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# Research of Power System Closest Voltage Collapse Boundary

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***Abstract*—The determination of power system voltage collapse boundary is very important to study voltage stability. The explicit expressions of power system voltage high and low voltage curves are obtained by solving the equations which are formed by the hybrid network equations and then get the characteristic equations of the voltage collapse boundary. For analyzing the closest boundary by the characteristic equations we define the one-node voltage collapse boundary and all-node voltage collapse boundary. The closest stability boundary of the power system has been found out by comparing the active power index. Because the Jacobian matrix is singular near the voltage collapse boundary the dimensionality reduced algorithm is proposed. This method can get the value of voltage collapse boundary effectively. The simulations show that the concepts and methods presented in this paper are correct.**

***Index Terms*—Power system analysis computing, power system faults, power system reliability, power system stability.**

## I. INTRODUCTION

A typical case that the power system loses its voltage stability is that the stable equilibrium point and the unstable equilibrium point overlap as the parameters change. The voltage collapse boundary (VCB) appears because the Jacobian matrix of power network equations is singular.

Many methods are available to determine the VCB. One of them is PU curve. The solutions of load voltages are often presented as a PU curve. But the PU curve can not be obtained near the VCB because the Jacobian matrix tends to be singular, and the conventional power flow algorithm is failure. Therefore, the calculation of the VCB often combines with morbid flow algorithm. Continuous power flow methods [1]–[5] track the trend of balanced solution by forecasting or correcting the power flow equations to improve the pathological phenomena and convergence. But, this method fails to give the accurate result if the step length is more. Though Interior Point method is efficient to solve the maximum loading problem [6], [7], this method has the limitation of starting and terminating conditions [8]. The Sequential Quadratic Programming algorithm includes the differentiation of the constraints, and converts collapse point conditions to optimized load, and solves it with Kuhn – Tucker optimality conditions [9]. This method is very slow as it involves many matrices during the iteration process. Fuzzy logic has been used to find the loadability limit in [10], this algorithm does not give the global optimal solution. Evolutionary algorithms have been applied to solve this problem. Particle swarm optimization is a computational intelligence-based technique that is not affected largely by the size and nonlinearity of the problem and can converge to the optimal solution in many problems [11]–[13].

Traditional power system analysis is based on node voltage equations in which the node voltage and power injection are as variables. This method is widely used because of its simple, practical, and intuitive physical meaning. The explicit expression of high and low voltage solution curve is difficult to obtain due to the interconnectedness among the nodes voltage, and numerical calculation or simulation can not show the characteristics of the VCB comprehensively. Because of the problems as described above the study of VCB can not carry out deeply and further reveal the nature.

The current study of VCB mainly focuses on the changes of single parameter (or two-dimensional parameter [14], [15]) and the directions of node injection power are fixed. However, when the power injection changes in different directions the VCB is likely to reach, which means that the VCBs are different from each other. From another perspective, the zero eigenvalue which leads Jacobian matrix to singular is not unique. If the number and the location of zero eigenvalue appear different the VCB should be different. The mechanism of VCB has not been deeply identified.

When the voltage collapse occurs, if there are two smooth solution curve through the collapse boundary is called one-node voltage collapse boundary (ONVCB), if there are more than two smooth solution curve through the collapse boundary is multi-node voltage collapse boundary. Compared to the ONVCB, multi-dimensional bifurcation point needs more stringent generation conditions, but also contains deeper meanings, especially when all nodes are VCBs (also called all-node voltage collapse boundary, ANVCB), ANVCB is also more practical significance.

In this context, we proceed from the state variables of power network equations represented by the branch-current and node-voltage and form the explicit expression of voltage high and low solution curves, then describe the characteristics of VCB. On this basis we analyze the feature of VCB and propose dimensionality reduced algorithm to calculate it.

