

## A Non-Linear System's Response Identification using Artificial Neural Networks

**B. Sokouti, M. Sokouti, S. Haghipour**

Faculty of Engineering, Islamic Azad University –Tabriz Branch,  
Tabriz, Iran, phone: +984113366634, e-mail: b.sokouti@gmail.com

**crossref** <http://dx.doi.org/10.5755/j01.eee.113.7.614>

### Introduction

Identifying the systems transient responses with different inputs and various settling times will take a lot of time and repetition of getting new responses with the maximum overshoot ( $M_p$ ) will be harmful for the related systems. Rapid and accurate weighing in industry and its applications has a vast need in laboratory conditions [1, 2].

There are various successful researches in speedy weightings and this depends on the filter being used. In this paper the output weight is as the output parameter (transient terms) and the responses will be taken in to consideration by the ideal step function (without oscillation) [3, 4]. An Artificial neural network is employed for predicting the transient term and system identification. Using a part of the weighing platform data, without the bad effects of transient terms, depending on the theoretical data and the initial learning, predicts the transient term of the system. The Multilayer Neural Network is a common method in Neural Networks. It is known that the Multilayer Perceptron (MLP) with one hidden layer and a non-linear function is one of the most common neural networks [5–10]. For instance the implementation of the multilayer neural network is shown in Macy, Rumelhart Haykin and Pandye.

### The RBF Network

The Radial Basis Function Networks are similar to the biological networks behavior; primarily the hidden layers contain almost sensing units and the output layer contains linear units. Usually for a common RBF network the transfer function in the hidden layer is a Gaussian function that is introduced in formula (1) as below

$$\varphi(x) = \exp\left(-\left(\frac{|x - \mu_i|}{h}\right)^2\right), \quad (1)$$

where  $\mu_i \in R^d$  is called the center vector, and  $h \in R$  is called the kernel width (or smoothing parameter). The

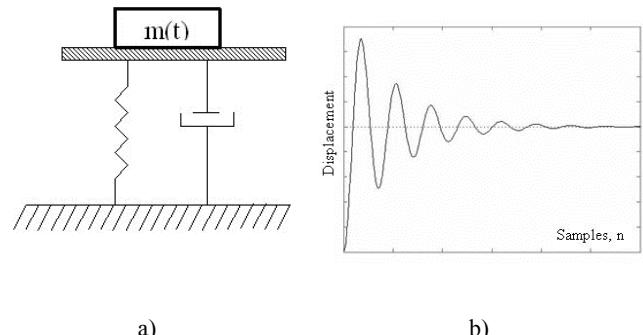
basic RBF network provides a nonlinear transformation of a pattern  $x \in R^d \rightarrow R^c$  according to formula (2)

$$f_j(x) = b_j + \sum_{i=1}^m w_{ji} \varphi\left(\frac{|x - \mu_i|}{h}\right), \quad (2)$$

where  $m$  is the number of basis functions,  $w_{ji}$  is a weight,  $b_j$  is a bias [8–11].

### Weighing system model

An ideal weighing platform can be modeled by a mass-spring-damping structure shown in (Fig. 1, a).



**Fig. 1.** a – weighing platform model with mass-spring-damping structure; b – a typical under damped ideal step response

It has a typical under damped ideal step response as illustrated in (Fig. 1, b).

It is governed by the solution of the following second order differential equation

$$(m(t) + m_p)y''(t) + C(n)y'(t) + K(n)y(t) = gm(t), \quad (3)$$

where  $y(t)$  is the deflection signal obtained from the strain gauge on the weighing machine;  $m(t)$  and  $m_p$  are applied mass and the platform mass respectively;  $C(n)$  is the damping factor;  $K(n)$  is the spring constant and  $g$  is the

gravitational constant. For a general applied mass function  $m(t)$ , this is a non-linear differential equation. However, for commonly encountered situation  $m(t)$  is a step function, which is assumed here. In this case the differential formula (3) is linear, for which the explicit solution is modeled by a constant term plus a transient term [12] which can be under damped ( $u$ ), critically damped ( $c$ ), or over damped ( $o$ ). Thus

$$y(t) = q_0 + \{F_u(q_u, t), F_c(q_c, t), F_o(q_o, t)\}. \quad (4)$$

The transient terms for under damped, critically damped and over damped cases are shown as formula (5),(6) and (7):

$$F_u(q_u, t) = e^{-q_{ul}t} q_{u2} \sin(q_{u3}t + q_{u4}), \quad (5)$$

$$F_c(q_c, t) = e^{-q_{cl}t} (q_{c2} + q_{c3}t), \quad (6)$$

$$F_o(q_o, t) = e^{-q_{ol}t} q_{o2} + e^{-q_{os}t} q_{o4}, \quad (7)$$

where the various  $q$  parameters are related to the initial platform displacement  $b_0$ , initial velocity  $b_1$ , the constant platform parameters  $C(n)$ ,  $K(n)$ ,  $m_p$  and the applied mass  $m(t)$  by the expressions given in the appendix.

