http://dx.doi.org/10.5755/j01.eee.20.9.5418 ELEKTRONIKA IR ELEKTROTECHNIKA, ISSN 1392-1215, VOL. 20, NO. 9, 2014

Influence of Users’ Density on the Mean Base Station Output Power

M. Mileusnic[1], M. Popovic[2], A. Lebl[1], D. Mitic[1], Z. Markov[1]
[1]*IRITEL A.D.,*
Beograd, Batajnicki put 23, 11080 Beograd, Serbia
[2]*Faculty of technical sciences, University in Novi Sad,*
Fruskogorska 11, Novi Sad, Serbia
mita@iritel.com

***Abstract*—In this paper we developed the method for determination of output power of base station as the function of distribution of users’ density in one GSM cell. The method consists of several steps: determination of the function of density distribution (PDF) and function of distribution (CDF) of distance between user and base station, finding PDF of output power of one GSM channel. In order to prove calculated results, we developed the program for simulating seizure and release channels in one GSM cell. It is necessary that inverse function of CDF of distance between user and base station exists in order to implement this program. This simulation program can be used to estimate CDF of total emission power of base station. Calculation of CDF of total emission power is, except in simplest cases, pretty complicate.**

***Index Terms*—Base station, cellular phones, GSM, mobile communication.**

I. INTRODUCTION

One of the main guidelines in the design of all telecommunication systems, including the systems in mobile network, is power saving, [1]−[7]. That’s why good knowledge of the characteristics of base station (BTS) output power is very important. The main characteristics of total output power are mean power and cumulative function (CDF). Total output power is random variable and presents the sum of random power of all channels, [8]. Generally, it can be very complicated to calculate the characteristics of the sum of random variables. Calculation of the output power of one BTS is complicated, because it is random variable which depends on two random variables, traffic values and users’ distance. The total output power also depends on channel attenuation and on number of channels, [8]−[10]. In [8] total output power is estimated when users’ density in the cell is uniform, i.e. has uniform distribution. In this paper we present how to calculate the mean value of total BTS output power in the case of non−uniform users’ density. In order to verify calculated results, we developed simulation method for traffic process in one GSM cell.

In Section II the considered model is presented and in Section III the principle of power estimation in one GSM channel is described. Section IV presents simulation method, which is used to verify the results from Section III. In Section V we give three numerical examples and the method of calculation and simulation for concrete numerical values. Section VI presents the results of simulation for CDF of total BTS output power.

II. MODEL, DESIGNATIONS AND ASSUMPTIONS

Let us consider round GSM cell with radius R and the group of N traffic channels, where emission power is adjusted according to the needs. BTS adjusts the output power (w) in each traffic channel according to the channel attenuation. The attenuation depends on the distance between the mobile station (MS) and BTS but also from other factors. The greatest value of this power for one channel is w_{max} and its mean value is w_m. The distribution density function (PDF) of random variable w will be designated by $f_w(x)$ and cumulative distribution function (CDF) by $F_w(x)$. The distance MS–BTS (d) is random variable with PDF and CDF $f_d(x)$ and $F_d(x)$ respectively. In order to investigate the dependence of output power on the users’ density, let us suppose that that output power of one channel depends only on distance MS–BTS. It means that output power of the channel, when connection is established on it, is random variable, which can be expressed in the function of d as

$$w = v(d) = a \times d^{¥}, \qquad (1)$$

where a is constant for adjusting values and dimensions of variables and γ has values between 2 and 5, [9]. The channel of BTS has maximal output power (w_{max}) if MS is situated at the maximal distance (end of the cell radius − R) from BTS, i.e. $w_{max} = a \cdot R^{\gamma}$. Here it is supposed that output power in one channel is continuous variable, although it is changed in steps of 2dB. But, considering the fact that number of steps is great (typically 15 steps), the assumption of continual change of power will not reduce accuracy of analysis.

The emission power of BTS is random variable designated by w_B and its PDF is $f_{wB}(x)$, CDF is $F_{wB}(x)$ and mean value is w_{Bm}.

Manuscript received March 9, 2014; accepted September 21, 2014.
This work is partially supported by the Ministry of Science and Technological Development of the Republic of Serbia within the Project TR32007.

