# Design of Output LCL Filter for 15-level Cascade Inverter

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Abstract—This paper presents the design procedure for the output LCL filter used in grid connected one-phase 15-level cascade voltage source inverter for photovoltaic application. Output power of a system is in kilowatt range due to one-phase connection. The filter design is based on detailed analysis as it is a multifold problem. Design is adopted for predictive current control technique with finite control set. Because of lack of modulation technique there is no detailed higher-order harmonic analysis. Key parameters that define filter performance are described. Step-by-step design of LCL filter is presented. The design procedure is done in pre-unit base so results can be used for wider range of power levels. No passive damping is considered as active damping is preferable with regard to high efficiency of photovoltaic inverter.

Index Terms—Cascade inverter, design, LCL filter, predictive control.

# I. INTRODUCTION

Modern photovoltaic inverters use high-frequency power converters modulated with pulse-width modulation (PWM). High-frequency converters have many advantages such as small dimensions, high efficiency, etc. However due to the high-frequency switching their output voltage and current contain high-frequency components.

The purpose of the output LCL filter in grid connected systems is to mainly create the inductive load for voltage source inverter and to filter higher order harmonics in the current supplied to the grid. The first condition is set by the inverter topology. The second one is set by grid codes. The output LCL filter also influences the dynamics of the grid-connected inverter. If there is high amount of energy stored in reactive components of LCL filter, the dynamic of the grid-connected inverter will be compromised.

As can be seen, there are several requirements to be met at the same time. The paper offers analysis of an LCL filter as well as design guidelines for the LCL filter used with 15-level cascade voltage source inverter controlled by predictive control method with finite control set.

There are many papers concerning design of the LCL filter, e.g. [1]–[6]. However, many of them are based on THD of grid current [2], [5], [6], which is impossible to calculate for predictive control with variable switching

Manuscript received December 14, 2012; accepted May 7, 2013. This work was supported by the Slovak Research and Development Agency under the contract No. APVV-0185-10.

frequency or design is very simplified [3], [4]. The paper presents the LCL filter design based on current ripple and stored energy, which is important for fast control techniques such as predictive current control.

#### II. SYSTEM DESCRIPTION

## A. Filter topology

The output LCL filter is connected between the one-phase 15-level cascade voltage source inverter and grid.

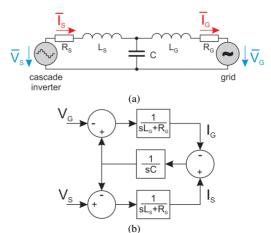


Fig. 1. LCL filter topology (a) and dynamic model (b).

The LCL filter is used in grid-connected inverters due to its high attenuation of high-frequency signals. The LCL filter is a system of 3<sup>rd</sup> order with three resonant frequencies defined by three reactive components

$$f_{0I_GV_S} = \frac{1}{2\pi} \sqrt{\frac{L_G + L_S}{L_G L_S C}} = \frac{1}{2\pi \sqrt{L_G C}} = \frac{1}{2\pi \sqrt{L_S C}}.$$
 (1)

The LCL filter has attenuation of 60 dB/decade for frequencies higher than  $f_{0I_GV_S}$ .

# B. Filter frequency characteristics

There are several transfer functions for LCL filter and their Bode characteristics which can be analyzed. For the design of LCL filter the relation between voltage  $V_S$  and current  $I_S$  (attenuation 40 dB/decade), the relation between voltage  $V_S$  and current  $I_G$  (attenuation 60 dB/decade) as well as the relation between current  $I_S$  and current  $I_G$  (attenuation 20 dB/decade) are important.

$$\frac{I_{S}(s)}{V_{S}(s)} = \frac{s^{2}L_{G}C + sCR_{G} + 1}{s^{3}L_{S}L_{G}C + s^{2}(L_{S}CR_{G} + R_{S}CL_{G}) + s(R_{S}CR_{G} + L_{S} + L_{G}) + R_{S} + R_{G}},$$
(2)

$$\frac{I_G(s)}{V_S(s)} = \frac{1}{s^3 L_S L_G C + s^2 (L_S C R_G + R_S C L_G) + s (R_S C R_G + L_S + L_G) + R_S + R_G},$$
(3)

$$\frac{I_G(s)}{I_S(s)} = \frac{1}{s^2 L_G C + s C R_G + 1}.$$
(4)

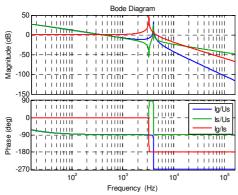


Fig. 2. Frequency characteristics of LCL filter.

