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Influence of Non-Magnetic Admixtures to the Signal of Electromagnetic Flow Meter

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Introduction

Electromagnetic flow meters EMFM are used for a fluid flow measurement. Different admixtures can be in this fluid. They can be solid, liquid or gaseous. Practically actual is the case when admixtures are solid or gaseous. In this case flow is two – phase. When the electric properties of admixtures are different than of the fluid the EMFM signal can vary. The variation of the EMFM electrode signal in the case when admixtures are composed of the solid magnetic particles was evaluated in [1], supposing that the shape of particles is a sphere and in [2], supposing that the shape of particles is an ellipsoid.

In this paper we evaluate the signal variation and measurement error of flow with non-magnetic admixtures in comparison with uniform fluid.

When flow is two-phase it can be actually for us the measurement of total two-phase flow, fluid flow or admixtures flow. The measurement error will be different in any of these cases. We investigate the measurement error of two-phase flow in this paper.

The electric properties of the EMFM sensor channel in the electrode signal expression are evaluated by the virtual current density [1, 2]. But the virtual current is a formal parameter. At first, we will show that virtual current distribution adequately presents real electric properties of the EMFM channel.

The reasoning of the electromagnetic flow meter electrode signal expression

The cause of the electrode signal origin is the fluid (as conductive body) movement traversing magnetic field lines. It is formed in sensor active zone. The sensor active zone is the fluid volume, in which the magnetic field is acted. This volume is not defined precisely. The mean contribution into electrode signal gives the magnetic flux density component $B\perp$, which is perpendicular to line, connecting the electrodes and to the sensor axis. Dependently on the EMFM metrological properties we can suppose that the active zone is composed of all sensor points in which $B\perp \geq (0{,}01 – 0{,}005)\, B\perp_{\max}$.

In his notable paper [3] Bevir relates the channel electric properties with some parameter which he names as virtual current density. He introduces this parameter nominally, as current density in the active zone with moveless fluid when current source with $I_0 = 1$ A is connected to EMFM sensor electrodes.

We can relate distribution of the virtual current density with real physical properties of the sensor active zone. When the fluid flows with velocity $\boldsymbol{v}$ inside EMFM sensor and it intersects the lines of the magnetic field with magnetic flux density $\boldsymbol{B}$, the electric field $\boldsymbol{E_F}$ arises. By Faraday's electromagnetic induction law the electric field strength can be expressed this way

$$\boldsymbol{E}_F = [\boldsymbol{v} \times \boldsymbol{B}]\,. \tag{1}$$

Because the raison of this field is mechanical fluid movement we have the voltage source. Electric field strength $\boldsymbol{E_F}$ characterizes the extraneous field, which separates the electric charges inside the fluid. These induced charges create the electric field with strength $\boldsymbol{E_i}$. The both fields, $\boldsymbol{E}_F$ and $\boldsymbol{E_i}$, create the electric current inside conductive fluid correspondingly with Ohm's law – the direction of current density in any point coincides with electric field direction in coordinate system, which moves with fluid. The density $\boldsymbol{J_i}$ of the current induced in the fluid moving across the magnetic field is

$$\boldsymbol{J}_i = \gamma\{\boldsymbol{E}_i + [\boldsymbol{v} \times \boldsymbol{B}]\}\,; \tag{2}$$

where γ is electric conductivity of fluid. The current density $\boldsymbol{J_i}$ is real parameter.

We express the virtual current density $\boldsymbol{J}$ in the moveless fluid created by the current source $I_0 = 1$A this way

$$\boldsymbol{J} = -\gamma\,\mathbf{grad}\,V; \tag{3}$$

where V is potential of electric field $\boldsymbol{E}$, created by current source I_0.

We note as S_1 and S_2 the areas of the first and second electrodes, correspondingly. The channel active zone is limited by perpendicular to channel axis planes which are sufficiently moved away of electrodes, one upstream and other downstream. The areas of these planes we note as S_p and S_a. The distances at electrodes to planes S_p and S_a must be chosen that conditions $\boldsymbol{B} \cong 0$ and $\boldsymbol{E}i \cong 0$ could be correct. Therefore the $\boldsymbol{E_F} \cong 0$ by (1) and $\boldsymbol{J_l} \cong 0$ by (2) on these planes. The total volume limited by channel walls area S_s, electrodes surfaces S_1 and S_2 and planes S_p and S_a is equal to τ. It presents the EMFM sensor active zone.

