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New Approach to Support Region Determination of the μ-law Quantizer

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***Abstract*—In this paper we address the optimization of the support region threshold of the μ-law companding quantizer designed for the Laplacian source of unit variance. We propose a new iterative method of determining the support region threshold, based on Newton's method. We provide a simple manner for determining the support region threshold. Numerical results show that the proposed new approach to the observed optimization problem outperforms the recently proposed method in terms of the relative error of determining the support region threshold. This indicates the usefulness of the proposed approach.**

***Index Terms*—Newton method, optimization methods, quantization.**

I. INTRODUCTION

The logarithmic compressor characteristic obtained by piecewise linear approximation to the μ-law characteristic is widely used as a design guideline for nonuniform quantization of speech signals in digital telephony [1]. Specifically, to achieve high-quality quantized speech signals, the contemporary public switched telephone networks utilize the piecewise linear approximation to the μ-law characteristic proposed by the G.711 Recommendation [1]. The problem that generaly occurs in designing quantizers, including μ-law companding quantizers, is how to provide the simple manner of determining the support region threshold of the quntizer to minimize distortion introduced in the quantization process. Namely, the quality of a quantized signal, measured by distortion or signal to quantization noise ratio (SQNR), is generally influenced by the width of the quantizer's support region and the number of quantization levels [2]–[4]. One of the main goals when designing quantizers is to determine the reproduction levels and the partition of the support region into cells, such as to provide the minimum possible distortion for a fixed number of quantization levels N, or a fixed resolution $R = \log_2 N$. It has been pointed out in [3], [4] that determining the support region threshold is a key issue in a quantizer designing. Since the support region threshold $x_{\max}$ and accordingly defined the support region of a quantizer $[-x_{\max}, x_{\max}]$, should be determined so that the support region is the interval where quantization errors are small, or at least bounded, a fast and accurate estimate of the support legion that minimizes the distortion is very useful in quantizer designing [3], [4].

The goal of this paper is to propose a closed-form formula for the optimal support region determination of the μ-law companding quantizer designed for the Laplacian source of unit variance. The observed optimization problem has already been addressed in [5], where optimization of the support region threshold of the μ-law companding quantizer has been performed under the constraint that compression factor μ has a large enough value. Observing this constraint, our goal is to provide a solution to the optimization problem that would be efficiently applicable for any compression factor value. In addition, our goal is to decrease the relative error of determining the support region threshold that has been calculated in [5]. In order to determine the optimal support region threshold the application of iterative numerical methods is required. In this paper, Newton's method of root-finding is applied with the goal to provide a simple manner of determining the optimal support region threshold of the μ-law companding quantizer in question. It is shown that Newton's method is a convenient iterative method for our optimization problem because it starts with an initial guess, which is reasonably close to the true root, and only after one iteration, for different values of the compression factor μ, it provides a smaller relative error of determining the support region threshold compared to the related relative error calculated in [5].

The rest of this paper is organized as follows. Section II provides a detailed description of the proposed simple new approach to the problem of optimizing the μ-law companding quantizer designed for the Laplacian source of unit variance. The achieved numerical results are the topics addressed in Section III. Finally, Section IV is devoted to the conclusions which summarize the contribution achieved in the paper.

II. OPTIMAL DESIGNING μ-LAW QUANTIZER USING NEWTON'S METHOD

An N-level scalar quantizer Q is defined by $Q: R \to Y$ [1],

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where R is a set of real numbers, and $Y \equiv (y_1, y_2, y_3, ..., y_N) \subset R$ is a set of representation levels that makes the code book of size $|Y| = N$. An N-level scalar quantizer partitions the set of real numbers into N cells $R_i = (t_{i-1}, t_i]$, $i = 1, …, N$, where t_i, $i = 0, 1, …, N$ are decision thresholds and where it holds $Q(x) = y_i$, $x \in R_i$.

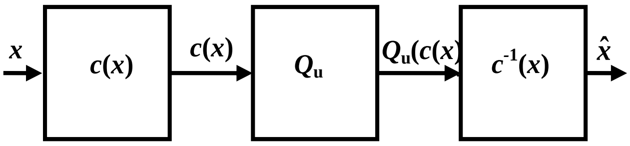

Fig. 1. Block diagram of companding technique.