It is necessary to use one channel for each connection. The offered traffic to the group of channels will be designated by A and the served traffic by Y. Y can be expressed as the function of A as $Y = (1 - B)\cdot A$, where B is the probability that call is lost due to the lack of free channels. The number of MSs (N_{ms}) is much greater than N, so traffic process in the group of traffic channels can be described by Erlang model with loss [11]. In this model the probability that j channels are busy, if the number of channels is N and the offered traffic is A, $P(j, A, N)$ can be expressed as

$$P(j,A,N) = ERL(j,A,N) = \frac{\frac{A^j}{j!}}{\sum_{i=0}^{N_t} \frac{A^i}{i!}}, \quad (2)$$

where $j = 0,1,2,...,N$ and it is

$$B = P(N,A,N) = ERL(N,A,N). \quad (3)$$

The mean output power of BTS (w_{Bm}) is equal to the product of served traffic (Y) and the mean power used by one channel as the contribution to the total power, $\omega_m = w_m/8$, [8].

The density of MS (number of MS per the unit of cell surface area) is designated by g. It is supposed that density is continuous variable, which depends only on the distance from BTS. Relative users' density in the cell will be treated as the function $g = g(x)$, where x is the distance between MS and BTS, Fig. 1.

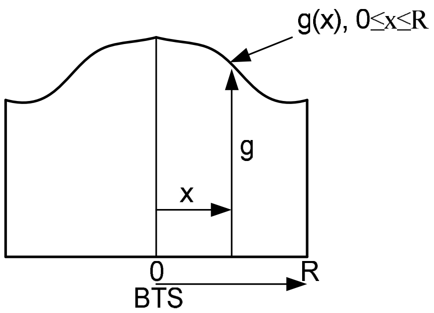

Fig. 1. Relative users' density as the function of distance MS−BTS.

Let us consider the network of mobile users, where users move inside the cell, but also enter and get out the cell (handover). In this paper we suppose that movement of users from the centre of cell is equal to the movement towards the centre and that the number of users in the cell is constant, i.e. that the number of users who get out the cell is statistically equal to the number of users who enter the cell. It means it is supposed that traffic intensity, caused by handover, is $\lambda_H = 0$, [12].

III. Estimation of the Power of One GSM Channel

The channel power is dependent random variable, which depends on the distance MS – BTS. The probability density of output power of one channel, $f_w(w)$, is, according to [13]

$$f_w(w) = \left[\frac{1}{\left|v'(d)\right|}\right] \times f_d(d). \quad (4)$$

Determination of $f_d(d)$, i.e. PDF of random variable, d, (distance MS – BTS) can be performed in the following way. Let us consider the number of MSs at the distance less than x from BTS, $N(x)$, and total number of MSs in the cell, $N(R)$. The number of MSs in some area is equal to the product of considered area and the density of MSs in that area.

This product can be conditionally equated with the volume of the body constructed above this area. The body is bordered by the area obtained rotating the function, which presents the density of MSs. In the considered case the relation of number of MSs, $N(x)/N(R)$, is equal to the ratio of volumes, Fig. 2. The probability that MS is on the distance less than x from BTS, i.e. CDF of distance, $P\{d<x\} = F_d(x)$ is

$$F_d(x) = \frac{V(x)}{V(R)}, \quad (5)$$

where $V(R)$ and $V(x)$ are the volumes of rotary bodies from Fig. 2(e)) and Fig. 2(f)).

In (5), $V(x)$ represents the number of MSs on the distance less than x from BTS, and $V(R)$ represents total number of MSs in the area covered by MS, i.e. probability that MS is on the distance less than x can be easily calculated as the ratio of the volumes of two rotary bodies.

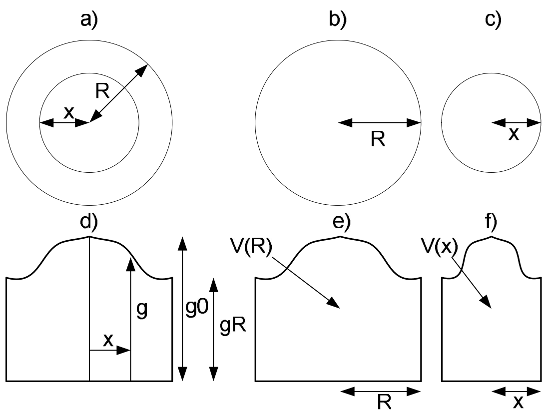

Fig. 2. Determination of the number of MSs as the body obtained rotating the function, which presents the density of MSs, above the area, presenting mobile cell.

In this way we can easily also obtain PDF of random variable d, as $f_d(d) = F'_d(d)$ and using (4) we can calculate PDF, then also CDF, $F_w(x)$, and mean output power of one GSM channel, w_m. It can be easily proved, [8] that mean output power of BTS is equal to the product of served traffic in the cell, Y, and the mean output power of one channel, which contributes to the total power, ω_m.

