

# Power System Topology Verification with Use of Reduced Unbalance Indices

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**Abstract**—Knowledge about a correct topology model is required for monitoring, control and analyses of a power system. There are many papers which deal with the examination of correctness of the mentioned model. Among approaches to solving a problem of examination of that correctness there is one assuming utilization of raw analogue measurement data. That approach does not require complex numerical computations. It is utilized in the paper. In the paper, a next stage of investigation on topology verification based on utilization of so-called unbalance indices is presented. In that stage, the original idea of reduced unbalance indices, being a modification of the earlier-mentioned unbalance indices, is introduced. Utilization of the new unbalance indices increases possibility of taking verification decision when there are multiple topology errors and decreases sensitivity of topology verification to a lack of a full set of measurement data. The topology verification can be performed locally (e.g. in a substation what is often now considered) as it is in the case of earlier-defined unbalance indices. Utilization of the proposed method is shown on an example of topology verification of the IEEE 14-bus test system.

**Index Terms**—Power system, topology error, topology model, verification.

## I. INTRODUCTION

A topology model of a Power System (PS), i.e. a description of the physical connections in PS [1] is the important component of knowledge required from the view point of monitoring, control or analyses of PS. A prerequisite for the usefulness of a PS topology model is its correctness. There are many papers which deal with the examination of correctness of the PS topology model [2]–[7]. Generally, three different approaches to solving the considered problem can be distinguished [3]. Those approaches are based on utilization of: (i) raw analogue measurement data, (ii) results of static State Estimation (SE), (iii) results of dynamic SE. It should be underlined, that in the last two approaches, results of the relatively complex PS SE calculations are required. When results of the static SE are utilized, a process of detection and identification of Topology Errors (TEs) is recursive in nature. The characteristic feature of the considered approach is time

consuming. In the third approach, innovations are used. The innovation is defined as difference between telemetered and predicted quantity. The innovations can be calculated in a pre-filtering stage of a dynamic SE. In the case of that approach, the time consumed for identification of TEs is less than in the case, which is earlier considered.

The approach assuming utilization of raw analogue measurement data allows avoiding heavy numerical computation. The approach leads to rule-based methods utilizing local information. The paper deals with that approach. In the paper, a next stage of investigation on Topology Verification (TV) based on utilization of so-called Unbalance Indices (UIs) is presented. The idea of utilization of UIs for the examination of correctness of the PS topology model is described in [8] and [9]. That idea is used for a design of a multi-agent system for PS TV [10] (and other papers of the authors).

The mentioned idea of PS TV based on utilization of UIs assumes that: (i) one considers UIs defined with use of exact relationships among active and reactive power flows in a power network, (ii) measurement data of active and reactive power flows on both ends of each branch, which is included in a topology model, and modules of voltages at each node of a power network are accessible. The shortcomings of the mentioned idea is inefficiency when there are multiple TEs and requirements of the full set of measurement data.

The aim of the paper is to present the idea of PS TV based on utilization of modified UIs, which has not the earlier-presented shortcomings.

## II. IDEA OF TV WITH UNBALANCE INDICES

The key assumption, which is adopted in the idea of TV with the use of UIs [8], [9], is that all branches of PS are taken into account independently of their inclusion in the topology model. For each node and branch, UIs are calculated. For nodes, they are as follows:

$$\begin{cases} W_{Pk} = \sum_{i \in I_k} d_{Pki}, \\ W_{Qk} = \sum_{i \in I_k} d_{Qki}, \end{cases} \quad (1)$$

where  $k$  – a number of the considered node;  $i$  – a number of the node which is connected with the node  $k$  by the branch  $k$

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-  $i$ ;  $I_k$  – the set of nodes connected with the node  $k$  in a topology model;  $d_{Pki}$ ,  $d_{Qki}$  – measurement data of active and reactive power flows, respectively, at the node  $k$  on the branch  $k-i$ .

For branches, UIs are following:

$$W_{Pxy} = -W_{Px} - W_{Py} + R_{kl}W_{xy}^u, \quad (2)$$

$$W_{Qxy} = -W_{Qx} - W_{Qy} + X_{kl}W_{xy}^u - W_{xy}^V, \quad (3)$$

where

$$W_{xy}^u = \left[ W_{Px}^2 + (W_{Qx} + B_{kl}V_x^2)^2 \right] / V_x^2, \quad (4)$$

$$W_{xy}^V = B_{kl}(V_x^2 + V_y^2), \quad (5)$$

here  $x, y \in \{k, l\}$ ,  $x \neq y$ ,  $k, l$  – numbers of the terminal nodes of the considered branch;  $V_k, V_l$  – voltage magnitudes at the nodes  $k$  and  $l$  respectively;  $R_{kl}, X_{kl}, B_{kl}$  – parameters of the model of the branch  $k-l$  (Fig. 1).

