Investigation of Microstrip Lines Dispersion by the FDTD Method

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Introduction

Microstrip transmission lines (MSTLs) are the most important components of microstrip integrated circuits (MICs). The accurate modeling of such MSTLs using various analysis techniques is one of the essential tasks in microwave computer aided design. The trial-and-error cycles in the design of MICs can be substantially reduced if the frequency-dependent characteristics of MSTLs will be calculated faithfully.

MSTLs are the linear information transmission systems in which electrical signals are distorted mainly due to their non-ideal frequency responses of amplitude and phase.

The length of MSTLs in the real MICs is very short, moreover MSTLs are manufactured using materials witch electrical parameters are of high quality therefore amplitude losses in MSTLs may be negligible and only phase distortion should be taken into account. So, while an electrical signal propagates along the MSTL, it becomes distorted mainly due to dispersion characteristic of the line – nonlinear dependency of propagated signal phase on frequency. In homogeneous transmission lines such as waveguides [1, 2], coaxial lines and strip lines [3, 4] electric and magnetic fields exist in one substance, whereas the MSTL is unclosed line and, respectively, the fields are partially in the air area and partially in the dielectric substrate [5]. The air-dielectric interface prevents propagations of a pure TEM mode and to describe phase velocity of propagated waves in the MSTL it is supposed to use effective dielectric permittivity $\varepsilon_{\mathrm{reff}}(f)$ instead of dielectric constant ε_{r} of a substrate. Wherefore, the frequency response of phase constant $\beta(\omega)$ is not linear function and it leads in dispersion of phase velocity.

The review of publications shows that the dispersion analysis of MSTLs has been studied by a number of researchers and by various methods. The investigation of MSTLs dispersion started in the early 1970s. For nearly a decade for MSTLs analysis so called spectral domain method and its modifications were mostly used [6]. These methods are intrinsically rigorous, but need long computation time. To reduce calculation time various semi-empirical techniques were suggested [7], which allow obtaining rather accurate results if frequency range and variation of constructional parameters are limited. All the above mentioned researches were done in frequency domain; it means the data are calculated in one frequency point at a time. It is an expensive approach when searching the results in a broad frequency range. This led researchers to use finite difference time domain (FDTD) method [8].

Analyses of the dispersion characteristics of MSTLs executed by various methods reveals that while the frequency f increases, the effective permittivity $\varepsilon_{\mathrm{reff}}(f)$ also increases from the values, obtained applying TEM approach, up to values approximate to dielectric constant of the substrate ε_{r} [7]. At the same time it should be noted that the increasing of permittivity at increasing of frequency obtained by different researchers can differs by 25 % and more in the same frequency range and practically at the same constructional parameters [8]. This scattering is explained that a large number of basis functions should be used to calculate dispersion characteristics numerically with a high accuracy. The results in many observed papers were calculated with a small number of basis functions to save computation time, and dispersion characteristics were calculated not accurately.

In this paper, based on the FDTD method, we propose analysis technique of the frequency dependency of effective dielectric permittivity – dispersion characteristic of MSTL. This technique demonstrates rather fast computational efficiency and good accuracy.

Backgrounds of the FDTD method

According to the FDTD method the partial differential form of time dependent Maxwell's equations are approximated, using central difference approximations, to calculate values of electric and magnetic fields component at every point in a problem area for every increment of time. For electromagnetic 3D field analysis the Yee algorithm [9] is used. In this algorithm each field component depends on its value of the previous time step and the surrounding components.

The MSTL is an open region structure (Fig. 1) and because of limits of computational recourses, the absorbing boundary conditions (ABC) must be applied. ABCs are

developed to prevent wave reflections from boundaries of the problem area. In our technique the modification of the perfectly matching layer – an uniaxial perfectly matching layer (UPML), introduced by Gedney [10], is used as the ABC. The UPML surrounds the entire MSTL except the ground conductor below the dielectric substrate (Fig. 1) and it will absorb any incident waves.