We introduce nominally the vector $\boldsymbol{K}$, which acts in the volume τ

$$\boldsymbol{K} = V_i \boldsymbol{J} - V \boldsymbol{J}_i \,; \tag{4}$$

where V_i is potential of any active zone point, created in the fluid moving across magnetic field, and V is the potential of the same point, created in the moveless fluid by current source I_0. Accordingly with the Gauss' theorem

$$\int_{S_\tau} (\boldsymbol{K} \cdot \mathbf{d}\boldsymbol{S}) = \int_{\tau} \mathrm{div}\boldsymbol{K} \mathrm{d}\tau \,; \tag{5}$$

where $S_\tau = S_1 + S_2 + S_s + S_p + S_a$ is the area of the surface, which limits active zone.

Let analyze the left side of equality (5). The inner walls of the channel are made of the insulator. The induced current with density $\boldsymbol{J_i}$ and created by current source I_0 current with the density $\boldsymbol{J}$ is not passed across the areas S_p ir S_a, too. Therefore, the currents with the densities $\boldsymbol{J}$ and $\boldsymbol{J_i}$ can flow across the electrodes surfaces S_1 and S_2, only. The left side of the equality (5) can be expressed

$$\int_{S_\tau} (\boldsymbol{K} \cdot \mathbf{d}\boldsymbol{S}) = V_{i1} \int_{S_1} (\boldsymbol{J} \cdot \mathbf{d}\boldsymbol{S}) + V_{i2} \int_{S_2} (\boldsymbol{J} \cdot \mathbf{d}\boldsymbol{S}) - V_1 \int_{S_1} (\boldsymbol{J_i} \cdot \mathbf{d}\boldsymbol{S}) - V_2 \int_{S_2} (\boldsymbol{J_i} \cdot \mathbf{d}\boldsymbol{S}). \tag{6}$$

The sensor electrodes are connected to the processor measurement unit with very large input resistance. Therefore the induced by fluid flow current is not passed across the electrodes

$$\int_{S_1} (\boldsymbol{J_i} \cdot \mathbf{d}\boldsymbol{S}) = \int_{S_2} (\boldsymbol{J_i} \cdot \mathbf{d}\boldsymbol{S}) = 0 \,. \tag{7}$$

Vice versus the all current created by the source $I_0 = 1$ A passes at one electrode to other

$$\int_{S_1} (\boldsymbol{J} \cdot \mathbf{d}\boldsymbol{S}) = - \int_{S_2} (\boldsymbol{J} \cdot \mathbf{d}\boldsymbol{S}) = 1 \,. \tag{8}$$

Evaluating (7) and (8) we obtain

$$U_e = V_{i1} - V_{i2} = \int_{S_\tau} (\boldsymbol{K} \cdot \mathbf{d}\boldsymbol{S}) = \int_{\tau} \mathrm{div}\boldsymbol{K} \mathrm{d}\tau \,. \tag{9}$$

The potentials V_i and V are the scalar coordinate functions, the current densities $\boldsymbol{J}$ and $\boldsymbol{J_i}$ are the vector coordinate functions. Differentiating (4) and evaluating the current continuity property $\mathrm{div}\boldsymbol{J} = \mathrm{div}\boldsymbol{J_i} = 0$, we obtain

$$\begin{aligned} \mathrm{div}\boldsymbol{K} &= (\mathbf{grad} V_i \cdot \boldsymbol{J}) + V_i \cdot \mathrm{div}\boldsymbol{J} - (\mathbf{grad} V \cdot \boldsymbol{J_i}) - \\ &- V \cdot \mathrm{div}\boldsymbol{J_i} = (\mathbf{grad} V_i \cdot \boldsymbol{J}) - (\mathbf{grad} V \cdot \boldsymbol{J_i}). \end{aligned} \tag{10}$$

In common case the fluid electric conductivity $\gamma(x,y,z)$ can be anisotropic. In this case the current density vectors can have different directions than vectors presented in the right side of the (2) and (3) expressions. This vector turning can be expressed by quadratic 3x3 matrix $[\gamma]$. The common element γ_{lk} of this matrix shows the fluid conductivity for current density l coordinate when electric field acts along k coordinate. Equation (2) can be written in matrix form

$$[\boldsymbol{J_i}] = [\gamma]\{[\boldsymbol{E}_i] + [\boldsymbol{E}_F]\}. \tag{11}$$