For different cases of inclusion of the distinguished branch  $k-l$  to a PS topology model, values of UIs for nodes and branches create characteristic sets. Analysing the formulas for UIs, we can ascertain, that:

1. When there are no incorrectly modelled branches connected with the node  $k$  as well as with the node  $l$  then UIs for the branch  $k-l$  and also for other branches connected with the nodes  $k$  and  $l$  are relatively near to zero;
2. When among the branches connected with the nodes  $k$  and  $l$  only the branch  $k-l$  is incorrectly modelled then UIs for the branch  $k-l$  are equal to zero and for other branches connected with the nodes  $k$  and  $l$  are different from zero;
3. When among the branches connected with the nodes  $k$  and  $l$  not only the branch  $k-l$  is incorrectly modelled then for all mentioned branches UIs are not equal to zero.

The ascertainments 1–3 are utilized for taking verification decisions.

Taking a decision about correctness of modelling a branch in a PS topology model is possible from the view point of each terminal node of this branch. A final decision is taken on the basis of two decisions. It is a decision “Branch is correctly modelled” (*Decision C*) or “Branch is incorrectly modelled” (*Decision B*), if the mentioned decisions are not contradictory, i.e. both decisions are the same, or one of them is the neutral decision (*Decision N*). *Decision N* stands for suspension from *Decision C* or *Decision B*. If a possible verification decisions are contradictory or each of both the decisions is *Decision N*, then the final decision is *Decision N*.

### III. REDUCED UNBALANCE INDICES

In each of cases distinguished in Section II, UIs for correctly modelled branches connected with the same node are different.

Let us assume that two branches  $k-m$  and  $k-n$  are correctly modelled. Let us consider UIs for these branches,

which are as follows:

$$W_{Pkm} = \sum_{i \in I_k^a} P_{ki} + \sum_{i \in I_m^b} P_{mi} + R_{km}W_{xy}^u \quad x, y \in \{k, m\}, \quad (6)$$

$$W_{Qkm} = \sum_{i \in I_k^a} Q_{ki} + \sum_{i \in I_m^b} Q_{mi} + X_{km}W_{xy}^u - W_{xy}^V \quad x, y \in \{k, m\}, \quad (7)$$

$$W_{Pkn} = \sum_{i \in I_k^a} P_{ki} + \sum_{i \in I_n^b} P_{ni} + R_{kn}W_{xy}^u \quad x, y \in \{k, n\}, \quad (8)$$

$$W_{Qkn} = \sum_{i \in I_k^a} Q_{ki} + \sum_{i \in I_n^b} Q_{ni} + X_{kn}W_{xy}^u - W_{xy}^V \quad x, y \in \{k, n\}, \quad (9)$$

where  $P_{zi}$ ,  $Q_{zi}$  – active and reactive power flows, respectively, at the node  $z$  on the branch  $z-i$ ,  $I_z^b$   $z \in \{k, m, n\}$  – the set of nodes which are connected with the node  $z$  in PS but are not connected with the node  $z$  in the model.

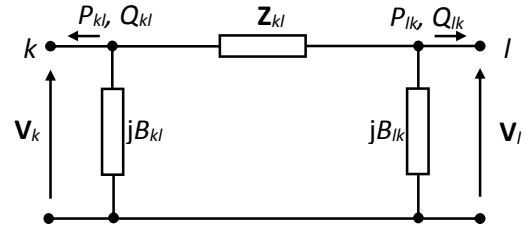


Fig. 1. The assumed model of the branch.  $Z_{kl} = R_{kl} + jX_{kl}$ ,  $B_{kl} = B_{lk} = B$ .  $B$  is a half of the capacitive susceptance of the branch.