Sampled data signals are assumed, for which,  $t = nT$ , where  $T$  is the sample interval. Thus  $y(t)$  is written as  $y(n)$  and the weighing platform system.

### Neural network technique

The Input/Output model describes a dynamic system based on the input and output data; in this model it is supposed that the output of the system can be predicted with system's previous input data. If the typical system (discrete-time) has one input and one output, the model will be as below

$$W(n) = f(y(n), y(n-1), \dots, (y(n-N+1)), \quad (8)$$

where  $y(n)$  and  $W(n)$  are the output values in the time instant  $n$  and  $N$  is the input samples and  $f$  is the non-linear function that make a relationship between the old input data and the new input data. Basically, the purpose of the network is to approximate the applied designs in the dynamic system. The formula (8) is represented as a diagram in (Fig. 2) in which the delay times between two samples are shown by  $z^{-1}$ .

As it is clear, one of the RBF learning algorithms is the Least Mean Square (LMS) which tries to reduce the errors between the actual and desired outputs.

### Simulation results

The Artificial Neural Network presented in (Fig. 2) is used for this purpose, the neural network is being trained in a way that can adapt itself with the dynamic weighing platform behavior that  $W(n)$  is an approximation of the transient mode of dynamic system where the input  $m(t)$  is

applied. The proposed RBF network model shown in (Fig. 2) is the owner of the following characteristics:

- Number of input samples –  $y(n), y(n-1), y(n-2), \dots, y(n-N+1); N = 100$  ;
- Number of output samples in  $W(n)$  is 3;
- Number of layers is 3: input layer, hidden layer and output layer;
- Total number of neurons used for this network is 133, where 100 for input neurons, 30 for our hidden neurons and 3 for the output neurons;
- Momentum rate is 0.95 and Learning rate is 0.5.

A data set of input samples ( $y(n), y(n-1), y(n-2), \dots, y(n-99)$ ) was taken from the MATLAB programming language to simulate the formula (4).

The weighing platform parameters in all simulations are  $m(t) = 5\text{kg}$ ,  $m_p = 0\text{kg m}$ ,  $g = 10\text{m/s}^2$ , sampling interval  $t_s = 0.04\text{ms}$ , initial platform displacement  $b_0 = 0$ , initial velocity  $b_1 = 0\text{mm/s}$ . 30 data patterns were used to train the network; To be sure of the trained network performance, the recalling process includes two stages: the first stage is tested with the seen patterns and the next stage is tested with the unseen patterns to the network and are seen to the user, after these two stage have been completed we can be sure that the network is reliable upon the unseen patterns to both the network and the user. The applied masses to the network are selected in the range of 0.23-1.33 kg in which for achieving the accurate results around the critical point, the more ranges of mass are introduced. The neural network was trained with the patterns that were noise-free. The obtained results show that the neural network can model the non-linear relationship between the continuous data from the weighing platform and the kind of system.

In the simulation, outputs are being considered each of which by taking the 0, 1 values predict the transient mode of the system.

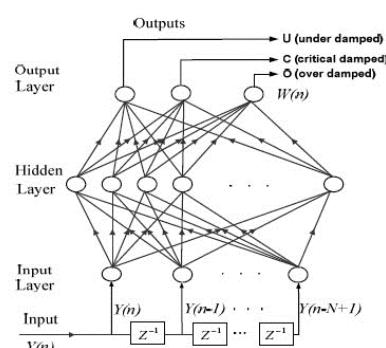
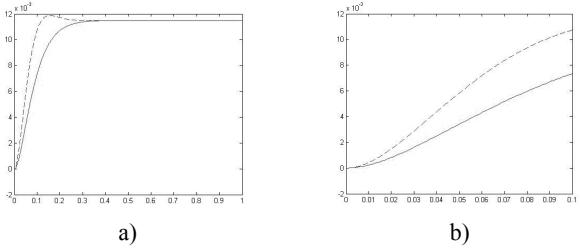