In general form Maxwell's curl equations in the UPML medium can be written [11] as

$$\nabla \times \vec{H} = \mathrm{j}\omega\varepsilon\overline{s}\vec{E} , \qquad \nabla \times \vec{E} = -\mathrm{j}\omega\mu\overline{s}\vec{H} , \qquad (1)$$

where $\vec{H}$ and $\vec{E}$ are the vectors of the magnetic and the electric field respectively; $\omega = 2\pi f$ is the angular frequency; μ and ε are respectively the permeability and permittivity of a medium; $\overline{s}$ is the diagonal tensor which is defined by

$$\overline{s} = \begin{bmatrix} s_x^{-1}s_y s_z & 0 & 0 \\ 0 & s_x s_y^{-1} s_z & 0 \\ 0 & 0 & s_x s_y s_z^{-1} \end{bmatrix} , \qquad (2)$$

where s_x, s_y and s_z are the relative complex permittivity tensors in x, y and z directions

$$s_x = K_x + \frac{\sigma_x}{\mathrm{j}\omega\varepsilon} , \ s_y = K_y + \frac{\sigma_y}{\mathrm{j}\omega\varepsilon} , \ s_z = K_z + \frac{\sigma_z}{\mathrm{j}\omega\varepsilon} . \qquad (3)$$

In the problem area tensors $\overline{s}$ are defined by setting real parts of (3) equations to one, i.e. $K_x = K_y = K_z = 1$ and conductivities σ_x, σ_x and σ_z are equal to the conductivity of the cell material. In absorbing boundary regions K_x, K_y, K_z and σ_x, σ_x, σ_z are varied.

Gradual attenuation of incident waves on the UPML obtained by polynomial grading of σ and K parameters. Polynomial grading can by defined as follows:

$$\sigma_x(x) = (x/d)^m \sigma_{x,\mathrm{max}} , \qquad (4)$$

$$K_x(x) = 1 + (K_{x,\mathrm{max}} - 1)\cdot(x/d)^m , \qquad (5)$$

where σ_x increases from zero at the inner surface of the UMPL ($x = 0$) to the $\sigma_{x,\mathrm{max}}$ at the outer boundary ($x = d$) of the UPML and K_x increases correspondently from unity to $K_{x,\mathrm{max}}$. Two parameters $\sigma_{x,\mathrm{max}}$ and m can be
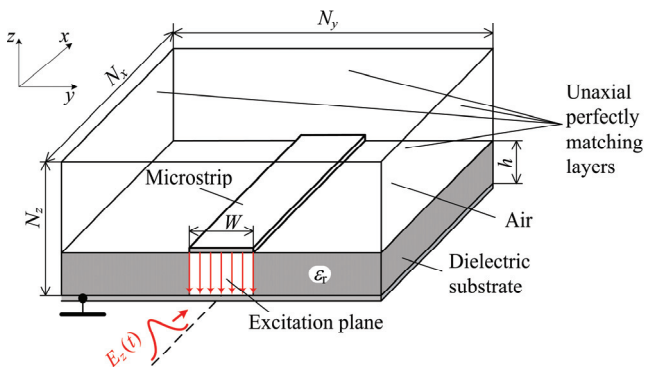

Fig. 1. Microstrip structure for analysis by FDTD method

predefined in the UPML of the fixed size d. It is recommended to choose $3 \le m \le 4$ [11], for the MSTL calculations m chosen to be 4. $\sigma_{x,\mathrm{max}}$ can be obtained from

$$\sigma_{x,\mathrm{max}} = -(m+1)\ln\left[R(\theta)\right]/(2\eta d) , \qquad (6)$$

where η is the impedance of the incident wave, and $R(\theta)$ is a reflection error. For the MSTL calculations the reflection error was chosen to be $R(\theta) = \exp(-16)$ and thick of the polynomial graded UPML to be 10 cells.

The precision and stability of the FDTD method depends on values of Δt and cell's sizes Δx, Δy, Δz. To minimize dispersive effects, cell's size should satisfy [11]

$$\Delta x = \Delta y = \Delta z = \Delta \le \lambda_{\mathrm{min}}/10 , \qquad (7)$$

where λ_{min} is the shortest wave length of a given signal.

