Optimal Position of Buried Power Cables

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***Abstract*—The optimum position of parallel underground cables will be calculated numerically. The criterion is how much joule losses should be dissipated in each cable so that the temperature increases of all cables are equal. A simple analytical formula is also given.**

***Index Terms*—Analytical analysis, numerical analysis, power cables, temperature control.**

I. INTRODUCTION

Underground power cables are thermally well insulated. At a depth of about 1 m the amount of ground surrounding the cable is quite huge so that it behaves as a high thermal resistance. As a consequence the temperature of the cable can be relatively high even for moderate values of the Joule heat produced in the cable as compared to the same cable in air with natural convection cooling. Due to the increasing demand for electric power, the thermal load of many cables is growing continuously because it is not possible in practice to install new cables any time the power demand is increasing. Several papers have treated the thermal problems of underground cables [1]–[17].

If cables have to be deposited underground, a well has to be excavated and several cables are installed parallel next to each other. Normally the power in all cables will be different. Hence one can ask the question in which order one has to put the cables so that the temperature distribution is optimal, i.e. the maximum temperature should be as low as possible. It can be proved that this problem is equivalent to the following one: how to interchange the position of the cables so that the temperature distribution is as uniform as possible. Needless to say, a uniform temperature distribution is the optimal situation.

Intuitively, it is clear that if all the cables have exactly the same Joule power, the cable in the middle will have the highest temperature. The outer cables will be at the lowest temperatures. If the transmitted power of each cable is different, one should install the cables with the highest power at the ends and the cable with the lowest power in the middle to approach the optimal situation as good as possible.

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The reason for this investigation is obvious. If several cables are buried next to each other, it is of practical use to arrange them in such a way to reduce the extreme temperatures as much as possible.

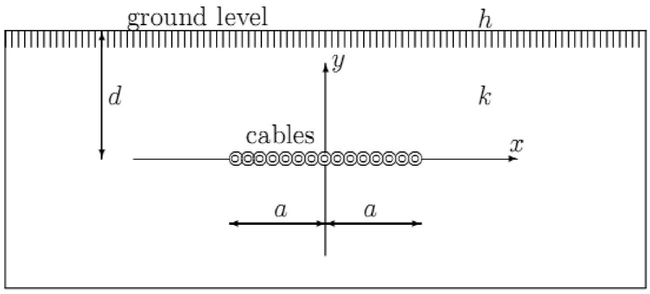

Fig. 1. Cross sectional view of the layout of the underground cables.

Related problems exist in other fields such as microelectronics. If several heat dissipating components are placed on a single substrate, a different layout can give rise to a more uniform temperature distribution and hence to a reduced peak temperature [18]–[22].

II. SIMULATION RESULTS

The problem of underground cables has been simulated numerically using the COMSOL multiphysics software [23]. A steady state analysis has been carried out using the geometry shown in Fig. 1.

As the cables are very long only a cross section has to be considered. This gives rise to a two dimensional analysis. The dimensions of the box are 5 m $\times$ 2 m. The cables are at a depth of d = 1 m. The bottom and the two sides are modelled as adiabatic boundary conditions. The ground has a thermal conductivity k = 0.83 W/m.K. The ground level is cooled convectively. A heat transfer coefficient h = 20 W/m^{2}K has been used and the ambient air temperature was set to $T_{air} = 20^{\circ}C$.

Each cable has an external diameter of 5 cm. The metal conducting part with a diameter of 3 cm was assumed to be made of copper with a thermal conductivity k = 400 W/m.K. The electrical insulation around each cable has a thermal conductivity k = 0.28 W/m.K. At the copper insulation and insulation ground interfaces, the continuity of the temperature and the heat flux were used as boundary conditions.

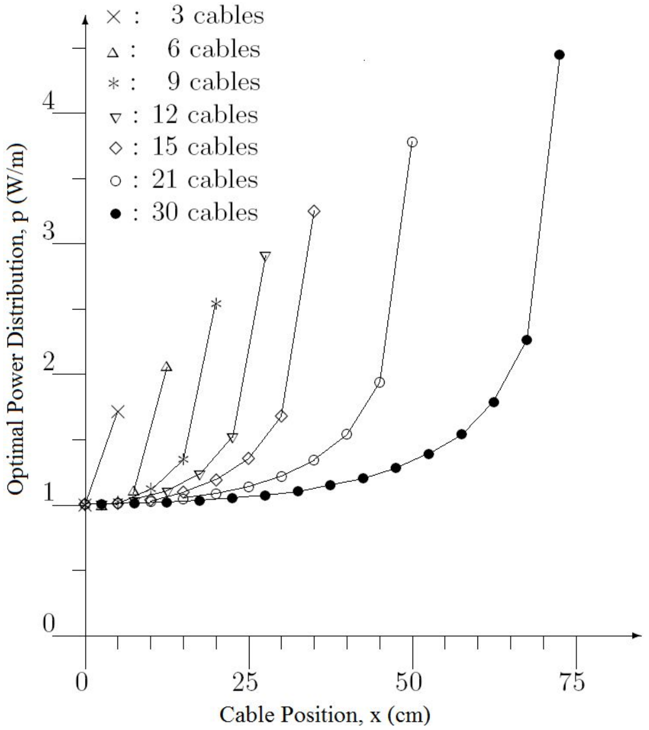

Fig. 2. Optimal power distribution in the cables as a function of the position of the cables.