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## II. THE EXPLICIT EXPRESSION OF EQUILIBRIUM SOLUTION CURVE

The line (or transformer) of electric power system can be simulated by the $\pi$ equivalent circuit model, as shown in Fig. 1, called loop. The loop is composed of three branches: an impedance branch and two grounded branch.

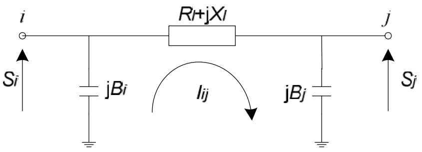


Fig. 1. The $\pi$ equivalent circuit.

In Fig. 1: $i$, $j$ are two nodes on both sides of branch $l$, $\dot{s}_i = p_i + \mathrm{j}q_i$, $\dot{s}_j = p_j + \mathrm{j}q_j$ are node power injections of $i$ and $j$, node voltages are as follows: $\dot{u}_i = e_i + \mathrm{j}f_i$, $\dot{u}_j = e_j + \mathrm{j}f_j$, branch $l$ current is $\dot{i}_l = i_l^a + \mathrm{j}i_l^r$, $R_l + \mathrm{j}X_l$ is the impedance of branch $l$, $\mathrm{j}B_i$ and $\mathrm{j}B_j$ are the grounded susceptance of node $i$ and $j$. The grounded conductivity is ignored for simple calculation in this paper.

For a grounded branch, the current trend is in two ways including grounded capacitor branch and load branch. The current flows in the load branch are not only the current of the circuit itself but also the adjacent loop current. The analytical method of load branch is the same principle with node voltage. Node $i$, for example, the voltage of equivalent voltage source of the load branch is

$$\overset{*}{u_i} = \frac{p_i - \mathrm{j}q_i}{\sum\limits_{l\in i} i_{li} - \mathrm{j}u_i \sum\limits_{l\in i} B_l}, \tag{1}$$

where $l = 1,2,\cdots,L$ is branch collection, $i, j = 1,2,\cdots,N$ is node collection, $\sum\limits_{l\in i} i_{li}$ is the sum of the injection current of node $i$, $\sum\limits_{l\in i} B_l$ is the sum of the grounded susceptance of node $i$, $\mathrm{j}u_i \sum\limits_{l\in i} B_l$ is the sum of the susceptance current of node $i$, $\sum\limits_{l\in i} i_{li} - \mathrm{j}u_i \sum\limits_{l\in i} B_l$ represents the load branch current of node $i$. Derive from (1):

$$\begin{cases} e_i \sum\limits_{l\in i} i_l^a + f_i \sum\limits_{l\in i} i_l^r = p_i, \\ e_i \sum\limits_{l\in i} i_l^r - f_i \sum\limits_{l\in i} i_l^a + (e_i^2 + f_i^2) \sum\limits_{l\in i} B_l = -q_i. \end{cases} \tag{2}$$

At the same time in the Cartesian coordinate system, the electric power network can be described as a mixed equation of branch currents and node voltage:

$$\begin{cases} i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0, \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0. \end{cases} \tag{3}$$

Suppose $x_i = \sum\limits_{l\in i} i_{li}^a$, $y_i = \sum\limits_{l\in i} i_{li}^r$ denote node injection current real part and imaginary part (excluding the grounded branch currents), and $B_{i0} = \sum\limits_{l\in i} B_l$. Derive from (2):

$$\begin{cases} e_i = \dfrac{\begin{pmatrix} \left[2B_{i0}p_i x_i - y_i(x_i^2 + y_i^2)\right] \mp \\ \pm y_i \sqrt{(x_i^2 + y_i^2)^2 - 4B_{i0}q_i(x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2} \end{pmatrix}}{2B_{i0}(x_i^2 + y_i^2)}, \\ f_i = \dfrac{\begin{pmatrix} \left[2B_{i0}p_i y_i + x_i(x_i^2 + y_i^2)\right] \pm \\ \pm x_i \sqrt{(x_i^2 + y_i^2)^2 - 4B_{i0}q_i(x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2} \end{pmatrix}}{2B_{i0}(x_i^2 + y_i^2)}. \end{cases} \tag{4}$$