Time step Δt is determined by Courant condition. This stability criteria shows relation between the time step, mesh cell size and the speed of wave propagation. Using three dimensional area cells with $\Delta x = \Delta y = \Delta z = \Delta$, time step can be defined [11]

$$\Delta t = \Delta/(c\sqrt{3}) , \qquad (8)$$

where c is the velocity of propagation in free space.

For analysis of dispersion characteristics of MSTL in the broadband frequency range a broadband excitation source should be used. For simulations we used Gauss excitation pulse

$$E_z(t) = \exp\left[(t-t_0)^2/t_\mathrm{w}^2\right] , \qquad (9)$$

where t_0 is the pulse delay and t_w is the pulse width. Excitation pulse introduced at the input of the MSTL below the microstrip (Fig. 1) and output is taken below microstrip at the end of the problem area. Microstrip is extended to the UPML in both ends to prevent reflections due to microstrip discontinuity. The frequency response of the MSTL can be obtained by taking the Fourier transform of input $E_z(0,t)$ and output $E_z(l,t)$ time responses. From the input and output frequency response ratio we can get wave propagation phase coefficient $\beta(\omega)$. Dispersion characteristic of the MSTL can be found knowing phase coefficient $\beta(\omega)$

$$\varepsilon_{\mathrm{r\,eff}}(\omega) = \left[\beta^2(\omega)\right]/(\omega^2 \varepsilon_0 \mu_0) . \qquad (10)$$

Calculation algorithm

Flowchart of the algorithm for calculation of the effective permittivity of the MSTL is presented in Fig. 2.

The algorithm consists of 8 steps:
1. Constructional and electrical parameters of the MSTL are entered: dimensions of the analysis area; thickness of the UPML boundaries; dielectric constant and height of the substrate; width of the microstrip and its length l which is equal to the analysis area in x direction.
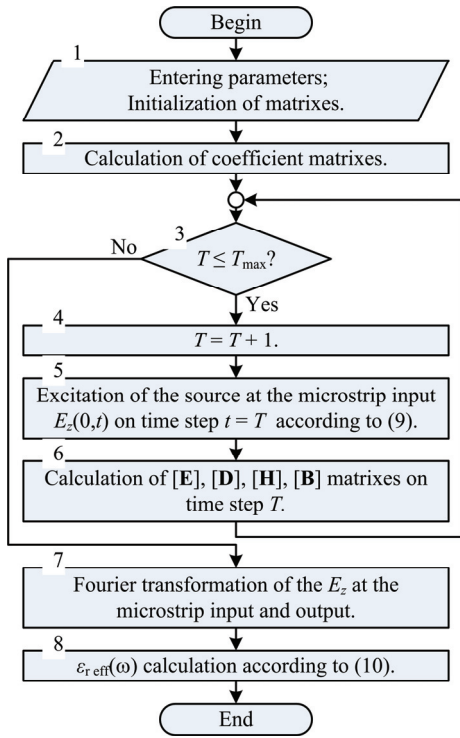
Fig. 2. Flowchart of the algorithm for calculation of frequency dependency of the effective permittivity of the MSTL

Using the entered parameters, update equations coefficient matrixes and $[\mathbf{E}]$, $[\mathbf{D}]$, $[\mathbf{H}]$, $[\mathbf{B}]$ matrixes are initialized.

2. Update equations coefficient matrixes are filled up in the problem area, and are calculated in the UPML absorbing boundary.
3. A time stepping cycle begins. This cycle ends when the maximum time step T_{max} is reached.
4. The time stepping cycle begins at time step $T = 0$ and it is incremented in each stepping cycle.
5. Gauss excitation pulse is calculated at the MSTL input at each time step according to (9).
6. At each time step matrixes $[\mathbf{E}]$, $[\mathbf{D}]$, $[\mathbf{H}]$, $[\mathbf{B}]$ are recalculated and values of the E_z component at the MSTL input $E_z(0,t)$ and output $E_z(l,t)$ are saved.
7. Fourier transformations of saved values of components $E_z(0,t)$ and $E_z(l,t)$ are performed and from values of transformation the wave propagation phase coefficient $\beta(\omega)$ is calculated.
8. Dispersion characteristic of the MSTL is calculated according to (10).