UIs for the branches  $k-m$  and  $k-n$  are different due to different values of parameters of these branches, i.e. different values of  $R_{km}, X_{km}, B_{km}$  and  $R_{kn}, X_{kn}, B_{kn}$  and different values of UIs for the nodes  $m$  and  $n$ , respectively. If for a branch  $k-y$ , UIs are defined as follows:

$$\begin{cases} W_{Pky}^r = -W_{Pk} - W_{Py}, \\ W_{Qky}^r = -W_{Qk} - W_{Qy}, \end{cases} \quad (10)$$

$$\begin{cases} W_{Py} = 0, \\ W_{Qy} = 0, \end{cases} \quad (11)$$

when  $y \in \{m, n\}$ , then  $W_{Pkm}^r = W_{Pkn}^r$ ,  $W_{Qkm}^r = W_{Qkn}^r$ .

UIs defined with the use of (10) will be further called as Reduced (Branch) Unbalance Indices (RBUIs). If the conditions (11) are not fulfilled, then  $W_{Pkm}^r \neq W_{Pkn}^r$ ,  $W_{Qkm}^r \neq W_{Qkn}^r$  even if the branches  $k-m$  and  $k-n$  are correctly modelled.

If the branch  $k-l$  is incorrectly modelled, then its RBUIs are as follows:

$$W_{Pkl}^r = -W_{Pk} - W_{Pl} = P_{kl} + P_{lk} + \sum_{i \in I_k^{b-}} P_{ki} + \sum_{i \in I_l^{b-}} P_{li}, \quad (12)$$

$$W_{Qkl}^r = -W_{Qk} - W_{Ql} = Q_{kl} + Q_{lk} + \sum_{i \in I_k^{b-}} Q_{ki} + \sum_{i \in I_l^{b-}} Q_{li}, \quad (13)$$

where  $I_k^{b-} = I_k^b - \{l\}$ ,  $I_l^{b-} = I_l^b - \{k\}$ .

If only the branch  $k-l$  is incorrectly modelled among the branches incident to the nodes  $k$  and  $l$ , then RBUIs for the branch  $k-l$  from the view point of the node  $k$  are as follows:  $W_{Pkl}^r = P_{kl} + P_{lk}$ ,  $W_{Qkl}^r = Q_{kl} + Q_{lk}$ . For other branches incident

to the node  $k$  we have:  $W_{P_{kl}}^r = P_{kl}$ ,  $W_{Q_{kl}}^r = Q_{kl}$ .

The analogical formulas for RBUIs can be written from the view point of the node  $l$  for the branch  $k-l$  and for other branches incident to the node  $l$ .

Considering real measurement data, we should take into account measurement noise. In this situation, instead of the conditions (11), the following conditions should be tested:

$$\begin{cases} W_{P_y} \in [-u_{P_y}, u_{P_y}], \\ W_{Q_y} \in [-u_{Q_y}, u_{Q_y}], \end{cases} \quad (14)$$

where  $y \in \{m, n\}$ ,  $p_y$ ,  $q_y$  – positive constants dependent on characteristics of measurement noise burdening measurement data of active and reactive power flows, respectively.

If the conditions (14) are fulfilled, then we say that the UIs  $W_{P_y}$ ,  $W_{Q_y}$  are “zero” ones and zero is inserted into (10) instead of real values of  $W_{P_y}$ ,  $W_{Q_y}$ .

#### IV. PROPERTIES OF THE DESCRIPTION OF A PS TOPOLOGY MODEL WITH THE USE OF RBUIs

Taking into account RBUIs, one can ascertain that:

1. When UIs for all nodes are “zero” ones, and in consequence, RBUIs for all branches are equal to zero then all branches are correctly modelled;
2. When for more than one branch having the same terminal node, RBUIs are the same and for each of these branches only for common terminal node, UIs are not “zero” ones, then these branches are correctly modelled;
3. When only for one branch, RBUIs are different from RBUIs calculated for other branches connected with the same node, then the mentioned branch is incorrectly modelled;
4. When there is more than one branch with RBUIs, which are different from RBUIs calculated for other branches (at least two branches with the same RBUIs) connected with the same node, then the mentioned branches may (but need not) be incorrectly modelled.

#### V. PRINCIPLE OF THE METHOD

In this section, the principle of the method for PS TV is presented under assumption that only UIs for active power are taken into account. In this situation, the method is simpler. It is easier to assess the power flows at the ends of branches, what is essential element of the method.