Obtain the explicit expressions of node voltage in which the branch current is as parameter. From (4)

$$(x_i^2 + y_i^2)^2 - 4B_{i0}q_i(x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2 \geq 0. \tag{5}$$

Derive (5)

$$\left[(x_i^2 + y_i^2) - 2B_{i0}q_i\right]^2 \geq 4{B_{i0}}^2{q_i}^2 + 4{B_{i0}}^2{p_i}^2. \tag{6}$$

Only if meet '$\geq$' in (6) the power network equations have solutions existing. The physical meaning of (6) is that the power network equations have solutions existing if the amplitude square of the node injection current distributes out of the circle of which the $2B_{i0}q_i$ is center and $2B_{i0}\sqrt{q_i^2 + p_i^2}$ is radius. If it distributes on this circle (only '=' meet) two solutions coincide, the system is in the critical state of the VCB. The conditions of solution existing have been found.

In (4), the '$\pm(\mp)$' symbols indicate that the electricity network equations exist two solution curves in per node, one is high voltage solution, the other is low.

But when $B_{i0} = 0$ happens, the node is called degraded node. If there is a degraded node the (2) becomes the following form:

$$\begin{cases} e_i \sum\limits_{l\in i} i_{li}^a + f_i \sum\limits_{l\in i} i_{li}^r = p_i, \\ e_i \sum\limits_{l\in i} i_{li}^r - f_i \sum\limits_{l\in i} i_{li}^a = -q_i, \end{cases} \tag{7}$$

$$\begin{cases} e_i = \dfrac{p_i x_i - q_i y_i}{x_i^2 + y_i^2}, \\ f_i = \dfrac{p_i y_i + q_i x_i}{x_i^2 + y_i^2}. \end{cases} \tag{8}$$

In this case, the node voltage does not have two solutions, but only a single solution, so we need not to face the problem of saddle-node bifurcation.

## III. VCB CHARACTERISTIC EQUATION AND THE DEFINITION OF ONVCB AND ANVCB

If the equality of (6) is met the two solutions curves intersect, that is ONVCB. Suppose the quantity of PQ node is $N_L$ in power system. The VCB generating condition can be derived as

$$(x_i^2 + y_i^2) = 2B_{i0}\gamma_i, \tag{9}$$

where $i \in N_L$, $\gamma_i = q_i + \sqrt{p_i^2 + q_i^2}$ .

The PU node has the similar equation, but we do not discuss that of the PU node in this paper.

The node voltage changes to:

$$\begin{cases} e_i = \dfrac{p_i x_i - y_i \gamma_i}{2B_{i0}\gamma_i}, \\ f_i = \dfrac{p_i y_i + x_i \gamma_i}{2B_{i0}\gamma_i}, \end{cases} \tag{10}$$

where $i \in N_L$. The (9) is called node characteristic equations of VCB. So can deduce that the establishment of (9) on any node will cause voltage collapse, that is the occurrence of VCB is corresponding to the critical conditions of the power network equation solution existing.

The Jacobian matrix is singular and there is zero eigenvalue on the VCB. If the characteristic (9) only meets on one node, and only one pair solution curve intersects, the VCB is called ONVCB, it is called the ANVCB if (9) meets on all nodes at the same time.

## IV. THE CLOSEST POWER SYSTEM VOLTAGE STABILITY BOUNDARY

VCB represents the power system voltage stability boundary. From ONVCB to ANVCB the voltage stability boundary is different. ONVCB is the power system voltage stability boundary of single node. Because ONVCB achieves the stability critical conditions on just one of nodes, it can not represent the static voltage stability boundary of the whole system, and only ANVCB represents the whole system voltage stability boundary.

