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A Type of Fuzzy T-S Controller for a Nonlinear MIMO Dynamic Plant

P. Dworak[1]
[1]*Department of Control Engineering and Robotics, West Pomeranian University of Technology, 26 Kwietnia 10; 71–126, Szczecin, Poland*
pawel.dworak@zut.edu.pl

***Abstract*—In the paper a control system utilizing Takagi-Sugeno (T-S) fuzzy rules for a MIMO nonlinear dynamic plant is presented. In the proposed control system use is made of a set of linear modal controllers that create a multi-controller structure from which a group of controllers appropriate to given operation conditions is chosen and used to calculate, by employing T-S fuzzy rules, control signals. Stability conditions of the closed-loop system may be checked by the use of the simple, well-known techniques thus the proposed method allows one to simplify the synthesis of the controller. Problems of the controller practical realization and its implementation in, for example, constrained memory of programmable automation devices are discussed. The final part of the paper includes simulation results of system operation with an adaptive controller of (stepwise) varying parameters along with conclusions and final remarks.**

***Index Terms*—Nonlinear MIMO plant, stability analysis, Takagi-Sugeno controller.**

I. INTRODUCTION

Control of a nonlinear dynamic plant is a very difficult task, especially for multi input multi output (MIMO) plants. Despite many remarkable theoretical achievements in analysis of nonlinear control and first attempts to employ nonlinear algorithms in industry, a number of problems still remain to be solved. Among them, controller synthesis, checking its stability and possibility of realization of nonlinear control algorithms in typical industrial controllers should be mentioned. A lot of different methods has been proposed for many years, including those of design a nonlinear controller [1], [2], linear robust controllers [3] or predictive controllers [4]. Ones of the most extensively studied are the adaptive [5]–[7] and beginning from [8] those based on fuzzy logic theory [9]–[12].

The T-S fuzzy systems gain its popularity partially because its stability conditions can be easily tested by the use of linear matrix inequalities (LMIs). However, the proposed stability conditions are usually quite restrictive, and no common Lyapunov function exists for many stable fuzzy systems. On the contrary many theorems provided do not take into account e.g. shapes of membership functions [13] so they offer sufficient conditions only. Relaxation and/or finding of the necessary conditions are intensively studied [9]–[12] but so far they are either not constructive or very complicated, thus usually unfeasible in practice. Promising results may be obtained by the use of the nonquadratic and piecwise Lyapunov functions [14], [15].

What many multi-controller structures, where not all controllers at the moment are utilized, have in common is that all controllers employed in these structures must be stable by themselves. This means that system strong stability conditions should be fulfilled [16].

To soften these problems a method for reducing the largeness of a T-S fuzzy structure working with a big amount of linear controllers, necessary to cover all nonlinearities of the plant is proposed. An adaptive modal MIMO controller with varying parameters in the process of operation is studied. The controller can be physically realized as a multi-controller structure of modal controllers and the current control signals calculated using the T-S rules from the appropriate group of controllers. The group is selected during system operation on the basis of auxiliary signals measured, on which the operating points of the nonlinear plant are dependent. A vast reduction of the running controllers allows one to soften stability conditions as there is only a small group of controllers running at the same time. Local controllers do not have to cover all plant nonlinearities. So, as not all controllers at the moment are utilized in a closed-loop system, they do not have to be stable by themselves, which considerably softens synthesis constraints. The stability checking procedure does not demand to calculate a huge number of LMI and very simple fuzzy rules adopted allow one to decrease the size of the controller and make the control system easy to implement in any programmable controller.

The paper is organized as follows. Section II presents a mathematical background of T-S modelling of continuous nonlinear plants. In Section III stability conditions of the closed-loop system with a T-S controller are derived. The structure of the proposed control system as well as its stability conditions are presented in Section IV. A simulation example of the synthetized control system for a nonlinear model of a drilling vessel is presented in Section V. The paper ends with a conclusion in Section VI.

II. FUZZY MODEL OF A PLANT AND A CONTROLLER

A fuzzy model of a dynamic plant may be described as a set of rules [8], [13], [17]