IV. Simulation

The results, obtained for CDF of output power of one GSM channel, $F_w(x)$, can be tested by simulation. The simulation consists of the known simulation model *Monte Carlo* or *roulette*, [14], and the added part for determination of random distance MS−BTS for each connection. The principle of the flow−chart of simulation program is presented in Fig. 3.

The variables, which are dedicated before the process of simulation, are the chosen number of iterations in simulation (*Lmax*), total number of channels (N) and offered traffic value (A). The first step in program loop is to determine whether the number of program iterations (L) has reached *Lmax*. If no, program loop starts.

The generated random number RN1U, according to blocks 1 and 2 in Fig. 3, is in the range 0 to $A + N$. If its value is less than A, (block 3), new call is generated in the case that number of busy channels (j) is less than N (block 4). Then blocks 5 to 7 present the process of calculating the new BTS output power. In the case that all channels are busy, the call is not generated (block 12 in Fig. 3).

If RN1U is greater than A, the call has to be ended. In order to determine whether the call is really ended, the area of random numbers between A and $A + N$ is divided in N sub−areas. Each of these N sub−areas corresponds to one of N channels. The call is really ended if the generated RN1U (which chooses some channel K) determines previously occupied channel (blocks 8 to 10). New total power in this case is calculated in block 11.

Addition to *Monte Carlo* model for determination of output power is the block 6. This block determines random distance MS−BTS, i.e. random power for each connection. The block 6 consists of four parts, Fig. 4. The first part (6.1.) is random number generator, RN2U(0, 1). These numbers have uniform distribution.

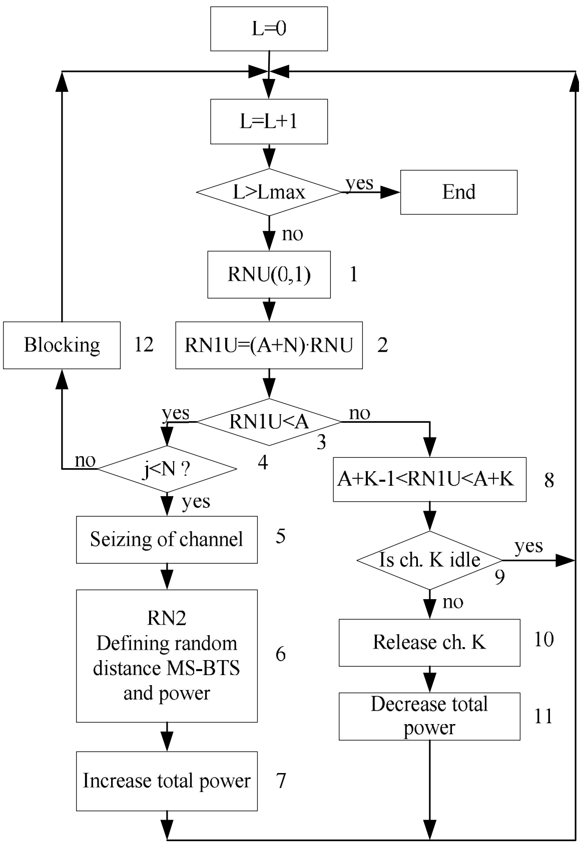

Fig. 3. Flow−chart of simulation program.

The second part (6.2.) generates random numbers RN2 from the range (0, 1), which satisfy desired CDF corresponding to the MSs density in considered cell. Random numbers from the range (0, 1), which satisfy arbitrary distribution (F_d) can be obtained from uniformly distributed random numbers using inverse function F_d^{-1}, [15].

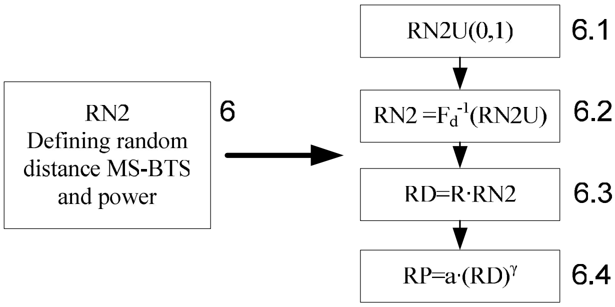

Fig. 4. The part of simulation program, which generates random distance MS−BTS.

The third part (6.3.) determines random distance of some connection (RD), multiplying the cell radius (R) with the random number RN2.