Expressing virtual current density $\boldsymbol{J}$ by Ohm's low in matrix form $[\boldsymbol{J}] = [\gamma][\boldsymbol{E}]$, too, and evaluating relation $\mathbf{grad}V = -\boldsymbol{E}$ the equation (10) can be expressed as follows

$$\begin{aligned} \mathrm{div}\boldsymbol{K} &= [\boldsymbol{E_i}]^T \cdot [\boldsymbol{J}] - [\boldsymbol{E}]^T \cdot [\boldsymbol{J_i}] = [\boldsymbol{E_i}]^T \cdot [\gamma] \cdot [\boldsymbol{E}] - \\ &- [\boldsymbol{E}]^T \cdot [\gamma] \cdot [\boldsymbol{E_i}] - [\boldsymbol{E}]^T \cdot [\gamma] \cdot [\boldsymbol{E_F}]. \end{aligned} \tag{12}$$

Matrix $[\gamma]$ is symmetric, therefore $[\gamma] = [\gamma]^T$, $[\boldsymbol{E}_i]^T[\gamma][\boldsymbol{E}] = [\boldsymbol{E}]^T[\gamma]^T[\boldsymbol{E}_i] = [\boldsymbol{E}]^T[\gamma][\boldsymbol{E}_i]$ and final expression of equation (10) will be

$$\mathrm{div}\boldsymbol{K} = -[\boldsymbol{E}]^T [\gamma][\boldsymbol{E}_F] \,. \tag{13}$$

Using equality $[\boldsymbol{E}]^T[\gamma] = \{[\gamma]^T[\boldsymbol{E}]\}^T = \{[\gamma][\boldsymbol{E}]\}^T$ we obtain

$$\mathrm{div}\boldsymbol{K} = -[J]^T [\boldsymbol{E}_F] \,. \tag{14}$$

Evaluating (1), (9) and (14) we can write

$$U_e = V_{i1} - V_{i2} = -\int_{\tau} (\boldsymbol{J} \cdot [\boldsymbol{v} \times \boldsymbol{B}]) d\tau = \int_{\tau} (\boldsymbol{v} \cdot [\boldsymbol{B} \times \boldsymbol{J}]) d\tau = \int_{\tau} \boldsymbol{v}\boldsymbol{W} d\tau \,; \tag{15}$$

where $\boldsymbol{W} = \boldsymbol{B} \times \boldsymbol{J}$ is weighting vector.

The expression (15) of the electromagnetic flow meter electrode signal is one of the fundamental results of electromagnetic flow meter theory. We can do some important conclusions by the presented derivation of expression (15). The electric current with the density $\boldsymbol{J_i}$ acts in the real EMFM sensor only. It is induced by fluid flowing across magnetic field. The regularities of this current density depend on the boundary conditions, i.e., on the channel and electrodes shape and on the fluid conductivity homogeneity. But the virtual current could be real when the stable current source $I_0 = 1$A could be connected to the electrodes of EMFM. The distribution of the virtual current density $\boldsymbol{J}$ varies analogically to the induced current density when the electric properties of the channel vary. It depends on the channel and electrodes shape and on the electric conductivity distribution in the active zone, too. The virtual current density $\boldsymbol{J}$ characterizes fully the electric channel properties agreeably with the expression (15). Therefore we can obtain the real variation of the electrode signal adequately investigating the virtual current distribution in the channel active zone.

The electrode signal for the flow with non-magnetic admixtures

Using virtual current conception we can investigate effectively the influence of non-magnetic admixtures with different than fluid electric conductivity to the measurement error. In [4] there was investigated the virtual current distribution in fluid with a bubble. But this investigation was performed for some fixed positions of the bubble. We suppose that admixtures are distributed uniformly and without restrain. When measurement time is suitably long the admixture positions can be any and we must investigate the mean values of virtual current density and velocity. Supposing that magnetic flux density is directed perpendicular to channel axis and to the line connecting the electrodes ($\boldsymbol{B}=\boldsymbol{e}_y B_y$) we can write of (15)

$$U_e = \bar{B}_y \cdot \bar{J}_x \cdot \bar{v}_z \cdot \tau_a \,; \tag{16}$$

where τ_a is volume of active zone, $\bar{J}_x, \bar{B}_y$ and $\bar{v}_z$ are the mean values of components J_x, B_y and v_z in the active zone.

We suppose that velocity in all volume of two-phase flow is distributed analogically to uniform flow. Therefore any variation of signal U_e will be measurement error.