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$$R_o^i : if\ p_1(t)\, is\, A_1^{q_1}\ and\ p_2(t)\, is\, A_2^{q_2}$$
$$and \ldots and\ p_{no}(t)\, is\, A_{no}^{q_{no}}\ then\, \Sigma_o^i, \tag{1}$$

for $i = 1, 2, \ldots, ro$, with $p_k(t)$ denoting variables of antecedents and $A_k^{q_k}$ fuzzy sets representing each of them, for $k = 1, \ldots, no$. Each fuzzy set $A_k^{q_k}$ is an element of a set of fuzzy sets (linguistic values) of a variable p_k

$$A_k^{q_k} \in \Omega_k = \left\{\Omega_k^1, \Omega_k^2, \ldots, \Omega_k^{r_k}\right\}, \tag{2}$$

where r_k is a number of elements of the set Ω_k, for $q_k = 1, \ldots, r_k$. When a grid partition of the domain of input variables is used one obtains a fuzzy system with the number of rules

$$ro = r_1 \cdot r_2 \cdot \ldots \cdot r_{no} = \prod_{j=1}^{no} r_j. \tag{3}$$

Functional consequent of each rule Σ_o^i is a local linear model of the plant described by the state and output equations:

$$\begin{cases} \dot{\mathbf{x}}_o(t) = \mathbf{A}_{oi}\mathbf{x}_o(t) + \mathbf{B}_{oi}\mathbf{u}(t), \\ \mathbf{y}(t) = \mathbf{C}_o\mathbf{x}_o(t), \end{cases} \tag{4}$$

where $\mathbf{x}_o(t) \in R^n$, $\mathbf{u}(t) \in R^m$ and $\mathbf{y}(t) \in R^l$ are the state, input and output vectors respectively.

Denoting normalized activation levels of the rules of the plant model (1) by

$$\tilde{w}_o^i(p) = \frac{w_o^i(p)}{\sum_{l=1}^{ro} w_o^l(p)}, \tag{5}$$

we obtain this model in the following form:

$$\begin{cases} \dot{\mathbf{x}}_o(t) = \sum\limits_{i=1}^{ro} \tilde{w}_o^i \left[\mathbf{A}_{oi}\mathbf{x}_o(t) + \mathbf{B}_{oi}\mathbf{u}(t)\right], \\ \mathbf{y}(t) = \mathbf{C}_o\mathbf{x}_o(t). \end{cases} \tag{6}$$

Assuming, in general, different number of inferring rules for the plant model (1) and controller $ro \neq r$, similarly one may take a set of rules for a continuous fuzzy controller T-S

$$R_c^j : if\ p_1(t)\, is\, A_1^{h_1}\ and\ p_2(t)\, is\, A_2^{h_2}$$
$$and \ldots and\ p_{nc}(t)\, is\, A_{nc}^{h_{nc}}\ then\, \Sigma_c^j, \tag{7}$$

for $j = 1, 2, \ldots, rc$, with $p_d(t)$ denoting variables of antecedents and $A_d^{h_d}$ fuzzy sets representing each of them, for $d = 1, \ldots, nc$. Functional consequent of each rule Σ_c^j is a local linear model of the nonlinear controller given in state and output equations:

$$\begin{cases} \dot{\mathbf{x}}_c(t) = \mathbf{A}_{cj}\mathbf{x}_c(t) + \mathbf{B}_{cj}\mathbf{e}(t), \\ \mathbf{u}(t) = \mathbf{C}_c\mathbf{x}_c(t), \end{cases} \tag{8}$$

where $\mathbf{e}(t) = \mathbf{y}_{ref}(t) - \mathbf{y}(t)$ is the control error.

Equations of the nonlinear model of the controller (8) take the form:

$$\begin{cases} \dot{\mathbf{x}}_c(t) = \sum\limits_{j=1}^{r} \tilde{w}_c^j \left[\mathbf{A}_{cj}\mathbf{x}_c(t) + \mathbf{B}_{cj}\mathbf{e}(t)\right], \\ \mathbf{u}(t) = \mathbf{C}_c\mathbf{x}_c(t), \end{cases}. \tag{9}$$

where $\tilde{w}_c^j(t)$ are normalized activation levels of the activation levels $w_c^j(t)$ of the rules (7) in time t.

The number of rules depends on and grows rapidly with the increase of dimensionality no of parameters p_k and the number r_k of fuzzy sets corresponding to each parameter p_k. This is called “curse of dimensionality” and that is why the domains of individual parameter p_k should be divided into a number of fuzzy sets as small as possible. However, in case of a strongly nonlinear plant the number of rules have to be large enough to obtain the demanded model accuracy, which may induce problems with implementation of such system and maintaining closed loop stability. It will be shown in detail in the following section.

III. STABILITY OF A CONTROL SYSTEM WITH THE T-S CONTROLLER

The structure of the closed–loop system with a T-S controller may be presented as in Fig. 1.

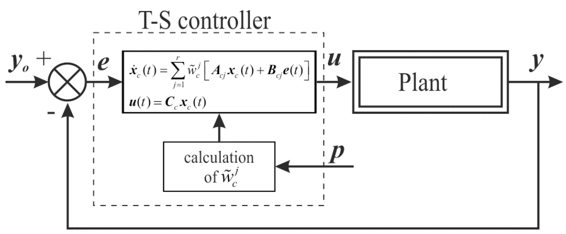


Fig. 1. General scheme of control system with TS controller.