The fourth part (6.4) determines random output power of some connection according to (1).

It is important to notice that random number generators RNU(0, 1) (which determines the type of event in traffic process) and RN2U(0, 1) (which determines random distance MS−BTS) are completely independent generators.

The main problem can be calculation of inverse function F_d^{-1}, which can have very complicate shape for some distributions of users' density. This can be exceeded using modern computer programs, which give approximate inverse function.

V. Numerical Examples

Example 1. Verification of calculated results by simulation is performed for the numerical example of one cell where holds: $\gamma = 3$, $R = 20\ km$, $w_{max} = 40\ W$ (class 4) in two cases. The first one is for linear decreasing MS density ($g_0 = 6$, $g_R = 1$) and the second one is for linear increasing MS density ($g_R = 6$, $g_0 = 1$), Fig. 1 and Fig. 2

$$g(x) = g_0 - (g_0 - g_R) \times \frac{x}{R}. \qquad (6)$$

Starting from (4)–(6), it can be proved that CDF of distance MS – BTS, $F_d(x)$, and CDF of power of one channel, $F_w(x)$, are:

$$F_d(x) = \frac{3 \times x^2 \times g_0 + \dfrac{2 \times x^3 \times (g_R - g_0)}{R}}{K}, \qquad (7)$$

$$F_w(x) = \frac{A_1 \times x^{\frac{2}{\gamma}}}{\frac{2}{\gamma}} + \frac{A_2 \times x^{\frac{3}{\gamma}}}{\frac{3}{\gamma}}, \qquad (8)$$

where:

$$\begin{cases} A_1 = \dfrac{6 \times g_0}{\gamma \times K \times a^{\frac{2}{\gamma}}}, \\ A_2 = \dfrac{6 \times (g_R - g_0)}{\gamma \times R \times K \times a^{\frac{3}{\gamma}}}, \end{cases} \quad (9)$$

where $K = 2 \cdot R^2 \cdot g_R + R^2 \cdot g_0$. For particular numerical values it is obtained

$$F_d(x) = 2.25 \times x^2 - 1.25 \times x^3, \quad (10)$$

if it is $g_0 = 6$, $g_R = 1$

$$F_d(x) = 0.23077 \times x^2 + 0.76923 \times x^3, \quad (11)$$

if it is $g_0 = 1$, $g_R = 6$.

Inverse functions can be calculated using programs MATHEMATICA and MATLAB. The implemented versions are 6.0.1.0. for Wolfram MATHEMATICA and 6.5.0. Release 13 for MATLAB (using Communications System Toolbox). The function according to equation (10) and its inverse function are presented in Fig. 5.

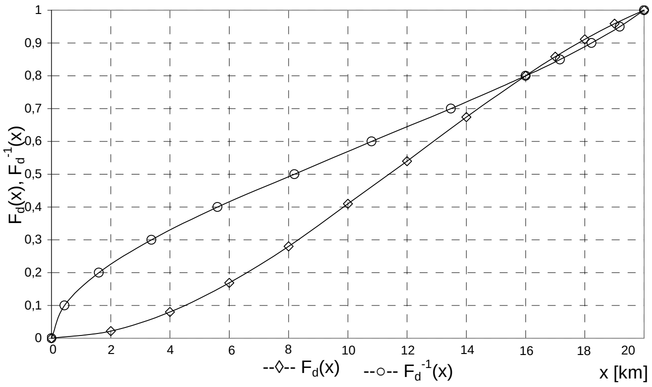

Fig. 5. Function $F_d(x)$ according to (10), (◊), and its inverse function $F_d^{-1}(x)$, (○).

The function according to (11) and its inverse function are presented in Fig. 6.

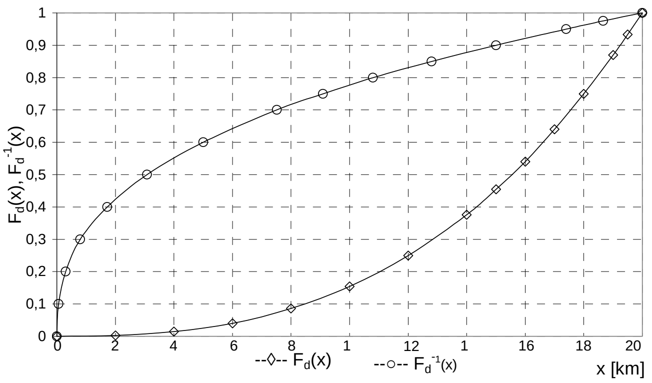

Fig. 6. Function $F_d(x)$ according to (11), (◊), and its inverse function $F_d^{-1}(x)$, (○).