In every cable i a certain amount of Joule heat is produced denoted with p_i (W/m). As a consequence each cable will have a temperature rise T_i above the ambient value. Our goal now is to find a set of power values p_i so that all the cable temperatures turn out to be equal or $T_1 = T_2 = \dots T_n$.

For the thermal conduction in the ground the Laplace equation has to be solved in order to determine the temperature distribution. The solution is carried out using the finite element package COMSOL. With this software package, as well as with other ones, it is quite easy to find a temperature distribution provided the heat generation is known. In this research, we want to solve the inverse problem: which should be the distribution oft the heat generation in order to get a uniform temperature distribution in the cables. The inverse problem will be approached using the superposition principle.

In order to find the power distribution $\{p_i \,|,\, i=1...n\}$ an arbitrary power p_0 is generated in cable 1 and zero power in all the other cables. After simulation we obtain the temperature values T_{11} in cable 1, T_{21} in cable 2, T_{31} in cable 3,… The same procedure is then repeated with a constant power p_0 in cable 2 and zero power in the all the remaining cables. The corresponding temperature values is then T_{12} in cable 1, T_{22} in cable 2, T_{32} in cable 3,…. This procedure is repeated for all n cables. If a power distribution p_1 ,p_2, p_3,… is applied, the temperature of cable i is found by superposition

$$T_i = T_{i1}\frac{p_1}{p_0} + T_{i2}\frac{p_2}{p_0} + T_{i3}\frac{p_3}{p_0} + \dots . \tag{1}$$

Requiring that all cable temperatures must be equal

$$T_1 = T_2 = T_3 = T_4 = \dots , \tag{2}$$

gives rise to an algebraic set of n-1 equations with n unknowns: p_1, p_2, p_3,…. However, one can always choose one; say p_1, because if all the power values are multiplied by the same constant, one still obtains a uniform temperature distribution. Hence, an algebraic set of n-1 equations and n-1 unknowns (p_2, p_3,…) remains. Obviously the problem can be simplified further on by taking the symmetry into account. It is clear that $p_1 = p_n$, $p_2 = p_{n-1}$,… so that $n/2$-1 unknowns remain in case n is even. For odd n, the number of unknowns is reduced to $(n-1)/2$. Simulations have been carried out for 3, 6, 9, 12, 15, 21 and 30 cables, the results of which are displayed in Fig. 2.

All power values have been normalised to the values obtained in the middle ($x = 0$). As expected, the outer cables allow a much higher power transmission. It is surprising to learn that the power in the outer cables can be several times the power value in the central cables. A more careful discussion of these results will be given further on in this paper.

III. Theoretical Analysis

As has been pointed out the optimal situation corresponds to a uniform temperature of the cables. For the theoretical analysis the problem will be reversed: given a constant temperature which the corresponding power distribution? The purpose of the theoretical section is to find a simple analytical formula which can be used on the numerical data so that a simple design rule could be obtained. As a consequence, the theoretical analysis can be largely simplified. First of all, the cables are now replaced by a horizontal flat plate BC at a uniform temperature T_0 as shown schematically in Fig. 3(a).

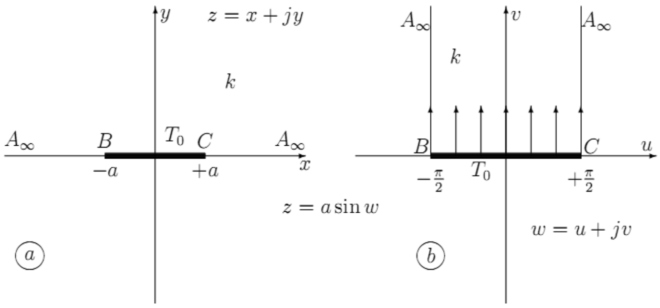

Fig. 3. Conformal mapping used to determine the optimal power distribution analytically.