The dynamics of such a system with a plant described by the model (4) and controller model (9) may by described by two equations

$$\dot{\mathbf{x}}_o(t) = \sum_{i=1}^{ro} \tilde{w}_o^i \left[\mathbf{A}_{oi}\mathbf{x}_o(t) + \mathbf{B}_{oi}\mathbf{C}_c\mathbf{x}_c(t)\right], \tag{10}$$

$$\dot{\mathbf{x}}_c(t) = \sum_{j=1}^{r} \tilde{w}_c^j \left[\mathbf{A}_{cj}\mathbf{x}_c(t) + \mathbf{B}_{cj}\left(\mathbf{y}_{ref}(t) - \mathbf{C}_o\mathbf{x}_o(t)\right)\right]. \tag{11}$$

As $\sum\limits_{i=1}^{ro} \tilde{w}_o^i(t) = 1$ and $\sum\limits_{j=1}^{r} \tilde{w}_c^j(t) = 1$ then

$$\dot{\mathbf{x}}_o(t) = \sum_{i=1}^{ro} \tilde{w}_o^i \sum_{j=1}^{r} \tilde{w}_c^j \left[\mathbf{A}_{oi}\mathbf{x}_o(t) + \mathbf{B}_{oi}\mathbf{C}_c\mathbf{x}_c(t)\right], \tag{12}$$

$$\dot{\mathbf{x}}_c(t) = \sum_{j=1}^{r} \tilde{w}_c^j \sum_{i=1}^{ro} \tilde{w}_o^i \left[\mathbf{A}_{cj}\mathbf{x}_c(t) + \mathbf{B}_{cj}\mathbf{y}_{ref}(t) - \mathbf{B}_{cj}\mathbf{C}_o\mathbf{x}_o(t) \right]. \quad (13)$$

and introducing the extended state vector as $\mathbf{v}^T(t) = \left[\mathbf{x}_o^T(t) \quad \mathbf{x}_c^T(t) \right]$ (13) may be described in the form

$$\dot{\mathbf{v}}(t) = \sum_{i=1}^{ro} \tilde{w}_o^i \sum_{j=1}^{r} \tilde{w}_c^j \mathbf{A}_{ij}\mathbf{v}(t) + \mathbf{B}_j \mathbf{y}_{ref}(t), \quad (14)$$

where:

$$\mathbf{A}_{ij} = \begin{bmatrix} \mathbf{A}_{oi} & \mathbf{B}_{oi}\mathbf{C}_c \\ -\mathbf{B}_{cj}\mathbf{C}_o & \mathbf{A}_{cj} \end{bmatrix}, \quad (15)$$

$$\mathbf{B}_j = \begin{bmatrix} 0 \\ \mathbf{B}_{cj} \end{bmatrix}. \quad (16)$$

Without loss of generality one can assume that $\mathbf{y}_{ref}(t) = \mathbf{0}$ and equations of the dynamics of the closed-loop control system take the form

$$\dot{\mathbf{v}}(t) = \sum_{i=1}^{ro} \sum_{j=1}^{r} \tilde{w}_o^i \tilde{w}_c^j \mathbf{A}_{ij}\mathbf{v}(t). \quad (17)$$

Theorem 1. The equilibrium point of the dynamic system (17) is globally asymptotically stable if there exists a symmetric positive defined matrix $\mathbf{P}$ such that for the state matrix $\mathbf{A}_{ij}$ of each local model the following inequality is satisfied

$$\mathbf{A}_{ij}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{ij} < 0, \quad (18)$$

where $i = 1, \ldots, ro$ and $j = 1, \ldots, r$.

Proof Let us define a scalar Lyapunov function in the form $V(\mathbf{v}(t)) = \mathbf{v}^T(t)\mathbf{P}\mathbf{v}(t)$ where $\mathbf{P}$ is a symmetric positive defined matrix. Then its derivative is

$$\begin{aligned} \dot{V}(\mathbf{v}(t)) &= \dot{\mathbf{v}}^T(t)\mathbf{P}\mathbf{v}(t) + \mathbf{v}^T(t)\mathbf{P}\dot{\mathbf{v}}(t) = \\ &= \mathbf{v}^T(t)\left[\sum_{i=1}^{ro}\sum_{j=1}^{r} \tilde{w}_o^i\tilde{w}_c^j \mathbf{A}_{ij}^T\mathbf{P} + \sum_{i=1}^{ro}\sum_{j=1}^{r} \tilde{w}_o^i\tilde{w}_c^j \mathbf{P}\mathbf{A}_{ij} \right]\mathbf{v}(t) = \\ &= \sum_{i=1}^{ro}\sum_{j=1}^{r} \tilde{w}_o^i\tilde{w}_c^j \mathbf{v}^T(t)\left[\mathbf{A}_{ij}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{ij} \right]\mathbf{v}(t). \end{aligned} \quad (19)$$