### III. DESIGN OF LCL FILTER

#### A. Basic considerations

The LCL filter must provide the inductive load for the output of voltage source inverter. The inductive load can be evaluated by current ripple. Thus from the inverter point of view, the LCL filter can be designed in time domain.

The LCL filter also has to limit higher frequency components in grid current. From the grid point of view the LCL filter should be designed in frequency domain.

The LCL filter is designed for 15-level cascade inverter with predictive control technique with finite control set. Such control systems does not have any modulator and the frequency spectrum of the voltage  $V_{\rm S}$  is unknown. Also the active dumping is desirable thus no passive dumping components design is included.

There are three components to be designed ( $L_s$ ,  $L_g$ , and C) to meet the above mentioned requirements. Calculations are made in per-unit (subscript pu) basis, the base values used in calculations are listed in Table I.

TABLE I. SYSTEM PU BASE VALUES

Parameter	Formula	Unit
Power S <sub>B</sub>	-	kVA
Voltage U <sub>B</sub>	=	V
Frequency f <sub>B</sub>	-	Hz
Current I <sub>B</sub>	$S_B/U_B$	A
Impedance Z <sub>B</sub>	$U_B/I_B$	Ω
Inductance L <sub>B</sub>	$Z_B/2\pi f_B$	mH
Capacitance C <sub>B</sub>	$1/Z_B 2\pi f_B$	μF
Energy E <sub>B</sub>	$U_BI_B/2\pi f_B$	J

# B. Design of inductor $L_S$

The function of LCL filter is to filter higher-order harmonics coming from the inverter. The lower order components in (2) have insignificant influence on high frequency signals (higher than resonant frequency  $f_{0I_GV_S}$ ) and can be omitted

$$\frac{I_S(s)}{V_S(s)}\Big|_{HF} = \frac{L_G}{sL_SL_G + L_SR_G + L_GR_S} \approx \frac{1}{sL_S + R_S}.$$
 (5)

From frequency analysis can be shown that the high-frequency current  $I_S$  is limited mainly by the inductor  $L_S$  because the resonant frequency  $f_{0I_GV_S}$  is lower than the inverter switching frequency. The design of the inductor  $L_S$  is based on the required current ripple which is usually 10-20%.

The 15-level cascade inverter is asymmetrical and has three voltage sources of 60 V, 120 V and 240 V (thus the DC link voltage  $V_{DC}$  is 420 V). Even though there is no modulator lets consider multilevel sinusoidal PWM modulation technique just for design of the inductor  $L_{\rm S}$  (It can be shown there is no big difference between waveform of  $V_{\rm S}$  generated by predictive controller and sinusoidal PWM modulator). The amplitude of ripple voltage for mentioned modulation technique and cascade inverter will be 60 V.

The peak ripple current  $I_S$  is defined by the difference between the peak volt-seconds and the average volt-seconds applied to the inductor  $L_S$ . It occurs when the duty cycle is 50% (average volt-seconds is zero). The voltage ripple of  $V_S$  for duty cycle of 50% will be 30 V (which is  $V_{DC}/14)$  and will last for a quarter of switching period  $(T_S/4)$  (Fig. 3). The amplitude of ripple current is

$$I_{S(ripple \max)} = \frac{T_S}{4} \frac{V_{DC}}{14} \frac{1}{L_S} = \frac{V_{DC}}{56L_S f_S}.$$
 (6)

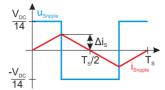


Fig. 3. Current ripple in L<sub>s</sub>.

For given rms value of current  $I_S$ , switching frequency  $f_S$ , DC link voltage  $V_{DC}$  and required current ripple in percentage, the required value of  $L_S$  in pu values is

$$L_{Spu} = \frac{V_{DCpu}\sqrt{2}\pi}{56f_{Spu}I_{RMSpu}ripple_{pu}}.$$
 (7)

The required inductance  $L_{Spu}$  for different current ripple and switching frequency is shown in Fig. 4.

