the output of SSC, by differentiating equation (10), is given by [11, eq. 9]

$$p_{SSC}(\lambda) = \begin{cases} f_{SSC}(\lambda), & \lambda \le z_T, \\ f_{SSC}(\lambda) + p_\lambda(\lambda), & \lambda > z_T, \end{cases} \quad (11)$$

where $p_\lambda(\lambda)$ is defined as (3) and $f_{SSC}(\lambda)$ is defined as

$$f_{SSC}(\lambda) = \int_0^{z_T} p_{\lambda_1\lambda_2}(\lambda, \lambda_2) d\lambda_2 =$$

$$= \sum_{k_1,k_2,l_1,l_2=0}^\infty A \times \left(S \frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}}\right)^{M+l_1} \frac{\lambda^{k_1}}{\left(\lambda + S \frac{\left(1-\sqrt{\rho_d}\right)}{\left(1-\sqrt{\rho_c}\right)}\right)^{M+k_1+l_1+1}} \times$$

$$\times B\left(\frac{z_T}{z_T + S\frac{\left(1-\sqrt{\rho_d}\right)}{\left(1-\sqrt{\rho_c}\right)}}, k_2+1, M+l_2\right) \quad (12)$$

The cdf of instantaneous SIR at the output of SSC can be expressed as [10, eq. 9]

$$F_{SSC}(\lambda) = \begin{cases} F_{\lambda_1\lambda_2}(\lambda, z_T), & \lambda \le z_T, \\ F_\lambda(\lambda) - F_\lambda(z_T) + F_{\lambda_1\lambda_2}(\lambda, z_T), & \lambda > z_T, \end{cases} \quad (13)$$

where $F_\lambda(\lambda)$ is given by (4) and $F_{\lambda_1\lambda_2}(\lambda, z_T)$ is given by

$$F_{\lambda_1\lambda_2}(\lambda, z_T) = \int_0^\lambda \int_0^{z_T} p_{\lambda_1\lambda_2}(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 =$$

$$= \sum_{k_1,k_2,l_1,l_2=0}^{\infty} A \times B\left(\frac{\lambda}{\lambda + S\frac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}}, k_1+1, M+l_1\right) \times$$

$$\times B\left(\frac{z_T}{z_T + S\frac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}}, k_2+1, M+l_2\right). \quad (14)$$

Outage probability

Outage probability is one of the relevant performance measures for diversity systems operating in fading environments, defined as the probability that the resulting SIR falls below a given outage threshold q, also known as a protection ratio. This performance is an important consideration in the design of cellular and mobile systems. Outage probability can be evaluated as

$$P_{out} = P_R(\lambda < q) = \int_0^q p_\lambda(t)dt = F_\lambda(q). \quad (15)$$

Fig. 2 shows the outage probability, P_{out}, versus the normalized switching threshold z_T/S for different number of present interferers. It is obvious that there is an optimal threshold that minimizes outage probability. For picked normalized outage threshold q/S=-10dB, the normalized switching threshold is also z_T/S=-10dB. So, for this threshold value, SSC combining can be observed as SC. This figure also shows that the higher number of existed interferers degrades the performance gain.

The outage probability, as a function of normalized parameter S/q for various correlation coefficient values of desired and interfering signals, is shown in Fig.3. We observe the SSC receiver as SC one ($q=z_T$). When the correlation coefficient of desired signal ρ_d increases, outage probability also increases, as expected. The same influence of coefficient ρ_d on P_{out} is in environment with one, two and three interferers. It is interesting to note that, when ρ_c increases (the correlation between interferers becomes weaker), the outage probability improves as number of interferers increases.