As $\tilde{w}_o^i > 0$ for every $i = 1, \ldots, ro$ and $\tilde{w}_c^j > 0$ for every $j = 1, \ldots, r$ then $\sum_{i=1}^{ro}\sum_{j=1}^{r} \tilde{w}_o^i\tilde{w}_c^j > 0$, and the derivative (19) along trajectories of (17) is less than zero $\dot{V}(\mathbf{v}(t)) < 0$ for $\mathbf{v}(t) \neq 0$ if only the condition (18) is satisfied. Then function $V(\mathbf{v}(t)) = \mathbf{v}^T(t)\mathbf{P}\mathbf{v}(t)$ is a Lyapunov one for the nonlinear dynamic system (17).

With a great number of rules verification of the conditions (18) of the Theorem 1 may be very troublesome or even computationally unachievable. Also strong plant nonlinearities may make the condition (18) unsolvable. Hence, there is a need to reduce the number of conditions, e.g. by assuming the same number of plant and controller rules with a controller calculated for the assumed fuzzy plant model or elimination of equations for which $\tilde{w}_o^i\tilde{w}_c^j = 0$. Theorem 1 requires also that the closed-loops for each pair (Σ_o^i, Σ_c^j) should be stable. Moreover, all controllers have to be stable by itself which puts additional constraints on their synthesis. This typical result is very restrictive and despite its simplicity is rather impractical. Nonetheless it does not mean that such a system cannot be synthetized for a strongly nonlinear MIMO plant. In the next section we show a T-S like control structure, which allows us to avoid all the mentioned above synthesis constraints.

IV. DESCRIPTION OF THE PROPOSED CONTROL SYSTEM STRUCTURE

Although all local controllers of the typical T-S controller (9) are running, only a few of them are practically used in calculation of the T-S controller outputs. This allows us to propose a T-S type controller where, by definition, only a reduced number of local controllers is running simultaneously. As we show further it softens the stability conditions of the closed-loop control system, thus reduce synthesis constraints, significantly reduce the number of simultaneously working controllers with maintaining the smoothness of operation and switching of the fuzzy structure.

Let us assume the most popular class of membership functions, the triangular ones. If triangular membership functions are consistent and complete, then, for a given input p_k, at most two membership functions, $«_{\Omega_k^i}(p_k)$ and $«_{\Omega_k^{i+1}}(p_k)$ are fired simultaneously. It means that the relation

$$«_{\Omega_k^i}(p_k) + «_{\Omega_k^{i+1}}(p_k) = 1 \quad (20)$$

is satisfied for every input p_k. Such assumption of shapes of the membership functions means that we may use only controllers in the direct vicinity of the controller most appropriate to the working point of the plant.

Let us assume the same number of antecedents and rules of the fuzzy plant and the controller models (1) and (7). Then the local linear plant model $\Sigma_o^{i_o}$ and its corresponding local controller $\Sigma_c^{j_o}$ may be described as current ones for every antecedent p_{kc} for which membership functions $«_{\Omega_k^{qk}}(p_{kc})$ for $k = 1, \ldots, no$, $q_k = 1, \ldots, r_k$ have the greatest values.

For the considered orthogonal membership functions at most 2^{no} rules are fired simultaneously. For example, for a system with two antecedents $no = 2$ presented in Fig. 2 for a current working point with $p_1(t) = p_{1c}$ and $p_2(t) = p_{2c}$ we obtain four controllers used.

Depending on the current values of the auxiliary signals p_{kc} for $k=1,\ldots,no$, the direct vicinity of the current controller $\Sigma_c^{j_o}$ are all controllers (rules) for which parameters p_k belong to fuzzy sets $\Omega_k^{q_k^o-1}$ and $\Omega_k^{q_k^o+1}$ for $k=1,\ldots,no$. It accounts to $rop=3^{no}$ fuzzy controllers and practically defines a *no* dimensional cube around the point for which the controller $\Sigma_c^{j_o}$ was synthetized.