The non-magnetic admixtures do not vary $\bar{B}_y$. Therefore the relative variation of signal U_e is equal to the relative variation δ_J of the virtual current density. We note the mean value of virtual current density in the uniform flow without admixtures as $\bar{J}_{x0}$. In any case the virtual current is equal to I_0=1 A. For the same volume of active zone τ_a the mean value of the virtual current density in the media with admixtures will be the same equal to $\bar{J}_{x0}$. For the two – phase media we can write

$$\bar{J}_{x0} = \bar{J}_{xp} k + \bar{J}_{xf}(1-k) \,; \tag{17}$$

where $k=\tau_p/\tau_a$ is volume concentration of admixture particles, when the total volume of the admixtures is equal to τ_p in active zone, $\bar{J}_{xp}$ - the mean value of virtual current density in the admixtures, $\bar{J}_{xf}$ - the mean value of the virtual current density in the fluid of two – phase flow. We evaluate the shape of particles by the ellipsoid.

The mean value of virtual current density inside elliptical particle can be expressed [2]:

$$\begin{cases} \bar{J}_{xp} = \bar{K}_p^{\gamma} \cdot \bar{J}_{x0}, \\ \bar{K}_p^{\gamma} = \dfrac{1}{3}(A_a^{\gamma} + A_b^{\gamma} + A_c^{\gamma}); \end{cases} \tag{18}$$

where the coefficients A_a^{γ}, A_b^{γ} and A_c^{γ} are:

$$A_a^{\gamma} = \frac{\gamma_p/\gamma_f}{1+L_a(\gamma_p/\gamma_f - 1)}; \tag{19}$$

$$A_b^{\gamma} = \frac{\gamma_p/\gamma_f}{1+L_b(\gamma_p/\gamma_f - 1)}; \tag{20}$$

$$A_c^{\gamma} = \frac{\gamma_p/\gamma_f}{1+L_c(\gamma_p/\gamma_f - 1)}; \tag{21}$$

where γ_m and γ_p are the electrical conductivities of the fluid and particles, accordingly, L_a, L_b and L_c are the depolarization factors which depend on ellipsoid semiaxes lengths a, b and c:

$$L_a = \frac{abc}{2}\int_0^{\infty}\frac{ds}{\sqrt{(s+a^2)^3(s+b^2)(s+c^2)}}; \tag{22}$$

$$L_b = \frac{abc}{2}\int_0^{\infty}\frac{ds}{\sqrt{(s+a^2)(s+b^2)^3(s+c^2)}}; \tag{23}$$

$$L_c = \frac{abc}{2}\int_0^{\infty}\frac{ds}{\sqrt{(s+a^2)(s+b^2)(s+c^2)^3}}. \tag{24}$$

We can express $\bar{J}_{xf}$ of (17) and (18)

$$\bar{J}_{xf} = [1 - \bar{K}_p^{\gamma} \cdot k] \cdot \frac{1}{1-k}\bar{J}_{x0}. \tag{25}$$

The relative variation of the virtual current density is composed of two components

$$\delta_J = \delta_p + \delta_f \,; \tag{26}$$

where δ_p is relative variation of virtual current density in the admixtures and δ_f is relative variation of virtual current density in the fluid around admixtures. These components can be expressed of (17), (18) and (25):

$$\delta_p = \frac{\bar{J}_{xp} - \bar{J}_{x0}}{\bar{J}_{x0}} = [\frac{1}{3}(A_a^{\gamma} + A_b^{\gamma} + A_c^{\gamma}) - 1] \cdot k \,; \tag{27}$$

$$\begin{aligned} \delta_f &= \frac{\bar{J}_{xf} - \bar{J}_{x0}}{\bar{J}_{x0}} = (\frac{1}{1-k} - 1) - \bar{K}_p^{\gamma} \cdot \frac{k}{1-k} = \\ &= -[\frac{1}{3}(A_a^{\gamma} + A_b^{\gamma} + A_c^{\gamma}) - 1] \cdot \frac{k}{1-k}. \end{aligned} \tag{28}$$

After summation we obtain

$$\begin{aligned} \delta_J &= [\frac{1}{3}(A_a^{\gamma} + A_b^{\gamma} + A_c^{\gamma}) - 1] \cdot (k - \frac{k}{1-k}) = \\ &= -[\frac{1}{3}(A_a^{\gamma} + A_b^{\gamma} + A_c^{\gamma}) - 1] \cdot \frac{k^2}{1-k}. \end{aligned} \tag{29}$$

This variation practically is equal to measurement error of the two-phase flow. When electric conductivity γ_p of particle is less than electric conductivity γ_m of fluid, the component δ_J varies between 0 and $\approx k^2$ and is independent on the particle shape, practically. But this component depends on the particle shape, when electric conductivity of particle is more than electric conductivity of fluid. In the case $\gamma_p >> \gamma_m$ the absolute value of the error component δ_J is minimal (equal to $\approx -2k^2$) for spherical particles and increases, when the ratios between semiaxes a/b and a/c increase. For example $\delta_J \approx -7k^2/(1-k)$ when a/c=9.

