Method for Wideband Signal Digitizing and their Real-time Reconstruction in Enlarged Dynamic Range

I. Bilinskis[1], K. Sudars[1], M. Min[2]
[1]*Institute of Electronics and Computer Science,*
Riga, Latvia,
[3] *Department of Electronics, Tallinn University of Technology,*
Tallinn, Estonia
bilinskis@edi.lv

***Abstract*—Various approaches to signal digitizing and real-time reconstruction in wide frequency and dynamic ranges are considered. They are discussed and compared on the basis of MATLAB simulation results. It is shown that the best results can be achieved at combination of pseudo-randomized sampling of the original signals at sub-Nyquist rates with low bit-rate pseudo-quantizing of the signal sample values obtained at high-frequency periodic sampling.**

***Index Terms* —Wideband signal digitizing, signal real-time reconstruction, nonuniform sampling.**

I. INTRODUCTION

While in many cases it is important to perform analog-digital conversions at high frequencies to support applications of digital electronics in various fields, it is not so easy to achieve this capability, as it is shown in [1] in regard to the specific case of software-based radio. The frequency range, where the currently available 10 to 12 bit ADCs are applicable, often is not wide enough. On the other hand, the dynamic range of the ADCs, applicable for analog-digital conversions in GHz frequency range, is limited by the achievable quantization bit rate usually not exceeding 4 bits [2], [3]. These typical problems, arising at attempts to convert analog signals into their digital counterparts in a wide frequency range extending up to GHz frequencies, are considered in this paper and a specific approach to resolution of them is suggested. In general, this approach is based on application of special Digital Alias-free Signal Processing (DASP) techniques [4], [5]. They are well suited for processing of signals digitally at much higher frequencies than it can be done on the basis of the classical Digital Signal Processing (DSP). Although it is easier to use them for estimation of wideband signal parameters, including their spectra, it has been shown that it is also possible to digitize wideband signals and reconstruct their waveforms [6]. However the so far developed methods for signal waveform reconstruction do not cover real-time applications. The method for waveform reconstruction discussed in [6], specifically, is based on the direct and inverse Discrete Fourier transforms.

Various options in resolution of the task of wideband signal digitizing and their waveform real time reconstruction in sufficiently wide dynamic range are considered in this paper under various digitizing conditions. Using of pseudo-randomized sampling of the original signals at sub-Nyquist rates is considered (Section II and III). As the precision achieved at iterative filtering of the non-uniformly digitised signals is not acceptable (Section III), it is suggested to combine additive pseudo-randomized sampling with pseudo-randomized quantizing of signal sample values obtained at high sampling rate periodic sampling (Section IV). In order to improve the results even more, it is suggested to introduce pseudo-randomized quantizing and additional filtering procedures (Section V). The obtained results are discussed in the conclusion.

II. ADDITIVE PSEUDO-RANDOMIZED SIGNAL SAMPLING

Whenever continuous time signals are digitized and then processed on the basis of Digital Signal Processing (DSP), the sampling rate f_s of the used periodic sampling limits the bandwidth of the original analog signals. Then the restrictions, defined by the Sampling Theorem, have to be satisfied to avoid the uncertainty due to the fact that all frequencies belonging to the sequence: f_o; f_s ï÷f_o; $2f_s$ ï÷f_o; $3f_s$ ï÷f_o;... nf_s ï÷f_o are indistinguishable. The obvious way to avoid this uncertainty is to require that the frequency $f_o < f_s/2$ and that all frequencies in the spectrum of the input signal above $f_{s/2}$ are taken out by analog low-pass prefiltering. Another possibility how to avoid the described uncertainty due to frequency overlapping or aliasing is based on application of nonuniform signal sampling [7]. Indeed, imagine that the signal frequencies are sampled non-equidistantly, obviously then all the digital replicas of signal frequencies will differ. That opens up the principal possibility of distinguishing between them without mentioned cutting-off of the analog signal spectra by low-pass pre-filtering of them. On the other hand, it is not so easy to gain from nonuniform sampling, to develop practically

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applicable designs of analog/digital systems successfully exploiting advantages of this technique. Whenever signals are sampled nonuniformly, processing of the obtained digital signals has to be done specifically and correctly, to avoid errors due to the cross-interference between signal components and to other negative effects related to the specifics of randomized sampling. These considerations have been taken into account at development of the suggested method.