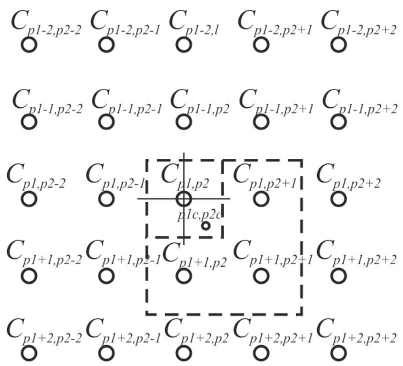


Fig. 2. Scheme of choosing of the moving controllers group.

Considering local stability of the T-S controller for the working points p_{kc}, $k=1,\ldots,no$ one has to check stability for each 2^{no} groups of 2^{no} controllers. From a practical point of view (accepting not full relaxation of the stability conditions) it seems reasonable to consider the whole neighbourhood of the $rop=3^{nc}$ controllers. For example, for a system with two antecedents $no=2$ presented in Fig. 3 we obtain a group of nine local controllers $rop=9$ utilized in calculating of the T-S controller stability. It pays the most for systems with small number of auxiliary signals p_k, $k=1,\ldots,no$ with a great number of fuzzy sets $A_k^{q_k}$, $q_k=1,\ldots,r_k$.

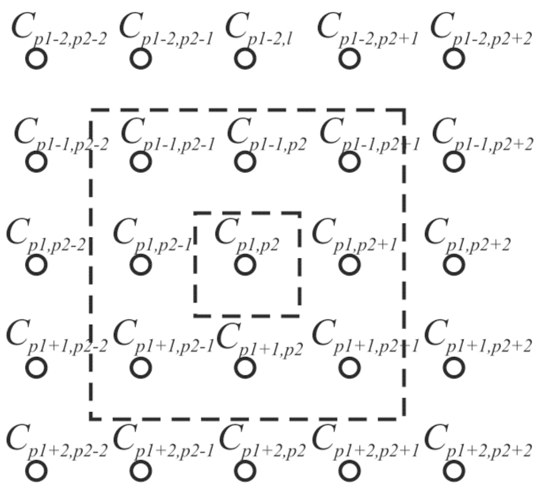


Fig. 3. Scheme of choosing of the moving controllers group.

Let us denote activation levels $w_{cc}^j(t)$ of the rules (1) and (7) for the *rop* neighbourhood of the current working point p_{kc}, $k=1,\ldots,no$, together with the corresponding subset $\mathbf{A}_{i_c j}$ of matrices (15), (16). Then the equations of the closed-loop system dynamics (17) for the current working point may be described as

$$\dot{\mathbf{v}}(t)=\sum_{j=1}^{rop}\tilde{w}_{cc}\mathbf{A}_{i_c j}\mathbf{v}(t). \tag{21}$$

Theorem 2 The equilibrium point of the dynamic system (17) is globally asymptotically stable if for each local linear plant model Σ_o^i of the dynamic system (6) there exists a symmetric positive defined matrix $\mathbf{P}_{ij}$ such that for each subset of state matrices $\mathbf{A}_{i_c j}$ the following inequality is satisfied

$$\mathbf{A}_{i_o j}^T\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{i_o j}<0. \tag{22}$$

where $i=1,\ldots,ro$ and $j=1,\ldots,rop$.

Proof The proof consists of two parts. The first, regarding stability conditions of the local system for the assumed current point p_{kc}, is similar to the proof of the Theorem 1. The second concerns stability conditions while switching between controllers.

Taking into account the plant dynamics, the number of fuzzy rules and the controller sampling time (of its digital realization) should be chosen in such a way as to avoid great “jumps” of working points p_{kc}, at subsequent moments, i.e. in subsequent sampling moments controllers Σ_c^j appropriate to the current working points should abut each other.

To show the stability of the above described closed loop system with “moving group” (gain-scheduled) controller the stability theory of the nonsmooth system given in [18], used successfully e.g. in [6] has been utilized.

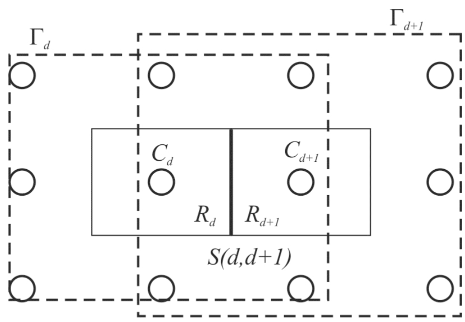


Fig. 4. Stability regions of two switching controllers.

If conditions of Theorem 2 for each controllers group in the grid are satisfied, then each two neighboring controllers C_d and C_{d+1} are stable in regions Γ_d and Γ_{d+1} that include region $R_d\in\Gamma_d$ and $R_{d+1}\in\Gamma_{d+1}$ for which they are used in control (see Fig. 4). The switching between these two controllers occurs at the switching boundary $S(d,d+1)$.