Companding technique, we consider in this paper, defines the following steps: compress the input signal x by applying the compressor function $c(x)$; apply the uniform quantizer on the compressed signal $Q_u(c(x))$; expand the quantized version of the compressed signal using an inverse compressor function $c^{-1}(Q_u(c(x)))$ [1] (Fig. 1). μ-law companding quantizer Q_μ, which we consider in this paper, is defined by μ-law compressor function $c(x)$: $[-x_{max}, x_{max}] \to [-x_{max}, x_{max}]$ [1]

$$c(x) = \frac{x_{max}}{\ln(1+\mu)} \ln\left(1 + \mu \frac{|x|}{x_{max}}\right) \mathrm{sgn}(x), |x| \le x_{max}, \quad (1)$$

where the parameter μ is the compression factor and x_{max} is the μ-law companding quantizer's support region threshold. The process of quantization performs as follows:

$$\hat{x}_u = Q_u(c(x)) \in \left\{ \hat{x}_{u_i} = -x_{max} + \frac{(2i-1)}{N} x_{max}, i \in \{1, ..., N\} \right\}, \quad (2)$$

$$\hat{x} = Q_\mu(x) = c^{-1}\left(Q_u(c(x))\right), \quad (3)$$

$$\hat{x} \in \left\{ \frac{x_{max}}{\mu} \left((1+\mu)^{\frac{\hat{x}_{u_i} \mathrm{sgn}(\hat{x}_{u_i})}{x_{max}}} - 1 \right) \mathrm{sgn}(\hat{x}_{u_i}), i \in \{1, ..., N\} \right\}, \quad (4)$$

where x and $\hat{x}$ denote an original and a quantized signal, respectively.

The total distortion is a quality measure of quantization process and can be found as a sum of the granular and the overload distortion. In general, distortion D, defined as a sum of a granular distortion D_g and an overload distortion D_o, is given by [1]:

$$D = D_g + D_o, \quad (5)$$

$$D_g = \sum_{i=1}^{N} \int_{R_i} (x - y_i)^2 p(x) dx, \quad (6)$$

$$D_o = \int_{-\infty}^{-x_{max}} (x - y_1)^2 p(x) dx + \int_{x_{max}}^{\infty} (x - y_N)^2 p(x) dx. \quad (7)$$

For the assumed Laplacian probability density function (PDF) [1]

$$p(x) = \frac{\sqrt{2}}{2\sigma} \exp\left(-\frac{|x|\sqrt{2}}{\sigma} \right), \quad (8)$$

the expressions for the granular and the overload distortion of the μ-law companding quantizer are as follows [5], [6]:

$$D_g(Q_\mu) = \frac{\ln^2(1+\mu)\sigma^2}{3N^2} \left[\frac{1}{\mu^2} \frac{x_{max}^2}{\sigma^2} + \frac{x_{max}}{\sigma} \frac{\sqrt{2}}{\mu} + 1 \right], \quad (9)$$

$$D_o(Q_\mu) = \sigma^2 \exp\left(-\frac{\sqrt{2} x_{max}}{\sigma} \right). \quad (10)$$

Distortion is then given by

$$D(Q_\mu) = \frac{\ln^2(\mu+1)\sigma^2}{3N^2} \left[\frac{1}{\mu^2} \frac{x_{max}^2}{\sigma^2} + \frac{x_{max}}{\sigma} \frac{\sqrt{2}}{\mu} + 1 \right] + \sigma^2 \exp\left(-\frac{\sqrt{2} x_{max}}{\sigma} \right). \quad (11)$$

Without diminishing the generality, in what follows the quantizer designing will be done for the reference input variance of $\sigma_{ref}^2 = 1$. Accordingly, we proceed to optimize distortion determined for the unit variance

$$D(Q_\mu) = c \left[\frac{x_{max}^2}{\mu^2} + \frac{\sqrt{2} x_{max}}{\mu} + 1 \right] + \exp\left(-\sqrt{2} x_{max} \right), \quad (12)$$

with respect to x_{max}, where $c = \ln^2(\mu+1)/(3N^2)$ is a constant. By setting the first derivate of the distortion given by (12) to zero with respect to x_{max}, we obtain:

$$\frac{\partial D(Q_\mu)}{\partial x_{max}} = f(x_{max}) = 0, \quad (13)$$

$$f(x_{max}) = \frac{2\, c\, x_{max}}{\mu^2} + \frac{\sqrt{2}\, c}{\mu} - \sqrt{2} \exp\left(-\sqrt{2} x_{max} \right) = 0. \quad (14)$$