Investigation of the proposed technique

The proposed technique for analysis of dispersion of the MSTL was investigated in the following order. Firstly, accuracy of the mathematical model used in the proposed technique was tested. Next, the dispersion characteristics of MSTLs with different width of microstrip and dielectric constant of substrate were investigated.

The search of publications concerning measurement of effective permittivity of MSTLs in the broad frequency

range was unsuccessful. Only two papers presenting such measurements were found. In [13] microstrip dispersion measurements for frequency range of 1–18 GHz, and in [13] measurements of the effective permittivity of the MSTL on glass for frequency range of 5–35 GHz are presented. Therefore, checking accuracy of the mathematical model used in the proposed technique, authors have been compelled to be limited to examples of the above named measurements. Comparison of dispersion characteristics calculated using the proposed technique and measured in [12, 13] papers are submitted in Fig. 3. For calculations according to the proposed technique the cubic cells were used and spatial discretization interval Δ was chosen ten times smaller than the smallest size of measured MSTLs. The chosen of such small value of Δ allow us to reach high frequencies without causing numerical dispersion. It is obvious, that the measured and calculated values presented in Fig. 3 in most cases agree within less than 3 %.

Further, the proposed technique was used to investigate dispersive characteristics of MSTLs with different width of microstrip and various substrates. Fig. 4 presents calculated normalized frequency dependencies of effective permittivity of such MSTLs. The presented curves show that effective permittivity increases when frequency increases, besides the permittivity of wide microstrips increases greater than narrow ones. To highlight changes of permittivity, when frequency changes,
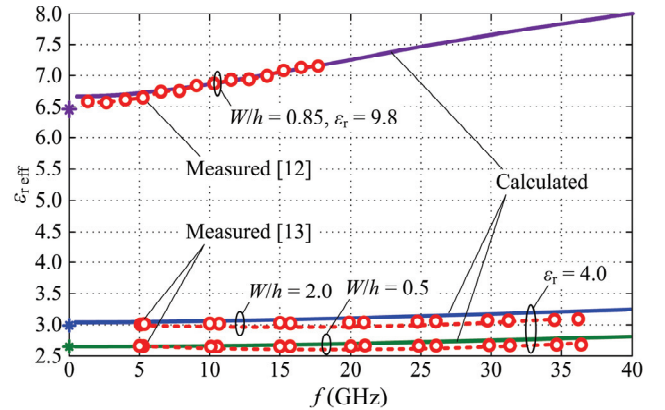

Fig. 3. Comparison the calculated (solid curves) and measured (dashed curves) in [12] and [13] works of dispersion characteristics of the MSTL. Values calculated using the method of moments for the TEM case are pointed by asterisks
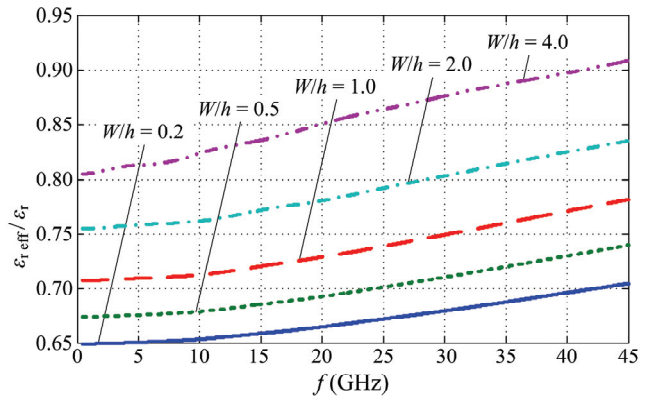