Denote $\mathbf{A}_{cl}$ as a state matrix of the closed-loop system for each plant-controller pair (when each controller is synthetized with e.g. use of pole placement method then each plant-controller pair the closed-loop system has the same dynamics described by matrix $\mathbf{A}_{cl}$). Then let us choose a continuous and parameter-independent Lyapunov function $V(\mathbf{x}_{cl},t)=\mathbf{x}_{cl}^T\mathbf{P}\mathbf{x}_{cl}$, where $\mathbf{x}_{cl}$ is the state of closed-loop system with state matrix $\mathbf{A}_{cl}$ and a symmetric positive-defined matrix $\mathbf{P}$ found when checking conditions of Theorem 2. As it was calculated in [19] the $\nabla V = 2\mathbf{P}\mathbf{x}_{cl}$ is continuous, $\dot{V}(\mathbf{x}_{cl},t)$ exists almost everywhere and $\dot{V}(\mathbf{x}_{cl},t) \in^{a.e.}$, $\dot{\tilde{V}}(\mathbf{x}_{cl},t) = \cap_{\P\in\partial V(\mathbf{x}_{cl},t)} \P^T K\left[\mathbf{A}_{cl}\mathbf{x}_{cl}\right]$.

Then we get $\dot{\tilde{V}}(\mathbf{x}_{cl},t) = \nabla V^T K\left[\mathbf{A}_{cl}\mathbf{x}_{cl}\right] = K\left[\nabla V^T \mathbf{A}_{cl}\mathbf{x}_{cl}\right] = K\left[2\mathbf{P}^T\mathbf{x}_{cl}^T\mathbf{A}_{cl}\mathbf{x}_{cl}\right]$ and since $\mathbf{P}$ is positive and the real parts of the eigenvalues of $\mathbf{A}_{cl}$ are all negative then $\mathbf{x}_{cl}^T\mathbf{A}_{cl}\mathbf{x}_{cl}<0$, which means $\dot{\tilde{V}}(\mathbf{x}_{cl},t)<0$. Thus $\mathbf{A}_{cl}$ with two switching controllers is quadratically stable in the region $R_d \cup R_{d+1}$. The same procedure applied to other directions and region of the parameter space allows one to prove the quadratic stability of the closed loop system in the whole operating parameter space.

The stability checking procedure requires solving *ro* (3) sets of $rop = 3^{no}$ equations, which is feasible for the most fuzzy systems, especially those with only few $p_k(t)$ (small no) parameters in plant and controller fuzzy models.

V. EXAMPLE

To show the effectiveness of the above described control structure we present a control system for a nonlinear state-space model of low-frequency 3DOF motions of the "Wimpey Sealab" ship [20]. The model can be presented as:

$$\begin{cases}\dot{x}_1 = x_4\cos x_3 - x_5\sin x_3 + V_c\cos\Psi_c,\\ \dot{x}_2 = x_4\sin x_3 + x_5\cos x_3 + V_c\sin\Psi_c,\\ \dot{x}_3 = x_6,\\ \dot{x}_4 = 0.088x_5^2 - 0.132x_4V_s + 0.958x_5x_6 + 0.958u_1,\\ \dot{x}_5 = -1.4x_5V_s - 0.978x_5^3/V_s - 0.543x_4x_6 +\\ +0.037x_6\left|x_6\right| + 0.544u_2,\\ \dot{x}_6 = (-0.764x_4x_5 + 0.258x_5V_s - 0.162x_6\left|x_6\right| + u_3)/a,\end{cases} \quad (23)$$

$$\mathbf{y} = \begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{bmatrix}\mathbf{x}, \quad (24)$$

where $V_s=\sqrt{x_4^2+x_5^2}$ - the linear speed of the ship with respect to water, $a = k_{zz}^2 + 0.0431$ - the ship's inertia moment, k_{zz}^2 - the square of the relative inertia radius with respect to the ship's length L_{pp}; V_c, Ψ_c - the speed and direction of the sea current (denoted in Fig. 5). All the signals in (23), (24) are dimensionless, related to the ship's dimensions and weight, with the dimensionless time $t = t_r/\sqrt{L_{pp}/g} \approx 0.32\, t_r$.