According to the flow−chart from Fig. 3 and Fig. 4, inverse functions are used in simulation, and the obtained values for CDF of power of one GSM channel are presented in Fig. 7(x). The calculated results according to (8) are presented in this figure for decreasing users' density $g_0 = 6$, $g_R = 1$, (◊) and for increasing users' density, $g_R = 6$, $g_0 = 1$, (□). The mean output power of one GSM channel during the connection is $w_m = 11$ W in the case of decreasing users' density, $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g_0 = 6$, $g_R = 1$ and $w_m = 19.2$ W in the case of increasing users' density from the centre to the periphery of the cell, $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g_0 = 1$, $g_R = 6$.

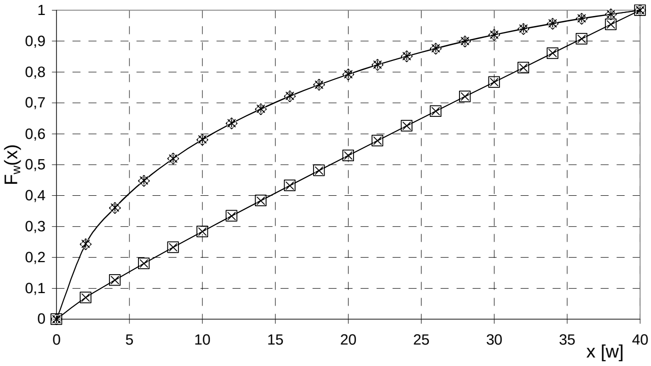

Fig. 7. Calculated CDF of output power of one GSM channel for $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g_0 = 6$, $g_R = 1$ (upper curve, ◊) and $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g_R = 6$, $g_0 = 1$ (lower curve, □), results of simulation are presented by (x).

Example 2. Let us consider the same cell, $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, but the users' density decreases exponentially from the centre to the periphery of the cell

$$g(x) = g_0 \times e^{-b \cdot x}, \quad (12)$$

where $g(0) = 6$, $g(R) = 1$.

Starting from (4), (5) and (12), CDF of distance MS – BTS is

$$F_d(x) = \frac{1 - (b \times x + 1) \times e^{-b \cdot x}}{1 - (b \times R + 1) \times e^{-b \cdot R}}, \quad (13)$$

and CDF of power of one channel is

$$F_w(x) = \frac{A \times b^2}{\gamma \times a^{\frac{2}{\gamma}}} \times \int_0^x z^{\frac{2-\gamma}{\gamma}} \times e^{-\frac{b}{a'} \times z'} dz, \quad (14)$$

where is $A = \dfrac{1}{1 - (b \times R + 1) \times e^{-b \cdot R}}$, $a' = a^{\frac{1}{\gamma}}$, $z' = z^{\frac{1}{\gamma}}$.

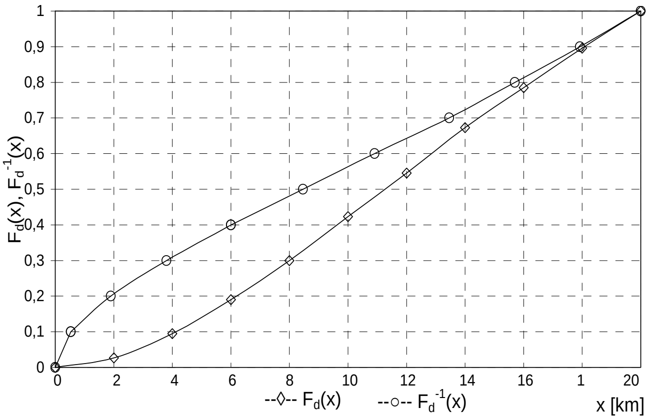

Fig. 8. Function $F_d(x)$ according to (13), (◊), and its inverse function $F_d^{-1}(x)$, (○).

Here CDF of distance MS–BTS is

$$F_d(x) = 1.87 \times (1 - (0.089588 \times x + 1) \times e^{-0.089588 \times x}), \quad (15)$$

and it is presented by the lower curve in Fig. 8, (◊). The inverse function $F_d^{-1}(x)$ is obtained using program MATHEMATICA and it is presented by the upper curve in Fig. 8. (○).

CDF of power of one channel in this case looks as in Fig. 9, calculation results (◊) and the results of simulation (x).