The real error for flow with non-conducting or spherical particles has appreciable value, when the volume concentration of particles exceeds 5%. The measurement error for conductive and elongate particles can be appreciable in the case of smaller particle concentration.

Comparison with formula of Bernier and Brennen

The expression (29) is suitable for gaseous admixtures, too. The electromagnetic flow meters with two-phases flow was investigated by Bernier and Brennen

[5]. They obtained the expression for electrode signal U_{2f} of electromagnetic flow meter, when the air bubbles are uniformly distributed in the fluid

$$U_{2f} = \frac{U_f}{1-k}; \quad (30)$$

where k – volume concentration of air in fluid, U_v – electrode signal of electromagnetic flow meter, when the air bubbles are absent in the fluid, and the volume flow is the same, as the volume flow of fluid in the two-phase flow.

The total volume τ_{2f} of both phases – fluid and air - is composed of uniform fluid volume flow τ_f and air volume flow τ_p

$$\tau_{2f}=\tau_f+ \tau_p=(1+k)\tau_f. \quad (31)$$

Therefore, the ratio of volumes τ_{2f} and τ_f is $\tau_{2f}/\tau_f=1+k$. This ratio is equal to the ratio of flows Q_{2f} and Q_f these volumes via the time unit

$$\frac{Q_{2f}}{Q_f} = \frac{\tau_{2f}}{\tau_f} = 1+k. \quad (32)$$

The ratio of electrode signal U_{2f} in the two-phase case and signal U_f in the uniform fluid case is $U_{2f}/U_f=1/(1-k)$. The relative error of suspension measurement can be evaluated as difference of these ratios

$$\delta_{BB} = \frac{U_{2f}}{U_f} - \frac{Q_{2f}}{Q_f} = \frac{1}{1-k} - (1+k) = \frac{k^2}{1-k}. \quad (33)$$

We can compare this expression with (29), remembering that electric conductivity of air bubbles is $\gamma_p \approx 0$ and $A_a^\gamma = A_b^\gamma = A_c^\gamma = 0$. Then the expression (29) becomes $\delta_J = k^2/(1-k)$.

Therefore the measurement errors are the same in the both cases: $\delta_n = \delta_{BB}$.

The expression (29) is obtained in [4] for the case of uniformly distributed air, only. By results obtained in this paper this expression can be generalized for any non-magnetic and non-conductive admixtures.

The obtained results can be used for investigating of the measurement of solid phase volume flow transporting by fluid.

Conclusions

1. The virtual current is formal parameter but its density distribution adequately characterizes the electric properties of the electromagnetic flow meter sensor.
2. By investigating the virtual current distribution in the active zone of electromagnetic flow meter we can obtain the expressions for measurement error which evoke the admixtures with any physical properties and any shape.
3. The obtained expression of measurement error coincides with the results obtained by Bernier and Brennen for the two-phase flow with the air bubbles.

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The virtual current is formal parameter but its density distribution adequately characterizes the electric properties of the electromagnetic flow meter sensor dependently on the electrodes and channel shape. By investigating the virtual current distribution in the active zone of electromagnetic flow meter we can obtain the expressions for measurement error which evoke the admixtures with any physical properties and any shape. The obtained expression of measurement error coincides with the results obtained by Bernier and Brennen for flow with the air bubbles. Bibl. 5 (in English; abstracts in English and Lithuanian).

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Virtualioji srovė yra formalus parametras, tačiau jos tankio pasiskirstymas kanale adekvačiai apibūdina elektrines elektromagnetinio srauto matuoklio jutiklio savybes priklausomai nuo skysčio elektrinio laidžio, elektrodų ir kanalo formos. Nagrinėjant virtualiosios srovės pasiskirstymą elektromagnetinio srauto matuoklio aktyviojoje zonoje gauta matavimo paklaidos, kurią sukelia bet kurių fizinių savybių ir formos nemagnetinių priemaišų dalelės, išraiška. Gautoji išraiška sutampa su rezultatais, kuriuos gavo Bernieris ir Brennenas, tyrinėdami srautą su oro burbulais. Bibl. 5 (anglų kalba; santraukos anglų ir lietuvių k.).