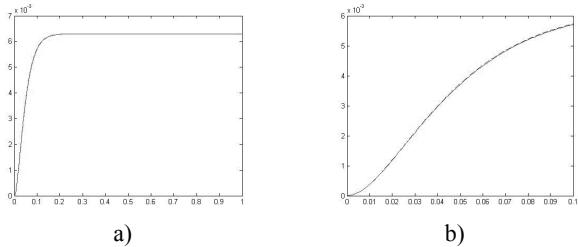
Fig. 2. RBF network transient term identification

(Fig. 3, a) is an example for the under damped mode of the system that can determine the kind of the system by sampling in a period of 0.1 seconds (Fig. 3, b). Additionally, an example of critical damped mode in (Fig. 4, a, Fig. 4, b) and an example of over damped mode in (Fig. 5, a, Fig. 5, b) are illustrated. Dashed graphs (--) in all case are presented as a comparison in system identification. The accepted error rate in system

identification is 2 % that finally the appropriate results are gained.



**Fig. 3.** a – (--) under damped response of the system, (-) Critical damped response of the system for comparison. b – (--) under damped response of the system, (-) Critical damped response of the system for comparison (both in the period of 0.1 seconds)



**Fig. 4.** a- Critical damped response of the system for comparison. b- Critical damped response of the system for comparison (in the period of 0.1 seconds)

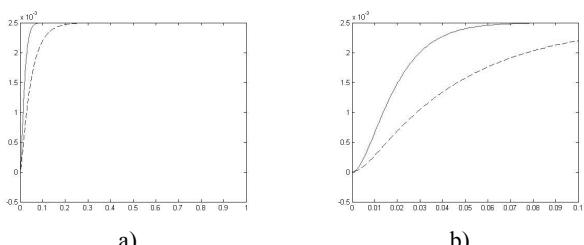
Different values from the results are mentioned in Table 1 in which  $m(t)$  is weight (kg),  $\zeta$  is damping rate and output will show the kind of the system

$$\zeta = \frac{c}{2} \sqrt{m(t) + m_p k} . \dots \quad (9)$$

**Table 1.** Simulation results using RBF

$M(t)$ kg	$\zeta$	Output[u, c, o]
115	0.7372	[1.0172, -0.0179, 0.0000]
95	0.8111	[0.9679, 0.0316, -0.0000]
85	0.8575	[0.9648, 0.0346, 0.0000]
75	0.9129	[1.0876, -0.0880, 0.0000]
69	0.9517	[0.9516, 0.0492, -0.0000]
63	0.9960	[0.0393, 0.9616, 0.0000]
61	1.0122	[0.1571, -0.1567, 1.0001]
35	1.3363	[0.0066, -0.0065, 1.0000]
25	1.5811	[0.0444, -0.0444, 1.0000]

Note:  $C$  is damping factor,  $k$  is spring constant,  $m(t)$  is the applied mass and  $m_p$  is the weighing platform mass ((1,0,0) as under damped; (0,1,0) as critical damped; (0,0,1) as over damped).



**Fig. 5.** a – (--) over damped response of the system, (-) Critical damped response of the system for comparison. b- Critical damped response of the system for comparison (in the period of 0.1 seconds)

## Conclusions

From the simulation results, it can be known that by system response sampling in a time less than %25 of settling time, the kind of the response (Over damped, Critical damped and Under damped) can be identified. In reality the system response identification in the related time can identify the probabilistic problems before their occurrence and unsuitable response effects that can be harmful for systems; additionally it can identify the response in a little period of time and introduce the relation between the input data and the system rapidly. It is clear that if the theoretical data are sampled in the laboratories will be able to identify the kind of response. The researchers can design a similar system for predicting the settling time, the Overshoot ( $M_p$ ) value, and etc. The more values we get from a system the more we can keep the system safe from using the non appropriate data in the laboratory testing.

By implementing this method in various non-linear systems, the obtained results can be used for system identification with different input data that has advantages of keeping the system safe form harm and process the system in a little period of time.

## Appendix

Model parameters of the weighing platform system with the parameters of  $K$  damping factor,  $C$  spring constant, and  $m(t)$  as the applied mass,  $m_p$  as the platform mass,  $b_0$  as the platform displacement and  $b_1$  as the initial velocity are defined as below in three states of the step response: under damped (u), over damped (o) and critically damped (c):