Assume that the PQ node $i$, for example, in the same generator power increasing scheme, the node power injections are $S_i^{(1)}$ and $S_i^{(N_L)}$ while calculating the ONVCB and ANVCB respectively. It must be

$$S_i^{(N_L)} \le S_i^{(1)}. \tag{11}$$

In conclusion: if regard the space distance of node injection power as a measure, the voltage stability boundary of ANVCB is the most adjacent to the normal state in the whole system, and the ONVCB is the farthest.

Get $\gamma_i, p_i$ according to the calculation results:

$$S_i = \frac{\gamma_i}{\sin\varphi_i + 1}, \tag{12}$$

$$P_i = p_i, \tag{13}$$

where $S_i = \sqrt{p_i^2 + q_i^2}$ , $\sin\varphi_i = q_i / S_i$, $\varphi_i$ is power factor angle. The $S_i$ or $P_i$ represents the establishing conditions of VCB, and also is the voltage stability judgmental index of node $i$.

## V. SOLVING STEPS OF DIMENSIONALITY REDUCED ALGORITHM

The power flow calculation based on Newton method can't be convergence because the jacobian matrix is singular near the VCB. From the analysis in this paper the main reason that jacobian matrix appears zero eigenvalue is that node characteristic (9) is met. In order to calculate VCB the dimension reduced algorithm is used to solve the network equations.

Solving steps:

1) Set the initial value of node voltage $U$ and branch current $I$ ;

2) Suppose node $m$ is voltage collapse point. Remove node $m$ 's equation out of node voltage equations when we form jacobin matrix by (2) and (3). The node voltage equations become one-dimension reduced equations and the voltage variable of $m$ is replaced by (10) in (3). So Newton iteration method can be used to calculate the equations simultaneously with (9);

3) Repeat steps 1 and 2 to calculate the ONVCB of each node;

4) Calculate node voltage collapse index with (12);

5) Similarly, calculate the ANVCB with (3), (10) and (9) in all nodes simultaneously and voltage collapse index;

*6) Judge the node stability margin according to each node $P_i$ of ONVCB and ANVCB.*

## VI. CASE STUDY

Define:

$Q_6$: The reactive power of node 6 in Ward & Hale 6 bus system;

$|U_6|_h$ : The high voltage amplitude of node 6;

$|U_6|_l$ : The low voltage amplitude of node 6;

$\Delta U_6$ : $|U_6|_h$ - $|U_6|_l$ ;

$P_s$ : Node load active power initial value;

$P_g$ : ONVCB node load active power calculation value.

$\Delta P_g$ : ONVCB node load active power critical closer degree. It is the degree that $P_s$ divided by $P_g$ , it can represent the stability redundance;

$P_a$ : ANVCB node load active power calculation value.

$\Delta P_a$ : ANVCB node load active power critical closer degree. It is the degree that $P_s$ divided by $P_a$ :

1) Calculate the multiple solutions with the method proposed for the Ward & Hale 6 bus system used in [16].

Change the Q value of node 6, and calculate the voltage amplitude with (4). The calculation results in Table I show that it is very close to that of in [16]. It proves that the method in this paper is available to track balanced solutions curve, and the explicit expressions of voltage make it easier to be observed.

TABLE I. CALCULATION RESULTS OF WARD & HALE 6 BUS SYSTEM.

| No. | $Q_6$ | $\lvert U_6 \rvert_h$ | $\lvert U_6 \rvert_l$ | $\Delta U_6$ |
|---|---|---|---|---|
| 1 | -1.3 | 1.2505 | 0.4918 | 0.7587 |
| 2 | -1.4 | 1.2411 | 0.5127 | 0.7284 |
| 3 | -1.5 | 1.1207 | 0.5894 | 0.5313 |
| 4 | -1.6 | 1.0882 | 0.6198 | 0.4684 |
| 5 | -1.7 | 0.9745 | 0.8086 | 0.1659 |

2) We carry on the calculation in IEEE-14 system. The number of node 1 and node 14 exchanges, node 14 is balance node, power factor is 0.9. Calculate the VCB in two ways: ONVCB (calculate node each to each) and ANVCB (assume all nodes achieve VCB at the same time). Table II shows some typical results.