The proposed method for PS TV comprises the following steps:

1. For each node in PS, calculate  $W_{P_i}$ , where  $i$  – a number of the node;
2. Define the set  $I_u$ , i.e. a set of numbers of nodes in PS, for which UIs are not “zero” ones. If the set  $I_u$  is empty, then stop;
3. Define the set  $I_B$ , i.e. a set of branches, from which each one have at least one terminal node belonging to the set  $I_u$ ;
4. Calculate RBUI for each branch from the set  $I_B$ ;
5. Take *Decision C* for each branch from the set  $I_B$ , satisfying the condition that there exist other branches

with the same values of RBUIs, which are connected with its terminal node;

6. Take *Decision B* for each branch from the set  $I_B$ , satisfying the condition, that the mentioned branch is the only one among branches connected with the considered node, for which *Decision C* has not been taken earlier;
7. Change the state of switching of each branch for which *Decision B* has been taken. If such a branch is the branch  $k-l$  and *Decision B* has been taken from the view point of the node  $k$ , then assess power flows on this branch as:  $P_{kl} = -W_{P_k}$  and  $P_{lk} = -P_{kl}$ . Change  $W_{P_k}$  and  $W_{P_l}$  as follows:  $W_{P_k} = 0$ ,  $W_{P_l} = W_{P_l}^* + P_{lk}$ , where  $W_{P_l}^*$  characterizes the situation before *Decision B* is taken;
8. Go to the step 2.

#### VI. PROBLEM OF LACK OF MEASUREMENT DATA FOR SOME OF BRANCHES

A branch, for which there are no measurement data, can be considered as the branch which is not included in a PS topology model. Let us assume that this is the branch  $k-l$ . In fact, if this branch is in operation in PS and only this branch is incorrectly modelled, then UIs for terminal nodes of the branch are different from zero and for this branch, UIs are equal to zero. In this situation, for the mentioned branch, *Decision B* can be taken. If the considered branch is off in PS, then for terminal nodes of the branch, UIs are equal to zero and for the branch, UIs are near to zero. In the verification process, for that branch, *Decision C* is taken.

Other situation is when two or more branches, for which there are no measurement data, are connected with at least one of the terminal nodes of the branch  $k-l$  and these branches are in operation in PS. In this case, UIs for the branch  $k-l$  are not equal to zero. UIs do not give the basis for taking any verification decision. Such the basis for taking a verification decision can take place when RBUIs are utilized. If only there is a node, for which the ascertainment 3 from Section IV is valid, for a suitable branch, *Decision B* is taken. Such possibility does not exist when approach based only on utilization of UIs is considered.

#### VII. CASE STUDY

In the case studies, the IEEE 14-bus test system [11], which in this section is presented, and Polish Power System has been considered. For the test system, it is assumed, that there are no measurement data for the branches 3-4 and 4-9 [11]. The nodal and reduced branch UIs for active power are calculated. UIs for a part of the test system, which is essential from the view point of the performed TV, are collected in Table I. That part of the test system includes nodes: 3, 4, 9 and all their neighbouring ones. Only in the mentioned part of the test system, there are nodes and branches with UIs which are not equal to zero. The set  $I_u$  is as follows: {3, 4, 9}. Also we can state, that:  $I_B = \{3-2, 3-4, 4-2, 4-5, 4-7, 4-9, 9-7, 9-10, 9-14\}$ . In Table I, only UIs for active power are shown, because for this power, when there is a power flow at one end of a branch, it is relatively easy to assess the power flow at the opposed end of this branch.

Analysing values of RBUIs, one can notice that for all

branches connected with the nodes 3, 4, and 9, RBUIs significantly differ from zero. On the basis of Table I, one can take *Decision C* for the following branches from the set  $I_B$ : 4-2, 4-5, 4-7, 9-7, 9-10, 9-14, and *Decision B* for the branch 9-4. In this situation, the branch 9-4 is considered as switched on in the topology model of the test system.

Taking into account results of verification of correctness of modelling branches connected with the node 9 and value of UI for this node, one can give the following estimates of the power flows at the ends of the branch 4-9:  $P_{94} = 16.1$  MW and  $P_{49} \approx -16.1$  MW.