Suppose a signal $x(t)$, continuous in time, is sampled at time instants t_k, $k = 1, 2, 3, \ldots$ and a sequence of sample values $x(t_k) = x_k$ taken at these time instants is obtained. It becomes possible to use this sequence of the sample values x_k for representing the original analog signal in the digital domain if certain requirements, depending on the specific sampling mode used, has been satisfied. While in the cases of periodic sampling the signal sample values have to be taken at a rate at least twice exceeding the upper frequency in the spectrum of the signal, it is less clear what requirements have to be satisfied in the cases where signals are sampled nonuniformly. They depend, in general, on the used specific method for nonuniform sampling, on various parameters characterizing the sample value taking process and also on the used approach to extraction of the information carried by the signal. So-called additive random sampling has proved to be a good nonuniform sampling option.

To achieve the possibility of using 10 to 12 bit ADCs for wideband signal digitizing at frequencies exceeding the mean sampling rate, the pseudo-randomized version of this type of sampling is used. A typical realization of the used pseudo-randomized additive sampling point sequences is shown in Fig. 1.

Bandwidth of the signals $x(t)$ is limited, the upper frequency f_u of the signal spectrum does not exceed the frequency limit f_{lim}, inequality $f_u < f_{lim}$ is satisfied. Nonuniform sampling is carried out according to the model of pseudo-randomized additive sampling [4]. The basic parameters characterizing this additive pseudo-randomized sampling scheme is the period T_c of the high frequency clock used for generation of the sampling point stream defining the sampling instants; the mean sampling rate 1/μ, where μ is the mean value of the sampling intervals equal to nT_c, $n = 1, 2, 3, \ldots$. Therefore «= nT_c and $f_{\lim} = \dfrac{1}{2T_c}$. Problems met at reconstruction of the original signal waveforms, under the conditions that the upper frequency in the spectra of input signals might exceed the mean sampling rate but not the indicated frequency limit defined by the clock frequency , are studied and approaches to resolution of them are looked for.

Figure 1 illustrates filtering of signal sample value sequences obtained in result of pseudo-randomized sampling process under these conditions. A realization of an input signal sample value sequence x_k is given in the upper part of this diagram. All of the sample values are placed on the time grid dictated by the used clock frequency. As can be seen, the intervals between these sample values are not constant, they are pseudo-random. Each of these sample values are multiplied by the respective filter coefficient and at each filtering cycle, at calculation of an output signal value at a given time instant.

Note that the filter coefficients in fact are sample values of the filter impulse response. This function is step-by-step shifted at filtering and the step size is equal to the clock period. While the whole filter coefficient set is used for filtering, the number of them used at specific filtering cycles is reduced several times. It is shown that different coefficient sets are used for obtaining two output signal values. Specifics of low-pass digital filtering of a randomly sampled signal are described in more detail in [8].

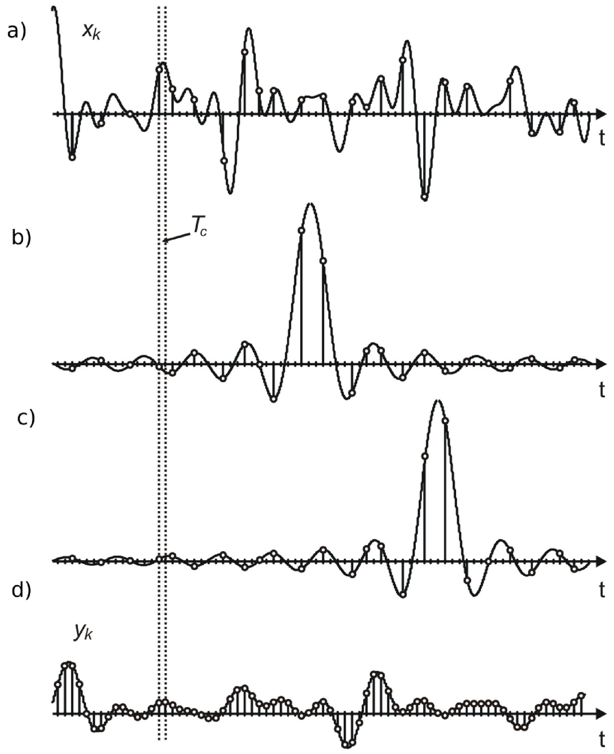

Fig. 1. Conditions for filtering signal sample value sequences obtained in result of pseudo-randomized sampling process; (a) input signal x_k; (b) and (c) illustrate the fact that at nonuniform sampling the filter impulse response, shifted in the filtering process in 2 particular positions, provide different sample values; (d) filter output signal y_k.