TABLE II. THE COMPARISION OF NODE LOAD ACTIVE POWER CRITICAL CLOSER DEGREE BETWEEN ONVCB AND ANVCB.

| Node | $P_s$ | $P_g$ | $\Delta P_g$(%) | $P_a$ | $\Delta P_a$(%) |
|---|---|---|---|---|---|
| 1 | 0.1490 | 1.1881 | 87.46 | 0.2373 | 37.21 |
| 4 | 0.4780 | 1.9984 | 76.08 | 0.6093 | 21.55 |
| 5 | 0.0760 | 2.1133 | 96.40 | 0.2212 | 65.64 |
| 7 | 0 | 1.0626 | 100 | 0.1254 | 100 |
| 9 | 0.2950 | 1.1850 | 75.11 | 0.4012 | 26.47 |
| 10 | 0.0900 | 0.8425 | 89.32 | 0.1184 | 23.99 |
| 11 | 0.0350 | 0.7553 | 95.37 | 0.0511 | 31.51 |
| 12 | 0.0610 | 0.6552 | 90.69 | 0.0887 | 31.23 |
| 13 | 0.1350 | 0.7877 | 82.86 | 0.1574 | 14.23 |

In Table II, $\Delta P_g$ , $\Delta P_a$ represent the distance between the node load active power initial value and ONVCB, ANVCB respectively, illustrate the stability margin size of each node. The initial load active power of node 7 is zero, so the load power margin is 100 %.

From Table II, the power critical closer degree of ONVCB is much larger than that of ANVCB, proves that ANVCB is the closest stability margin. This is mainly because only the node load of ONVCB grows largely and the load of other nodes is constant when calculate ONVCB. But, the load of all nodes is increasing when calculate ANVCB and the load stability margin of each node becomes small.

The ANVCB can observe the stability margin of each node from the global point of view, and can analyze the stability of all nodes comparatively in the same level. The ONVCB can represent the local situation, but sometimes can not truly reflect the whole stability conditions. For example, the stability margin of node 13 is the smallest in ANVCB, but not in ONVCB. If adjust the node power injection to improve the system stability, should be based on the calculation results of ANVCB. In this system, for example, should first adjust node like 13.

## VII. CONCLUSIONS

In this paper, power system network equations have been established by introducing branch current as a variable based on the traditional node voltage equations. The existing conditions of solution have been found by analyzing voltage high and low solution curve, and have proposed the characteristic equations of VCB. We define the ONVCB and ANVCB, and prove ANVCB is the closest stable margin of the power system. The conclusions of simulations are: 1) the method proposed can be applied to calculate VCB and analyze voltage stability; 2) comparing ONVCB and ANVCB, the calculating results of ANVCB can reflect the system stability information more richly and can embody more approaching to the reality; 3) the ANVCB can obtain system closest stability margin and observe system stability overall and provide the basis for the voltage stability adjustment.

## REFERENCES


[1] V. Ajjarapu, C. Christy, "The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis", *IEEE Trans. Power Syst.*, vol. 7, no. 1, pp. 416–423, 1992. [Online]. Available: http://dx.doi.org/10.1109/59.141737

[2] Z. Feng, V. Ajjarapu, D. J. Maratukulam, "Identification of Voltage Collapse Through Direct Equilibrium Tracing", *IEEE Trans. Power Syst.*, vol. 15, pp. 342–349, 2000. [Online]. Available: http://dx.doi.org/10.1109/59.852142