The mean power of one GSM channel during the connection in the case $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W and exponential decreasing relative users' density from the centre to the periphery of the cell, (12), is $w_m = 11.02$ *W*.

The mean power of BTS can be now estimated as

$$w_{Bm} = Y \times \mu_m = \mu_m \times A \times (1 - B). \quad (16)$$

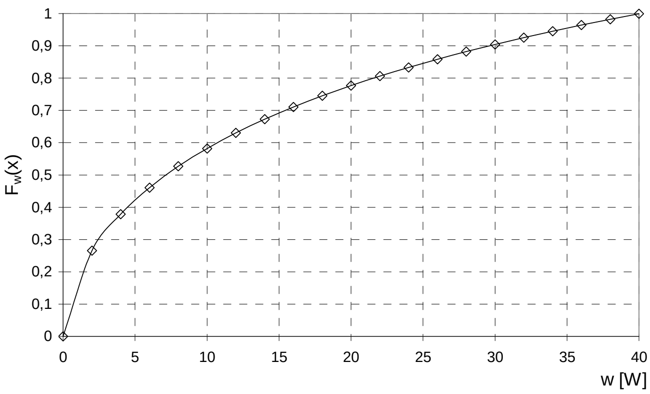

Fig. 9. CDF of output power of one GSM channel for $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g(x)$ according to (12) (◊) and the results of simulation (x).

Dependence of total BTS output power with $N = 16$ channels from the offered traffic for two cases is presented in Fig. 10. The first one is $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g(x)$ according to (12), curve W_{Bm1}. The second one refers to the linearly increasing users' density from the centre to the periphery of the cell, where is $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g(x)$, $g_R = 6$, $g_0 = 1$, curve W_{Bm2}.

Programs for simulation, used to get results in Fig. 7, Fig. 9 and Fig. 10, can be used to obtain CDF of total emission power of one BTS.

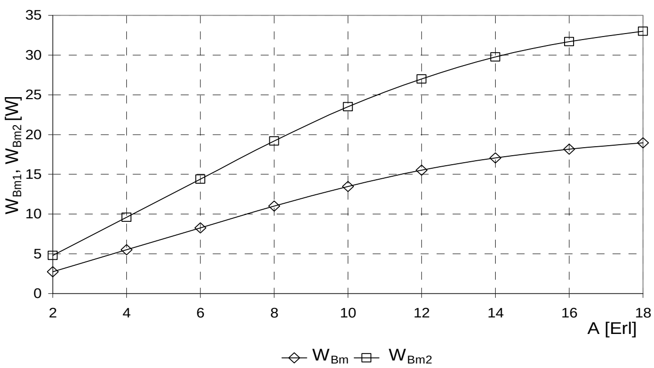

Fig. 10. Mean output power of BTS for $N = 16$, $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g(x)$ according to (12) (lower curve W_{Bm1}) and for $N = 16$, $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g(x)$ linearly increasing $g_R = 6$, $g_0 = 1$ (upper curve W_{Bm2}) as the function of the offered traffic.

VI. CDFs of the Base Station Output Power

Calculation of CDF of base station output power in general case can be very complicated. Total base station output power (random variable) is the sum of output power of all active channels, which are independent random variables. Calculation of PDF (and CDF) for random variable, which is the sum of several random variables is connected with the calculation of convolution of all PDFs for single random variables. Convolution calculation may be very complicate, except in the case of several simple distributions. That's why cumulative distribution of base station output power is rather estimated by simulation.

Figure 11(a) presents CDF obtained by simulation in two cases. The first one is linearly increasing users' density ($\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g_R = 6$, $g_0 = 1$, numerical example 1, case 2) for $N = 16$ channels and offered traffic $A = 12$ Erl. The second one is exponentially decreasing users' density from the centre to the edge of cell, ($\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, users' density according to (12), numerical example 2) also for $N = 16$ channels and offered traffic $A = 12$ Erl.

Figure 11(b) presents detailed CDF from Fig. 11(a) in the area of great probability values. It can be seen from this figure that for 95 % of connections it is necessary to have output power 26.4 W in the case of exponential distribution of users' density. In the case of linearly increasing users' density it is necessary to have 40.6 W for 95 % of connections, or 53.7 % more than in the previous case.