The modal controllers used in the proposed control system are multivariable dynamic systems synthesized by the pole placement technique. Detailed description of their synthesis may be found in [21]. The described method allows us to synthetize the control system with a group of linear controllers calculated for velocities $V_s \in [-4.9 \div 4.9]$ *knots* with the resolution of 0.2 *knot* and round angle $\Psi_c - x_{30} \in [0 \div 360^0]$ with the resolution of 5^0 (about 0.0873 *rad*), which results in a set of 3650 modal controllers. It determines directly the number of calculations to check stability of the system. The standard procedure satisfying conditions of Theorem 1 requires solving a set of 3650 nonlinearities defined by (18). Assuming a 3 × 3 moving group of controllers, according to the Theorem 2, to check stability of this positioning control system it is enough to solve 3650 sets of 9 nonlinearities. It does not last long and proves system stability. It does not require also that all local controllers be stable by themselves to ensure system stability.

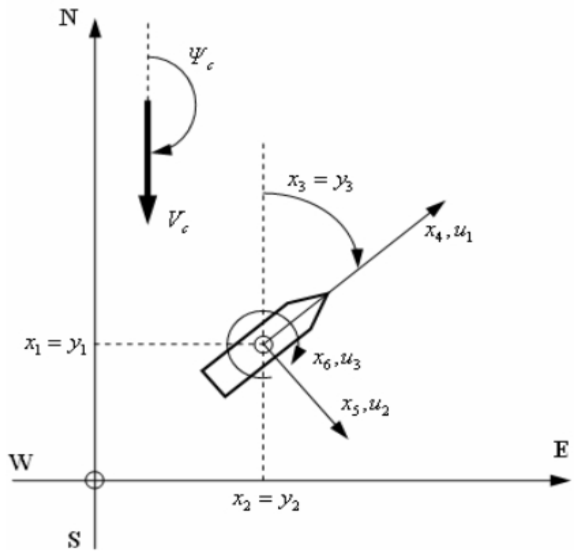


Fig. 5. Ship's co-ordinate systems.

Due to the assumed resolution of the ship's velocities and her yaw angle the membership functions for ship's velocity and yaw angle for a system with 3×3 controllers group, may be assumed as in the Fig. 6.

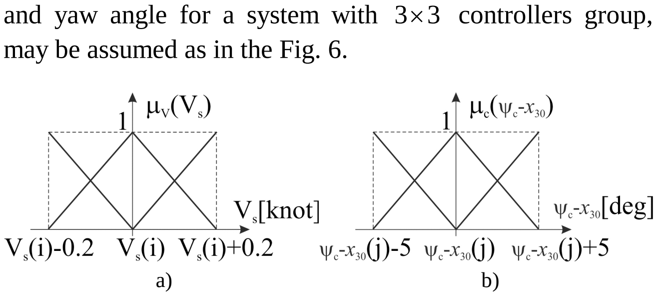


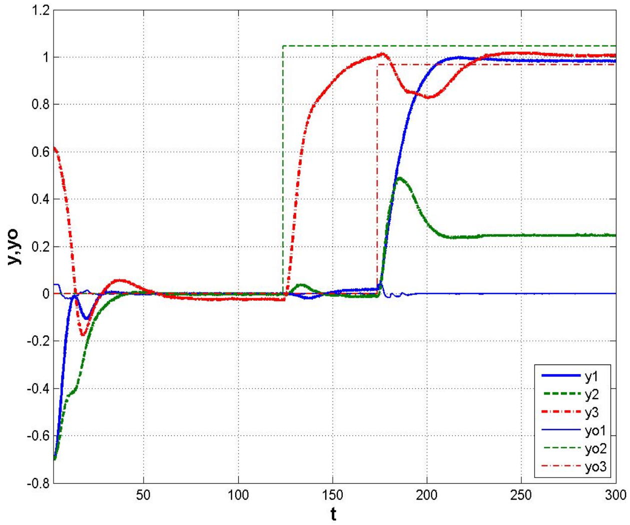
Fig. 6. Membership functions for ship's velocity V_s (a) and yaw angle $\Psi_c - x_3$ (b).

Bellow we present results of simulation tests of the stepwise switched controllers and a fuzzy controller carried out in Matlab/Simulink and in Hardware in the Loop (HiL) environment. The HiL test stand consist of two 1104 dSpace

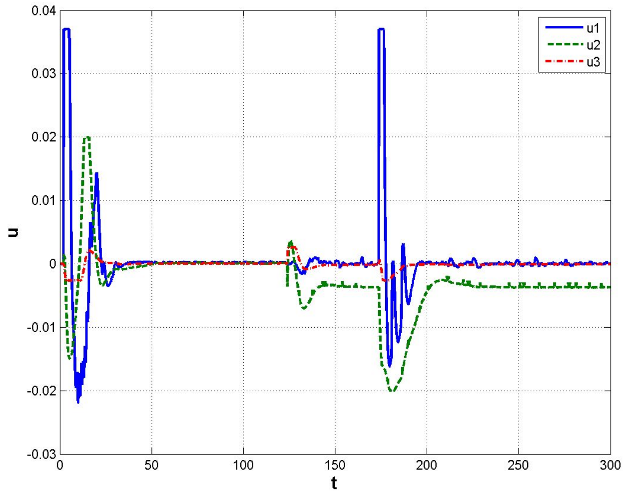
R&D controller boards which allows one to simulate in thereal time regime independently the Wimpey-Sealab model (23) and the controller. The ship's and controller inputs and outputs as well as the current ship velocity are transmitted between cards by wires with the use of 12 bit independent AQ and AI.