In the iteration 2, new values of UIs for the node 4 and 9 are observed. The UI  $W_{P9}$  is the “zero” one and the set  $I_u$  is following {3, 4}. Now, new values of RBUIs for the branches 4-2, 4-3, 4-5, 4-7 and 4-9 enable also to take *Decision C* for the branch 4-9 and *Decision B* for the branch 4-3. The basis for such decisions is the fact, that values of RBUIs for the branches 4-2, 4-5, 4-7 and 4-9 are the same and the value of RBUI for the branch 4-3 is different from the earlier-mentioned ones. Now, we can state that the branch 4-3 should be switched on in the topology model of the test system and that:  $P_{43} = -23.7$  MW,  $P_{34} \approx 23.7$  MW. UIs for the nodes 3 and 4 are 0 MW and 0.4 MW, respectively, and therefore  $I_u$  becomes empty. In consequence, the TV process ends.

TABLE I. DECISIONS ON CORRECTNESS OF MODELLING CONNECTIONS IN THE TEST SYSTEM.

Nodes		Iteration 1				Iteration 2				Iteration 3			
		$I_B$		Nodal unbalance indices, MW	Branch unbalance index, MW	Decision	$I_B$		Nodal unbalance indices, MW	Branch unbalance index, MW	Decision	$I_B$	
		$W_{P_i}$	$W_{P_j}$				$W_{P_i}$	$W_{P_j}$				$W_{P_i}$	$W_{P_j}$
$i$	$j$												
2	1	0	0	0		0	0				0	0	
	3	0	-23.3	23.3		0	-23.3	23.3			0	0.4	
	4	0	39.8	-39.8		0	23.7	-23.7			0	0	
	5	0	0	0		0	0				0	0	
3	2	<del>X</del>	-23.3	0	23.3	<del>X</del>	-23.3	0	23.3		0.4	0	
	4	<del>X</del>	-23.3	39.8	-16.5	<del>X</del>	-23.3	23.7	0.4		0.4	0	
4	2	<del>X</del>	39.8	0	-39.8	C	<del>X</del>	23.7	0	-23.7	C	0	0
	3	<del>X</del>	39.8	-23.3	-16.5	C	<del>X</del>	23.7	-23.3	0.4	B	0	0.4
	5	<del>X</del>	39.8	0	-39.8	C	<del>X</del>	23.7	0	-23.7	C	0	0
	7	<del>X</del>	39.8	0	-39.8	C	<del>X</del>	23.7	0	-23.7	C	0	0
5	9	<del>X</del>	39.8	-16.1	-23.7		<del>X</del>	23.7	0	-23.7	C	0	0
	1	0	0	0		0	0				0	0	
	2	0	0	0		0	0				0	0	
	4	0	39.8	-39.8		0	23.7				0	0	
7	6	0	0	0		0	0				0	0	
	4	0	39.8	-39.8		0	23.7				0	0	
	8	0	0	0		0	0				0	0	
9	9	0	-16.1	16.1		0	0				0	0	
	4	<del>X</del>	-16.1	39.8	-23.7	B	0	23.7			0	0	
	7	<del>X</del>	-16.1	0	16.1	C	0	0			0	0	
	10	<del>X</del>	-16.1	0	16.1	C	0	0			0	0	
10	14	<del>X</del>	-16.1	0	16.1	C	0	0			0	0	
	9	0	-16.1	16.1		0	0				0	0	
14	11	0	0	0		0	0				0	0	
	9	0	-16.1	16.1		0	0				0	0	
14	13	0	0	0		0	0				0	0	

Note: C - a branch is correctly modelled; B - a branch is incorrectly modelled

After the TV process is performed, the assumption, that branches 3-4 and 4-9 are switched off, proved to be false.

## VIII. CONCLUSIONS

In the paper, the original approach to PS TE identification is presented. The approach is based on the so-called reduced unbalance indices, which are introduced by the authors. In the earlier papers ([8], [9]), the so-called unbalance indices for nodes and branches have been used for PS TV. In this paper, other definition of indices for branches is presented. A consequence of that definition is different way of performing PS TV. As far as, the earlier unbalance indices are focused on identification of incorrectly modelled branches in a PS model, the reduced unbalance indices are rather aimed at identification of correctly modelled branches. The here-presented approach assumes that, at the beginning of PS TV process the correctly-modelled branches and then the incorrectly-modelled ones are pointed out. It appears that such realization of verification procedure enables identification of multiple incorrectly-modelled branches. When the unbalance indices are used, i.e. when it is assumed that at the beginning of PS TV process the incorrectly-modelled branches are pointed out, and there are multiple such branches, no verification decision can be taken, as far as neural networks are not employed [1], [9]. Other advantage of the proposed approach is possibility of performing TV in the conditions of lack of a full set of measurement data.

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