Fig. 4. Calculated normalized frequency dependencies of effective permittivity of MSTLs with different W/h, and $\varepsilon_r = 6.0$
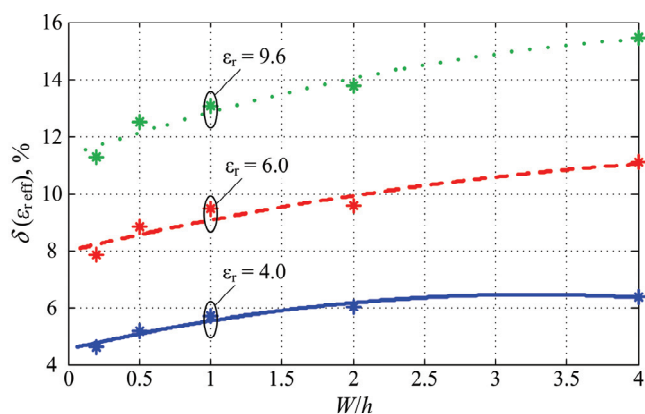
Fig. 5. Dependences of the relative change of effective permittivity of the MSTL at increase frequency, on the width of the microstrip and on a dielectric constant of the substrate

the simulation of MSTLs with various width of the microstrip and various dielectric constants of the substrate were performed. Results of these simulations are submitted on Fig. 5 as a relative difference of permittivity in case of high frequency $\varepsilon_{r\,eff}(45\,\text{GHz})$ and permittivity of the TEM case $\varepsilon_{r\,eff}(0)$. This relative difference was calculated according to such expression

$$\delta(\varepsilon_{r\,eff}) = \left[\varepsilon_{r\,eff}(45\,\text{GHz}) - \varepsilon_{r\,eff}(0)\right]/\varepsilon_{r\,eff}(0)\cdot 100\% . \quad (12)$$

Curves submitted in Fig. 5 show that when frequency increases, the effective permittivity of the wide microstrips formed on substrates of the high dielectric constant increases more ($\max\delta(\varepsilon_{r\,eff}) \cong 16\,\%$) than the narrow microstrips, formed on substrates of small constant ($\max\delta(\varepsilon_{r\,eff}) \cong 6\,\%$).

Conclusions

The technique based on FDTD method and Fourier transformation for calculation of dispersion characteristic of the microstrip transmission line (MSTL) is proposed.

The dispersion characteristics, calculated according to proposed technique, differ from measured and published by other researchers no more than 3 %.

Analysis of dispersion characteristics of MSTLs reveals that dispersive properties are manifested in the greater degree in the case of MSTLs with wide microstrips and on substrates with the larger dielectric constant.

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A. Krukonis, V. Urbanavicius. Investigation of Microstrip Lines Dispersion by the FDTD Method // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 9(115). – P. 51–54.

The technique based on the FDTD method for calculation of the dispersion characteristic of the microstrip transmission line (MSTL) is proposed. Accuracy of the suggested technique is checked up by comparison of results of calculation of the dispersion characteristics with results of measurements executed by other researchers. Analysis of dependency of dispersive properties of MSTL's from constructional parameters of MSTLs is carried out. Ill. 5, bibl. 13 (in English; abstracts in English and Lithuanian).

A. Krukonis, V. Urbanavičius. Mikrojuostelinės perdavimo linijos dispersinių savybių tyrimas taikant BSLS metodu // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 9(115). – P. 51–54.

Remiantis laikiniu baigtinių skirtumų metodu, siūloma metodika mikrojuostelinių perdavimo linijų (MJPL) santykinei efektinei dielektrinei skvarbai apskaičiuoti. Metodikos tikslumas patikrintas lyginant MJPL analizės rezultatus su kitų tyrėjų gautais matavimo rezultatais. Atlikti santykinės efektinės dielektrinės skvarbos dažninių charakteristikų priklausomybių nuo MJPL konstrukcinių parametrų tyrimai. Il. 5, bibl. 13 (anglų kalba; santraukos anglų ir lietuvių k.).