Secondly, it is assumed the plate BC is sufficiently deep below the ground level, so that convection can be neglected. Moreover, we are mainly interested in the temperature field near the plate BC. The plate BC with a width $2a$ is now placed in an infinite medium with thermal conductivity k. Due to symmetry it is sufficient to consider only the upper half part $y > 0$ of Fig. 3(a). This problem has been described in several textbooks mainly devoted to potential theory [24], [25]. The most obvious way to solve the problem of Fig. 3(a) is to use a conformal mapping from the $z = x + jy$ to the $w = u + jv$ plane

$$z = a\sin(w) \quad or \quad w = \arcsin\left(\frac{z}{a}\right). \tag{3}$$

The half upper plane $y > 0$ is then mapped into a strip in the w plane borned by $-® / 2 < u < +® / 2$ and $v > 0$ (Fig. 3(b)). In the w-plane the temperature distribution T depends only on v

$$T(v) = T_0 - \sim v, \tag{4}$$

which corresponds to a uniform heat flux $p(u) = k\sim$ in the plane. The complex temperature distribution in the w-plane is then

$$T(v) = T_0 + j\sim w, \tag{5}$$

which can be easily found because (4) is nothing else than the real part of (5). In the z-plane the complex temperature distribution is then given by

$$T(z) = T_0 + j\sim \arcsin\left(\frac{z}{a}\right). \tag{6}$$

The complex temperature gradient is found by taking the derivative of (6)

$$\frac{dT(z)}{dz} = j\sim \frac{d}{dz}\arcsin\frac{z}{a} = j\sim \frac{a}{\sqrt{a^2 - z^2}}. \tag{7}$$

Putting $z = x$ and bearing in mind that the imaginary part of (7) is the heat flux in the y direction, one gets the power density distribution along the plate

$$p(x) \propto \frac{1}{\sqrt{a^2 - x^2}}. \tag{8}$$

The proportionality constant is not important because the final solution can only be determined with respect to a constant.

IV. DISCUSSION

In this section it will be investigated how well the theoretical formulae (8) can be fitted to the simulation results shown in Fig. 2. To each cable a coordinate x_i can be assigned, given by

$$x_i = [i - \frac{n+1}{2}] \times 5\,cm, \tag{9}$$

where $n = 3, 6, 9, ... 30$, which corresponds to the centre of each cable. The half width of the cable set is then $a_n = (n/2) \times 5$ cm. According to the theoretical results (8), one has to to verify whether

$$p_i \propto \frac{1}{\sqrt{a_n^2 - x_i^2}}, \tag{10}$$

turns out to be valid or not. It is more convenient to verify that

$$\frac{1}{p_i^2} \propto (a_n^2 - x_i^2), \tag{11}$$

will be valid for all values of n used in the simulations.

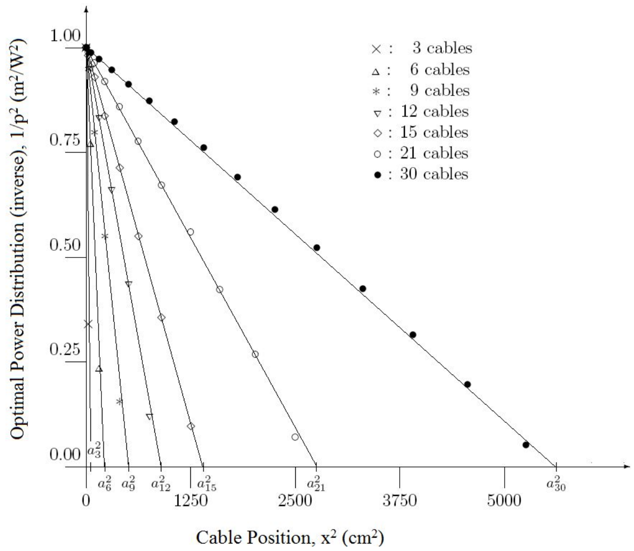

Fig. 4. Optimal power distribution (pi) versus cable position (x).1/pi2 is plotted versus x2 in order to obtain a linear relationship.

In Fig. 4 $1/p_i^2$, as obtained from the simulations, has been plotted as a function of x_i^2. It is remarkable that a linear relation appears which is in full agreement with the theoretical analysis. Moreover, all lines intersect the horizontal axis in points with abscissae given by a_n^2, in full agreement with (11).

Taking into account that the theoretical analysis, outlined in the previous section, assumed an infinitely thin flat plate as the heat source at a uniform temperature whereas the simulation used cables for the heat generation, the agreement between both can be described as surprisingly good.

In the theoretical section, convection on the ground level was not taken into account. The ground layer was assumed to be infinite. It proves that the power distribution is entirely determined by the thermal conduction in the near neighbourhood of the underground cables.

The same set of simulations has also been carried out for different cable diameters giving the exact same results.

V. CONCLUSION

The optimal power distribution for underground cables has been determined using numerical simulations. The criterion was that all cables should have the same temperature increase. The same problem was also solved using an analytical calculation and it was found that the numerical results agree very well with the theoretical analysis. As a consequence, one has now a simple formula to find the optimal power distribution for a set of underground cables.

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