To provide the solution of our optimization problem we apply Newton's method [7], [8]

$$x_{max}^{(i+1)} = x_{max}^{(i)} - \frac{f\left(x_{max}^{(i)}\right)}{f'\left(x_{max}^{(i)}\right)}. \quad (15)$$

Newton's method is a generalized process to determine a root of a system (or a single) of equations $f(x)=0$ [7], [8]. It is a method for determining successively better approximations to the roots (or zeroes) of a real-valued function. Newton's method is not only the method easy to comprehend, but it is a very efficient way to find the solution to the equation. Newton's method requires that the derivative be calculated directly. An analytical expression for the derivative may not be easily obtainable and could be expensive to evaluate. In our case, it is simple to determine the derivative of $f(x_{max})$ so that the application of Newton's method is suitable for our optimization problem.

We initialize the iterative method using the support region threshold of the μ-law companding quantizer derived in [5] for the cases when the compression factor has an arbitrary large value

$$x_{max}^{(0)} = \frac{1}{\sqrt{2}} \ln\left(\frac{3\mu N^2}{\ln^2(\mu+1)} \right). \tag{16}$$

By combining (15) and (16) we obtain

$$x_{max}^{(i+1)} = x_{max}^{(i)} \left(1 - \frac{1}{(\mu+1)} \right). \tag{17}$$

Geometrically, ($x_{max}^{(i+1)}$, 0) is the intersection with the x_{max}-axis of a line tangent to f at ($x_{max}^{(i)}$, $f($ $x_{max}^{(i)}$)).

The relative error of determining the support region threshold can be defined as follows

$$\delta^{(i)}[\%] = \left| \frac{x_{max}^{opt} - x_{max}^{(i)}}{x_{max}^{opt}} \right| \times 100 , \tag{18}$$

where x_{max}^{opt} is obtained by numericaly optimizing the support region threshold of the quantizer in question so that to provide the minimum of the distortion (12). Note that our goal is to provide the solution to the optimization problem that would be efficiently applicable for any compression factor value. In fact, our goal is to decrease the relative error of determining the support region threshold that has been calculated in [5]

$$\delta^{[5]}[\%] = \left| \frac{x_{max}^{opt} - x_{max}^{[5]}}{x_{max}^{opt}} \right| \times 100 . \tag{19}$$

This is of great interest especially for arbitrary small values of compression factor. In what follows discussion is provided about the relative error of determining the support region threshold by following the proposed approach and the one given in [5].

III. NUMERICAL RESULTS

This section provides us with a detail analysis of the numerical results that we have calculated for the considered μ-law companding quantizer and the proposed Newton's method of determining the support region threshold. Specifically, in this section, for some arbitrary values of the compression factor μ, the values of the support region thresholds, which we have ascertained by applying the proposed approach, are compared with the corresponding ones calculated following the approach from [5].

The relative error of determining a root value is usually appointed criteria for stopping an iterative algorithm. In our approach to support region determination, as a baseline, we initialize the iterative method, using the support region threshold $x_{max}^{(0)}$ derived in [5], for the cases when the compression factor has an arbitrary large value. Note that once we have $x_{max}^{(0)}$ [5], and we accept the accuracy of determining the support region threshold in the first decimal place, the applied Newton's method, stops after the first iteration in all of the observed cases of the assumed compression factor values, except in the case where μ has very small value (μ = 10).

TABLE I. THE VALUES OF x_{max}^{opt}, $x_{max}^{[5]}$ AND $x_{max}^{(1)}$ FOR μ =255 AND N = 256, N = 128, N = 64, N = 32, N = 16

N	x_{max}^{opt}	$x_{max}^{[5]}$	δ[5][%]	$x_{max}^{(1)}$	δ(1)[%]
256	10.076	10.115	0.381	10.075	0.011
128	9.1	9.134	0.379	9.099	0.013
64	8.123	8.154	0.384	8.122	0.008
32	7.146	7.174	0.391	7.146	0.001
16	6.170	6.194	0.384	6.169	0.008

TABLE II. THE VALUES OF x_{max}^{opt}, $x_{max}^{[5]}$ AND $x_{max}^{(1)}$ FOR μ =100 AND N = 256, N = 128, N = 64, N = 32, N = 16