The information carried by the nonuniform signal sample value sequences, obtained in cases where the signals have been sampled according to this type of sampling point processes, provides not only for elimination of the aliases but it could be also successfully used, under certain conditions, for obtaining accurate spectral estimates and recovered waveforms of a wide class of signals. That has been confirmed experimentally [4], [6]. As it has been demonstrated by using the iterative approach to signal processing based on direct and inverse DFT, not applicable for analog-digital conversions in real-time, a different approach to this task has to be found. Using iterative special digital filtering for that was considered.

III. Waveform Recovery by Digital Filtering

Consider using the classic low-pass digital filter (LPF in Fig. 2) for reconstruction of a signal from the sample values of it obtained in result of the described pseudo-randomized sampling. The calculation of the filter coefficients is based on a sampling model, according to which the signal is sampled periodically at the clock frequency. Relatively many signal samples are pseudo-randomly taken out as shown in Fig. 1. The remaining sample values then are to be filtered. They are placed on the time axis nonuniformly according to the additive sampling point process discussed. Then there are empty places on the time grid between each pair of successive sample values. These empty places have to be filled up (as it is explained in some detail in [6]) to provide conditions needed for standard digital low-pass filtering of equidistant signal sample sequences. Filtering is iterative. Structure of the electronic system, used for reconstructing a signal waveform from the sample values y_k, obtained in result of the described pseudo-randomized sampling, is shown in Fig. 2. Only the first filter stage is shown, the other following filter stages, needed for iterative filtering, are the same as the first one.

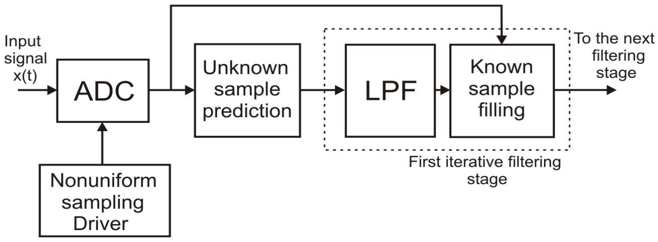

Fig. 2. Structure of the electronic system used for iterative reconstruction of signal waveforms.

Digitizing of the input signals is performed by using a 12 bit ADC. At the first stage of it the missing signal sample values are replaced by some values predicted for filling the mentioned empty places. The outputs of this iterative filtering are obtained in various ways. The empty places are filled either by zeroes (version1), by adding to each given sample values copies of it, placed on both sides of the known values (version 2) or by averaging each pair of the given sample values and using the result only for filling the empty places between the respective sample values (version 3). These sample values then are recovered with iteratively improved precision.

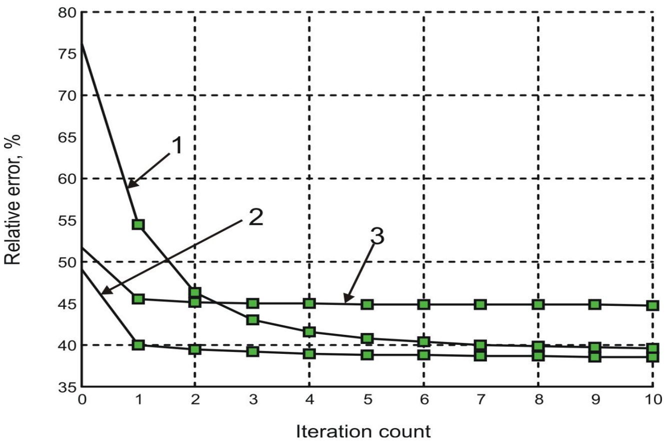

Fig. 3. Reduction of signal waveform reconstruction errors by filling in the empty places between the input signal values obtained at sample nonuniform analog/digital described versions conversion according to the 1, 2, 3.

Examples of the obtained results are given in Fig. 3 as relative reconstruction errors versus iterative filtering stage count. These errors are defined as

$$\varepsilon = \frac{1}{N}\sum_{k=1}^{N} 100\left(\frac{x_k - y_k}{x_k}\right), \tag{1}$$

where N – number of points; x_k – true signal values; y_k – estimated signal values.

While signal waveforms are reconstructed in this way, the precision obtainable at sufficiently low mean sampling frequencies is not acceptable. The basic reason why waveform reconstruction with high enough precision was not achieved is related to behaviour of the involved iterative filtering process. It does not tend to the zero error value. Some systematic error remains. It varies under variable sampling conditions but, in general, this error is too large. To achieve improved results, the option of using both high bit rate ADC and high sampling rate ADC in parallel was considered. The results obtained then indeed are much better. Discussion of this approach follows.