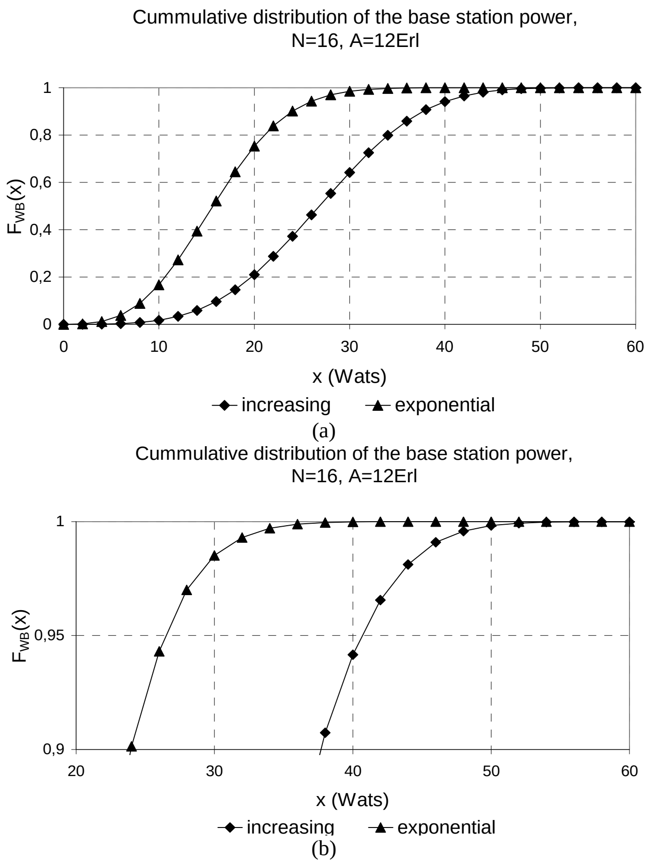

Fig. 11. Dependence of CDF of base station output power on users' density, $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $g_R = 6$, $g_0 = 1$, $N = 16$, $A = 12$ Erl, in the case of linearly increasing density (♦) and exponential density distribution (▲), compleate distribution (a) and detailed (b).

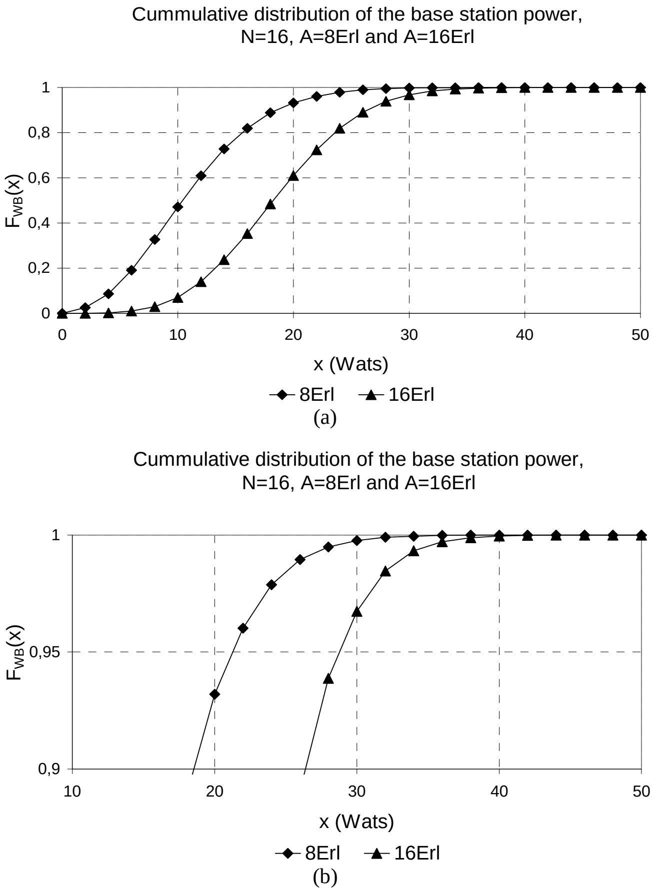

Fig. 12. Dependence of CDF of base station output power on offered traffic, $\gamma = 3$, $R = 20$ km, $w_{max} = 40$ W, $N = 16$, exponential distribution of users' density, in the case of $A = 8$ Erl (♦) and A = 16 Erl (▲),compleate distribution (a) and detailed (b).

It is interesting to notice that the dependence of base station output power on users' density is similar to the dependence on traffic. Figure 12(a) and Fig. 12(b) present CDFs for the case of exponential dependence of users' density ($\gamma = 3$, $R = 20$ km, users' density according to (12), numerical example 2), $N = 16$ channels, for two values of offered traffic, $A = 8$ Erl and $A = 16$ Erl. The first traffic value presents the load, which causes negligible loss ($B \approx 0.5$ %), while the second value presents very great load and the loss is also great ($B \approx 17.5$ %). It can be noticed that change of power in this case for 95 % of connections is slightly less (35.3 %), when traffic is changed from small to great value.