Figure 7 and Fig. 8 present results of simulation of an adaptive controller with stepwise changed parameters and a described T-S structure. The simulation has been carried out for the initial position that indicated ship's position about 100 *m* on the left below the drilling point with the initial course angle $x_3(0) = 35^0$ and velocities $x_4(0)$ and $x_5(0)$.

After 150 time units the course has been changed from $y_{30} = x_{30} = 0^0$ to $y_{30} = x_{30} = 60^0$ and after 200 time units the ship has been brought to the new drilling point about 100 *m* right over the old one and come to a standstill with a velocity of $V_s = V_c = 2$ *knots* relative to water.

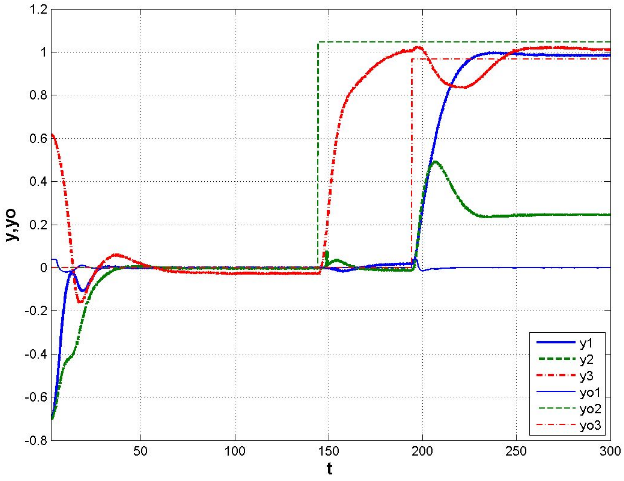


a)

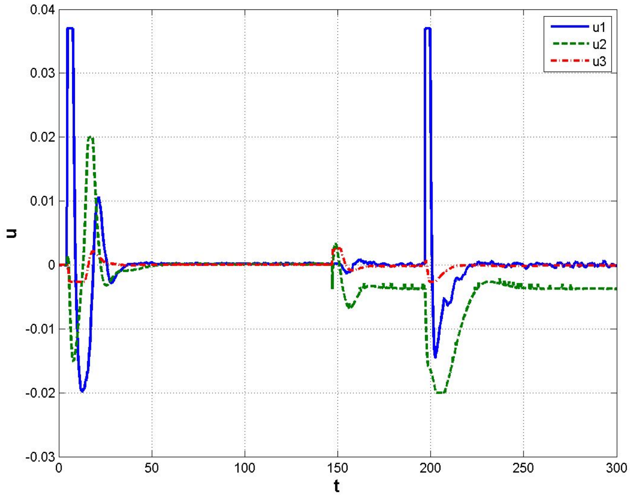


b)

Fig. 7. Ship's position and yaw angle $\mathbf{y}(t)$ (a) and control signals $\mathbf{u}(t)$ (b) for a multicontroller without fuzzy rules.



a)



b)

Fig. 8. Ship's position and yaw angle $\mathbf{y}(t)$ (a) and control signals $\mathbf{u}(t)$ (b) for a multicontroller with 3 × 3 fuzzy rules.

Both controllers behaved correctly yielding time responses without excessive oscillations experienced by the control signals $\mathbf{u}(t)$. Additionally, the fuzzy controller makes the control signals smoother.

VI. CONCLUSIONS

The presented results of simulation prove the feasibility of the proposed control system. A significant reduction of the running controllers in the proposed solution allows one to soften stability conditions. There is no necessity to calculate a huge number of LMI conditions to check system stability. It has to be checked for a series of defined $k \times p$ groups only, which is feasible and simple to carry out by the use of eg. Matlab Robust control toolbox and its LMI solvers. So, as not all controllers at the moment are utilized in a closed-loop system, they do not have to be stable by themselves, which considerably softens synthesis constraints. Moreover, very simple fuzzy rules adopted allow one to decrease the size of the controller and make the control system easy to implement in any programmable controller.

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