$$\begin{aligned} \text{Under damped (u): } q_0 &= \frac{(m(t) + m_p)g}{K}, \\ q_1 &= \frac{0.5C}{m(t) + m_p}, \quad q_2 = \sqrt{B_1^2 + B_2^2}, \\ q_3 &= \omega_d = \sqrt{K(m(t) + m_p)^{-1} - q_1^2}, \quad q_4 = \tan^{-1}\left(\frac{B_1}{B_2}\right), \\ B_1 &= q_0 - b_0, \quad B_2 = b_1 + \frac{B_1 q_1}{q_3} \\ \text{Critical damped (c): } q_0 &= \frac{(m(t) + m_p)g}{K}, \\ q_1 &= \frac{0.5C}{m(t) + m_p}, \quad q_2 = b_0 - q_0, \quad q_3 = q_1 q_2 - b_1, \quad \omega_d = 0. \\ \text{Over damped (o): } q_0 &= \frac{(m(t) + m_p)g}{K}, \\ q_1 &= \frac{0.5C}{m(t) + m_p} - \omega_d, \quad q_2 = -\frac{(q_0 - b_0)q_3 + b_1}{2\omega_d}, \\ q_3 &= \frac{0.5C}{m(t) + m_p} + \omega_d, \quad q_4 = \frac{(q_0 - b_0)q_1 - b_1}{2\omega_d}, \\ \omega_d &= \sqrt{\left[\left(\frac{C}{2(m(t) + m_p)}\right)^2 - \frac{K}{m(t) + m_p}\right]}. \end{aligned}$$

## References

1. **Shi W. J., White N. M., Brignell J. E.** Adaptive filters in load cell response correction. // Sensors and Actuators, 1993. – No. 37. – P. 280–285.
2. **Shu W. Q.** Dynamic Weighing Under Non-Zero Initial Conditions // IEEE Trans. on Inst. And Meas., 1993. – No. 4(42). – P. 806–811.
3. **Danaci M., Horrocks D.H.** A Non-linear Regression Technique For Improved Dynamic Weighing // Proc. Eur. Conf. on Circuit Theory and Design. – Istanbul: Turkey, 1995. – P. 507–510.
4. **Danaci M.** Signal Processing Techniques Applied to Dynamic Weighing Systems. – Ph. D. thesis, – School of Engineering: Cardiff University of Wales, UK, 1996.
5. **Pandya A. S., Macy R. B.** Pattern Recognition with Neural Networks in C++. – CRC Press Inc., 1996.
6. **Rumelhart D. E., Hinton G. E., William R. J.** Learning internal representation by error propagation // Parallel Distributed Processing. – MIT Press, Cambridge, MA, 1986. – P. 318–362.
7. **Taleb R., Meroufel A.** Control of Asymmetrical Multilevel Inverter Using Artificial Neural Network // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 8(96). – P. 93–98.
8. **Cybenko G.** Approximation by superpositions of a sigmoidal function // Math. Controls, Signals, Syst., 1992. – No.2. – P. 303–314.
9. **Hartman K., Keeler J. D., Kowalski J. M.** Layered Neural Networks with Gaussian hidden units as universal approximation // Neural Computation, 1990. – No. 2. –P. 210–215.
10. **Hornik K., Sitnichcombe M., White H.** Multilayer feedforward networks are universal approximators // Neural Networks, 1989. – No. 2. – P. 359–366.
11. **Bahar H. B., Horrocks D. H.** Dynamic weight estimation using Artificial Neural Network // Artificial Intelligence in Engineering, 1998. – No.12. – P. 135–139.
12. **Jonaitis A., Daunoras J.** Identification of Dynamic Model of Synchronous Generator High Frequency Excitation System, // Electronics and Electrical Engineering, – Kaunas: Technologija, 2011. – No. 1(107). – P. 25–29.

Received 2010 08 20

### **B. Sokouti, M. Sokouti, S. Haghipour. A Non-Linear System's Response Identification using Artificial Neural Networks // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 7(113). – P. 63–66.**

Identifying the kind of a non-linear system at the first initial times of applying different inputs can be useful in system identification; this identification will get important while the kind of the system's response is being predicted before reaching the %2 range of final value (time delay). As the Artificial neural networks provide the best approximation for non-linear system identification, so in this article the simulation is presented by the means of Radial Basis Function in MATLAB. The presented neural network is successfully able to identify the natural responses of a non-linear system in three modes: under damped, critical damped and over damped. Ill. 5, bibl. 12, tabl. 1 (in English; abstracts in English and Lithuanian).

### **B. Sokouti, M. Sokouti, S. Haghipour. Dirbtinių neuroninių tinklų taikymas atsakymams gauti netiesinėse sistemos // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 7(113). – P. 63–66.**

Pirminis netiesinių sistemų įvertinimas gali būti taikomas signalo vėlinimo trukmei nustatyti. Dirbtiniams neuroniniamams tinklams būdinga gera netiesinių sistemų aproksimacija. Pateikiamas radialinės bazinės funkcijos modeliavimas naudojantis programų paketu „Matlab“. Il. 5, bibl. 12, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).