N	x_{max}^{opt}	$x_{max}^{[5]}$	$\delta^{[5]}$[%]	$x_{max}^{(1)}$	$\delta^{(1)}$[%]
256	9.622	9.712	0.938	9.616	0.062
128	8.651	8.732	0.943	8.646	0.056
64	7.679	7.752	0.950	7.675	0.050
32	6.708	6.772	0.955	6.705	0.044
16	5.736	5.791	0.962	5.734	0.037

TABLE III. THE VALUES OF x_{max}^{opt}, $x_{max}^{[5]}$ AND $x_{max}^{(1)}$ FOR μ =50 AND N = 256, N = 128, N = 64, N = 32, N = 16

N	x_{max}^{opt}	$x_{max}^{[5]}$	δ[5][%]	$x_{max}^{(1)}$	δ(1)[%]
256	9.284	9.449	1.776	9.264	0.219
128	8.319	8.469	1.796	8.303	0.199
64	7.355	7.488	1.816	7.342	0.180
32	6.391	6.508	1.839	6.380	0.158
16	5.427	5.528	1.861	5.419	0.136

TABLE IV. THE VALUES OF x_{max}^{opt}, $x_{max}^{[5]}$ AND $x_{max}^{(1)}$ FOR μ =10 AND N = 256, N = 128, N = 64, N = 32, N = 16

N	x_{max}^{opt}	$x_{max}^{[5]}$	$\delta^{[5]}$[%]	$x_{max}^{(1)}$	$\delta^{(1)}$[%]
256	8.454	9.010	6.578	8.191	3.111
128	7.518	8.030	6.813	7.300	2.898
64	6.584	7.050	7.068	6.409	2.665
32	5.654	6.069	7.348	5.518	2.411
16	4.727	5.089	7.657	4.626	2.130

In that case, the relative error of determining the support region threshold for $N = 16$, $N = 32$, $N = 64$, $N = 128$ and $N = 256$ ranges from 2.130 % to 3.111 % and is much smaller than the one calculated following the approach from [5], which ranges from 6.578 % to 7.657 %. Since the initial value $x_{max}^{(0)}$ is derived for an arbitrary large value of compression factor μ, it is expectedly that our method converges fastest to the root value x_{max}^{opt}, with the smallest relative error $\delta^{(1)}$, in the case where the compression factor amounts to $\mu = 255$. Table I shows that, for a different number of quantization levels N ($N = 16$, $N = 32$, $N = 64$, $N = 128$ and $N = 256$), the near optimal support region thresholds $x_{max}^{(1)}$ can be determined by following the proposed approach

$$x_{\max}^{(1)} = x_{\max}^{(0)} \left(1 - \frac{1}{(\mu+1)}\right), \quad (20)$$

where reasonably higher precisions are achieved compared to the related precisions calculated in [5]. Obviously, in this paper we end up with the meaningful formula (20) that provides straightforward computation of the near-optimal support region threshold of the μ-law companding quantizer designed for the Laplacian source of unit variance.

By observing Tables II, III and IV, one can notice that for moderate and arbitrary small values of compression factor μ ($\mu = 100$, $\mu = 50$ and $\mu = 10$), the relative error of determining the support region threshold $\delta^{(1)}$ ranges approximately from 0.04 % to 2.13 %, for $N = 16$ and from 0.06 % to 3.11 %, for $N = 256$. One can also conclude that for a given value of the compression factor μ, the decrease of the number of quantization levels N results in the decrease of the relative error $\delta^{(1)}$. This indicates that the proposed approach can efficiently be applied for small number of quantization levels as well. Eventually, observe that there is the oposite effects in the case of the application of the approach from [5]. Specifically, in that case, for a given value of the compression factor μ, the decrease of the number of quantization levels N results in the increase of the relative error $\delta^{[5]}$, which possible limits the application of the approach from [5] not only for the cases of small values of compression factor μ, but also for small values of the number of quantization levels N. Based on the all above mentioned, there is evident advantage of the proposed method in comparison with the one given in [5].

IV. CONCLUSIONS

Since one of the main goals when designing quantizers is to provide as high as possible quality of the quantized signal, which is generally influenced by the width of a quantizer's support region and the number of quantization levels, the proposed approach, which has proven to be an effective method, capable of significantly increasing the precision for determining the support region threshold, is of great impotance. As the proposed method allows explicit and simply determination of the support region threshold of the μ-law companding quantizer having value near the optimal one, and for any compression factor value it outperforms the method proposed in [5], one can conclude that the importance of the proposed method is really high.

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