In order to verify analytical results, Monte Carlo simulations were performed and for comparison purposes were included in Fig. 2 and Fig. 3 It is evident that analytical results are coincide with simulation ones. The samples of correlated Rayleigh fading envelopes were generated by using the generation algorithm presented in [12]. The P_{out} values are estimated on the basis of 10^8 generated samples.
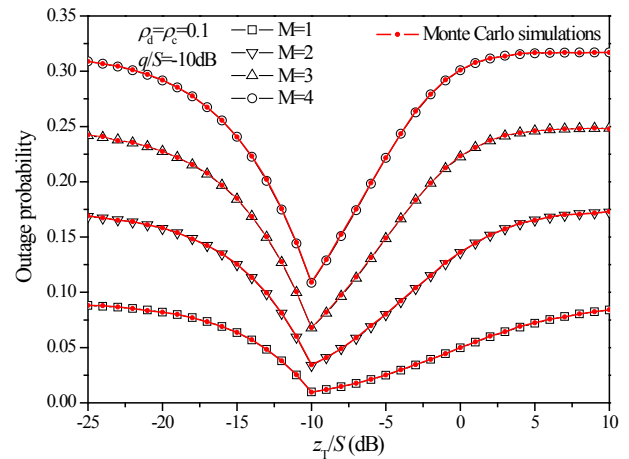

Fig. 2. Outage probability versus normalized switching threshold in environment with M=1,2,3,4 interferers
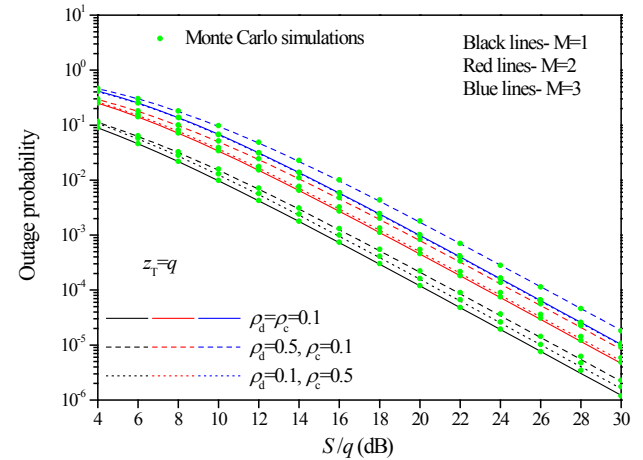

Fig. 3. Outage probability versus normalized average input SIR for various correlation coefficient values

Conclusions

In this paper, the performance analysis of dual branch switch and stay combining (SSC) receivers operating over correlated and identically distributed Rayleigh fading channels with M cochannel interferers was presented. Assuming this diversity technique, infinite series expressions were derived for pdf and cdf of the output SIR. The proposed analysis was used for evaluating outage probability. The effects of branch correlation and co-channel interferers were also considered and numerically presented. This analysis is important in design of a cellular mobile system and presented results can be used to determine optimal values of system parameters in order to achieve reasonable influence of interferers on the outage probability.

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In this paper switch-and-stay combining (SSC) receivers over the Rayleigh fading channels are observed. The case of interference-limited system in the environment with multiple cochannel interferers is considered. Proposed analysis gives new formula for evaluating the outage probability. Based on analytical results some numerical are obtained pointing out the influence of interferers and level of correlation to the outage probability. Monte Carlo simulations are also presented in order to confirm the validity and accuracy of the derived analytical expressions. Ill. 3, bibl. 12 (in English; abstracts in English and Lithuanian).

I. Petrovic, M. Stefanovic, J. Anastasov, S. Panic, G. Djordjevic, D. Milic. SSC imtuvų išjungimo tikimybė derinant Reilėjaus kanalų nykimą veikiant gretimų kanalų trikdžiams // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 10(106). – P. 67–70.

Apžvelgti SSC imtuvai su nykstančiais Reilėjaus kanalais veikiant gretimų kanalų trikdžiams. Nauja išjungimo tikimybės išraiška gauta atliekant pasiūlytąją analizę. Išjungimo tikimybės vertė apskaičiuojama įvertinant trikdžių įtaką ir koreliacijos laipsnį. Atliktas Monte Carlo modeliavimas, pagrindžiantis analitinių išraiškų tikslumą. Il. 3, bibl. 12 (anglų kalba; santraukos anglų ir lietuvių k.).