# C. Design of inductor $L_G$ and capacitor C

The transfer function (3) for low frequencies (neglecting

higher order terms) becomes

$$\frac{I_G(s)}{V_S(s)} = \frac{1}{s(R_S C R_G + L_S + L_G) + R_S + R_G} \approx \frac{1}{s(L_S + L_G) + R_S + R_G}.$$
 (8)

Equation (8) describes relation between low frequency grid current  $I_G$  and low frequency inverter's output voltage  $V_S$ . For given value  $L_S$  and  $L_G$  there is needed certain voltage  $V_S$  to maintain the required grid current.

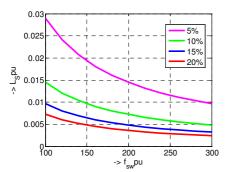


Fig. 4. Inductance of  $L_S$  versus switching frequency in pu basis for different current ripple.

The phasor diagram of LCL filter is shown in Fig. 5. For real LCL filter the reactance of capacitor is high for grid frequency and thus the capacitor current  $I_C$  can be neglected.



Fig. 5. Simplified phasor diagram of LCL filter.

Considering the phasor diagram the required inverter's voltage to maintain the nominal grid current  $I_G$  (the same  $I_S$  due to neglecting  $I_C)$  was calculated (Fig. 5). Maximal voltage  $V_S$  is limited by DC link voltage. As the net inductance of LCL filter (L=L\_G+L\_S) is increasing, required voltage  $V_S$  is increasing as well.

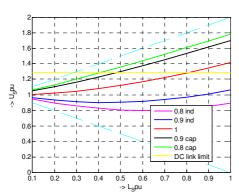


Fig. 6. Required voltage V<sub>S</sub> versus total inductance of the LCL filter for various power factors to maintain the nominal grid current.

The part of LCL filter formed by  $L_G$  and C is supplied by current  $I_S$ . The relation between grid current  $I_G$  and inverter current  $I_S$  is defined by (4). The  $L_G$ C part of LCL filter is responsible for attenuation of higher order harmonics supplied to the grid.

Simplified transfer function (neglected parts with small value) describes a second order system (attenuation 20 dB/decade)

$$\frac{I_G(s)}{I_S(s)} = \frac{1}{s^2 C L_G + 1}.$$
(9)

Product  $L_GC$  in (9) defines resonant frequency  $f_{0I_GI_S}$ . The lower the resonant frequency  $f_{0I_GI_S}$  the higher attenuation for particular higher-order frequency in  $I_S$ .

Components  $L_G$  and C influence also the resonant frequency  $f_{0I_GV_S}$  (1). To avoid resonance in the LCL filter it is advised to set the resonant frequency  $f_{0I_GV_S}$  in range of [5]

$$10f_g \le f_{0I_GV_S} \le 0.5f_s. \tag{10}$$

Because there is small difference in resonant frequencies  $f_{0I_GV_S}$  and  $f_{0I_GI_S}$  in a real LCL filter, (10) can be used for  $f_{0I_GI_S}$  as well.

The value of a capacitor C can be calculated using

$$C_{pu} = \frac{1}{\frac{L_{Spu}L_{Gpu}}{L_{Spu} + L_{Gpu}} n^2},$$
(11)

where  $n = f_{sw}/f_B$ .

Value of C in relation to  $L_G$  and  $L_S$  for different switching frequencies is shown in Fig. 7. As is the switching frequency lowered the required capacitance for the same values of  $L_G$  and  $L_S$  is increased. The reactive energy for capacitor is supplied by the inverter. It is thus advisable to limit the capacitance to around 5% of  $C_B$  [5].

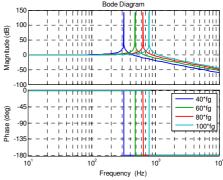


Fig. 7. Frequency characteristics of  $L_G C$  part of LCL filter for different resonant frequencies.

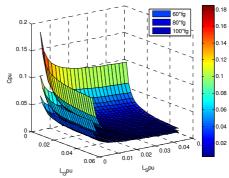


Fig. 8. Capacitance C versus filter inductances  $L_{\text{G}}$  and  $L_{\text{S}}$  for different resonant frequencies.