[3] C. A. Canizares, A. C. Z. de Souza, V. H. Quintana, "Comparison of Performance Indexes for Detection of Proximity to Voltage Collapse", *IEEE Trans. Power Syst.*, vol. 11, no. 3, pp. 1441–1450, 1996. [Online]. Available: http://dx.doi.org/10.1109/59.535685

[4] C. A. Canizares, F. L. Alvarado, "Point of Collapse and Continuation Methods for Large AC/DC Systems", *IEEE Trans. Power Syst.*, vol. 8, no. 1, pp. 1–8, 1993. [Online]. Available: http://dx.doi.org/10.1109/59.221241

[5] A. C. Z. de Souza, C. A. Canizares, V. H. Quintana, "New Techniques To Speed Up Voltage Collapse Computations Using Tangent Vectors", *IEEE Trans. Power Syst.*, vol. 12, no. 3, pp. 1380–1387, 1997. [Online]. Available: http://dx.doi.org/10.1109/59.630485

[6] G. D. Irisarri, X. Wang, J. Tong, S. Mokhtari, "Maximum Loadability of Power Systems using interior point non-linear optimization methods", *IEEE Trans. Power Systems*, vol. 12, no. 1, pp. 162–172, 1997. [Online]. Available: http://dx.doi.org/10.1109/59.574936

[7] F. D. Gallana, "Load Flow Feasibility and the Voltage Collapse problem", in *Proc. 23rd Conf. Decision and Control*, Las Vegas, Nevada, Dec. 1984. [Online]. Available: http://dx.doi.org/10.1109/CDC.1984.272043

[8] V. Ajjarapu, B. Lee, "Bibliography on Power Systems", *IEEE Trans. Power Systems*, vol. 13, no. 1, 1998.

[9] R. V. Schlueter, "A Voltage Stability Security Assessment Method", *IEEE Trans. Power Systems*, vol. 13, no 4, Nov. 1998. [Online]. Available: http://dx.doi.org/10.1109/59.736286

[10] C. K. Babulal, P. S. Kannan, J. M. Anita, B. Venkatesh, "Determination of Voltage Stability Analysis Limit using Fuzzy Logic", *Int. J. Power and Energy Systems*, vol. 28, no. 1, pp. 3651–3692, 2008. [Online]. Available: http://dx.doi.org/10.2316/Journal.203.2008.1.203-3651

[11] J. P. Chiou, F. S. Wang, "A hybrid method of differential evolution with application to optimal control problems of a bioprocess system", in *Proc. IEEE on Evolutionary Computation Conf.*, 1998, pp. 627–632.

[12] K. O. Jones, "Comparison of Genetic Algorithm and Particle Swarm Optimization", in *Proc. Int. Conf. Computer Systems and Technologies*, 2005, pp. 1–6.

[13] A. A. El-Dib, H. K. M. Youssef, M. M. El-Metwally, Z. Osman, "Load Flow Solution Using Hybrid Particle Swarm Optimization", in *Proc. Int. Conf. Electrical Electronic and Computer Engineering (ICEEC)*, 2004, pp. 742–746.

[14] K. Kim, H. Schattler, V. Venkatasubramanian et al., "Methods for Calculating Oscillations in Large Power Systems", *IEEE Trans. Power Systems*, vol. 12, no. 4, pp. 1639–1648, 1997. [Online]. Available: http://dx.doi.org/10.1109/59.627870

[15] A. A. P. Lerm, C. A. Canizares, A. S. Silva, "Multiparameter Bifufcation Analysis of the South Brazilian Power System", *IEEE Trans. Power System*, vol. 18, no. 2, pp. 737–746, 2003. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2003.811195

[16] Y. Tamura, H. Mori, S. Iwamoto, "Relationship between Voltage Instability and Multiple Load Flow Solution in Electric Power Systems", *IEEE Trans. Power Apparatus and Systems*, vol. 102, no. 5, pp. 1115–1123, 1983. [Online]. Available: http://dx.doi.org/10.1109/TPAS.1983.318052