VII. Conclusions

In this paper it is proved that output power of BTS, besides traffic and signal attenuation, depends on the distribution of users' density in the area of cell. In order to determine the mean power of GSM channel it is necessary to know the distribution of users' density and to use it to find distribution of power for one channel as dependent random variable. The mean output power of BTS can be calculated by multiplying the contribution of one channel with the served traffic. These results can be proved by the simulation of traffic process in the group of channels from one cell. It is necessary in this case to add the part, which simulates random distance MS−BTS, to the program for traffic simulation. The obtained results for mean value of BTS output power must be increased, if intra−cell traffic cannot be neglected [16]. At the contrary, mean value of power may be decreased if the impact of finite number of users is great [17].

References

[1] E. Oh, B. Krishmamachari, X. Liu, Z. Niu, "Toward dynamic energy−efficient operation of cellular network infrastructure", *IEEE Communications Magazine*, vol. 49, no. 6, pp. 56–61, 2011. [Online]. Available: http://dx.doi.org/10.1109/MCOM.2011.5783985

[2] Z. Niu, "Advances in Green Communicatios and Networks", *VTC2012−Spring*, 2012.

[3] J. Choi, "Green Radio, Approaches and Performance Analysis", 2012, [Online]. Available: http://ccit.kaist.ac.kr/Lecture%20Notes/Green%20Radio%202.pdf.

[4] J. Lorincz, T. Garma, G. Pretrovic, "Measurement and modelling of base station power consumption under real traffic loads", *Sensors*, vol. 12, no. 4, pp. 4281–4310, 2012. [Online]. Available: http://dx.doi.org/10.3390/s120404281

[5] F. Richter, A.−J. Fehske, G.−P. Fettweis, "Energy efficiency aspects of base station deployment strategies for cellular networks", *IEEE Proceedings of VTC*, 2009.

[6] K. Staniec, G. Debita, "An optimal sink nodes number estimation for improving the energetic efficiency in wireless sensor networks", *Elektronika ir Elektrotechnika*, vol. 19, no. 8, pp. 115–118, 2013.

[7] O. Blume, H. Eckhardt, S. Klein, E. Kuhn, W. M. Wajda, "Energy savings in mobile networks based on adaptation to traffic statistics", *Bell Labs Technical Journal*, vol. 15, no. 2, pp. 77−94, 2010. [Online]. Available: http://dx.doi.org/10.1002/bltj.20442

[8] P. Jovanovic, M. Mileusnic, A. Lebl, D. Mitic, Z. Markov, "Calculation of the mean output power of base transceiver station in GSM", *Automatika*. (to be published)

[9] J. Eberspracher, H.−J. Vogel, C. Bettstetter, *GSM, Switching, Services and Protocols*. John Wiley & Sons, 1999.

[10] G. Heine, *GSM Networks: Protocols, Terminology and Implementation*. Artech House, 1999.

[11] W. B. Iversen, *DTU Course 34340, Teletraffic Engineering and Network planning*. Technical University of Denmark, 2011.

[12] E. Kacerginskis, L. Narbutaite, "Capacity and handover analysis in mobile WiMAX", *Elektronika ir Elektrotechnika*, no. 3, pp. 23–28, 2012.

[13] A. Papoulis, *Probability, Random Variables and Stochastic Processes*. McGraw−Hill, 1991.

[14] L. Kosten, "Simulation in teletraffic theory", *6th ITC*, Munich 1970.

[15] M. C. Jeruchim, P. Balaban, K. S. Shanmugan, *Simulation of Communication Systems, Modeling, Methodology and Techniques*, Kluwer Academic Publishers, 2002.

[16] P. Jovanovic, T. Suh, A. Lebl, D. Mitic, Z. Markov, "Influence of Intra−cell connections on the traffic calculation of radio resources in mobile network", *Frequenz*, vol. 67, no. 9−10, pp. 315−320, 2013.

[17] T. Suh, P. Jovanovic, A. Lebl, D. Mitic, Z. Markov, "Comparison of the influence of intra−cell traffic and finite number of mobile phones on the determination of number of channels in the BTS of GSM network", *Frequenz*, vol. 68, no. 3−4, pp. 171−176, 2014.