To achieve good dynamics of the LCL filter, it is important to know the total energy stored in the filter [2]. The higher the stored energy is, the less dynamic the filter

becomes.

Energy stored in inductor L<sub>S</sub>

$$E_{L_S pu} = \frac{1}{2} L_{Spu} I_{Spu}^2. {12}$$

Energy stored in inductor L<sub>G</sub>

$$E_{L_G pu} = \frac{1}{2} L_{Gpu} I_{Gpu}^2.$$
 (13)

Energy stored in capacitor C

$$E_{Cpu} = \frac{1}{2} C_{pu} U_{Cpu}^2. {14}$$

Total energy stored in LCL filter

$$E_{pu} = E_{L_S pu} + E_{L_G pu} + E_{Cpu}.$$
 (15)

Total energy stored in the LCL filter is calculated with regard to the phasor diagram in Fig. 5 and is different for each resonant frequency of the filter. The grid current  $I_G$  is considered constant. By changing the inductance of  $L_S$ ,  $L_G$  and capacitance of C to maintain the constant resonant frequency, the energy stored in filter is varying (Fig. 9).

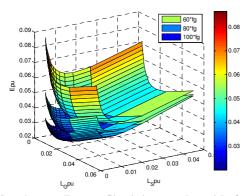


Fig. 9. Stored energy E versus filter inductances  $L_{\text{G}}$  and  $L_{\text{S}}$  for different resonant frequencies.

Form Fig. 8 it would be advisable to limit the capacitance of the LCL filter to as low as possible. It would force the reactive energy stored in capacitor to by minimal. However Fig. 9 indicates that lowering capacitance means increasing the total energy stored in the filter. The reason is increase of energy stored in inductors  $L_{\rm S}$  and  $L_{\rm G}$ . It is thus advisable to set the capacitance C somewhere near the abrupt change in Fig. 8. This would set the total energy to be minimal.

## IV. DESIGN STEPS OF LCL FILTER

The output LCL filter needs to be designed to meet grid standards for higher-order harmonics being supplied to the grid. The exact solution can be found for given modulation technique. However even in that case the only possibilities to by adjusted are current ripple of  $I_S$  and a resonant frequency of the filter.

The inductor  $L_S$  must deal with a high frequency current and is more expensive than grid side inductor  $L_G$  which mostly deals with a low frequency grid current. Thus saturation of a core material of an inductor  $L_S$  due to the

current ripple needs to be considered as well.

Lowering the switching frequency brings higher attenuation of high frequency current but on the other hand means increase of the total stored energy in the filter as well as capacitance of C.

Step-by-step procedure for LCL filter design is presented here.

Step 1. The PU basis values need to be calculated first and switching frequency  $f_{SW}$  of the inverter needs to be defined.

Step 2. Required current ripple is defined and inductance of  $L_s$  is calculated using (7).

Step 3. Resonant frequency of filter is defined according. This should be done with respect to Fig. 8 as a low resonant frequency could result in high capacitance of C. On the other hand, high resonant frequency would lead to poor attenuation of high frequencies.

Step 4. The inductance of  $L_G$  is set. The value of  $L_G$  will be always lower than the value of  $L_S$ . Plot of a total stored energy (Fig. 9) can be useful in this step, as a low value of  $L_G$  would result in higher energy stored in the filter. The value of  $L_S/2$  is good starting point.

*Step 5*. Capacity of C is calculated using (11) to meet the required resonant frequency.

Finally, the frequency characteristics of designed filter are verified.

#### V. CONCLUSIONS

Detailed design procedure for the output one-phase LCL filter is presented in the paper. Multifold problem of LCL filter design is described. Aspects as current ripple, required inverter's voltage, minimizing capacitance and stored energy are considered. From efficiency point of view (component size, losses and stored energy) it is desirable to set the resonant frequency of the filter as high as possible. However, it is shown, that high switching frequency will lower the filter attenuation. Presented step-by-step design procedure and filter analysis offer possibility to design LCL filter with good attenuation and minimized stored energy.

To verify the designed LCL filter it is required to have predictive control algorithm capable of active dumping of resonances in LCL filter.

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