IV. Using Precise Undersampling and Low-Bit Fast Periodic Sampling in Parallel

Suppose the iterative filtering procedure is repeated when results of 4 bit fast periodic sampling are used for filling the spaces remaining empty after the precise signal sample values have been taken at time instants according to the described additive random sampling. Then the conditions for waveform reconstruction are more favourable and the obtained results also are significantly improved. They are displayed in Figure 4.

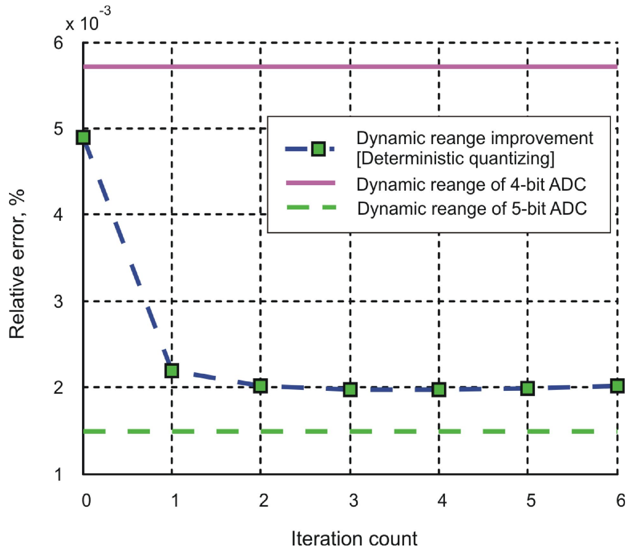

Fig. 4. Results of waveform reconstruction when precise undersampling and low-bit fast periodic sampling procedures are used in parallel.

As it can be seen, only a few iterations have to be made to get the results, obtainable under the given conditions. While these results are relatively good, the waveform reconstruction error is not suppressed to a level that would be achievable if the same signal would be digitized by a hypothetical 12 bit ADC that would be capable of taking signal sample values at the clock frequency. To improve the waveform reconstruction, using of yet another one of the randomized procedures, namely, randomized quantizing is suggested.

V. Pseudo-Randomizing of Low-Bit Rate Quantizing

Introduction of pseudo-randomized quantizing of the sample values, obtained in result of considered fast 4 bit periodic sampling, is considered. That is aimed to increase the achievable waveform reconstruction quality even more.

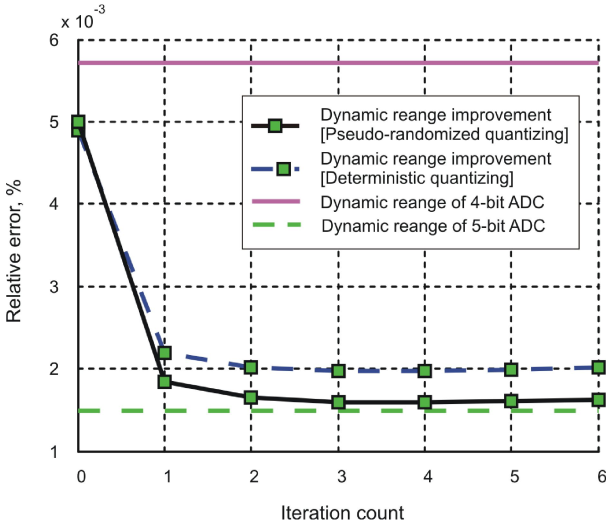

Fig. 5. Waveform reconstruction precision improvement due to pseudo-random quantizing of the 4 bit signal sample value sequences used for filling in the empty spaces between the input signal sample values obtained at nonuniform sampling.

The obtained results are shown in Fig. 5. The precision improvement, achieved in this way, is evident. While it seems to be relatively small, the actual quality improvement of the waveforms reconstructed in this way is more significant, as introduction of randomized filtering eliminates also the spurious frequencies in the spectra of the reconstructed signals usually distorting the spectra of 4 bit digital signals.

VI. Conclusions

Problems complicating digitizing of analog signals in a wide frequency range, extending up to GHz frequencies, are considered in this paper and application of DASP techniques for resolution of them is suggested. Although it is easier to use them for estimating wideband signal parameters, including their spectra, it has been already shown that it is also possible to digitize wideband signals and reconstruct their waveforms. However the earlier developed waveform reconstruction methods, based on the direct and inverse Discrete Fourier transforms, do not cover real-time applications. The suggested method for waveform reconstruction allows that to be done. The best results can be achieved at combination of pseudo-randomized sampling of the original signals at sub-Nyquist rates with low bit-rate pseudo-quantizing of the signal sample values obtained at low bit rate high-frequency periodic sampling.

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