

# Investigation of the Lamb Waves Generation in Isotropic Plates Using Ultrasonic Broadband Transducers

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**Abstract**—In the case of non-destructive testing or structural health monitoring of plate-like structures measurements are often performed using broadband ultrasonic transducers for generation and reception of guided waves. In some cases, the measurements could be performed having access just from one side of the object and it affects the excitation efficiency of generated asymmetric and symmetric guided waves modes. The experimental investigation was performed in order to evaluate how the change of the thickness of the object under investigation will influence the efficiency of generated guided waves asymmetric and symmetric modes.

**Index Terms**—Ultrasonic measurement, Lamb waves, dispersion curves, wave structure.

## I. INTRODUCTION

The guided waves are often used in ultrasonic applications of non-destructive testing (NDT) or structural health monitoring (SHM) of large engineering constructions, pipes, rails or plate-like structures [1], [2]. The main benefits of the guided waves making them so attractive for NDT and SHM applications is that these waves enable inspection over long distances with relatively small losses; fast detection of the internal non-homogeneities or defects; ability to inspect structures under water, coatings, multi-layered structures: inspection simplicity and speed [3]. However the main difficulties of the use of the guided waves are related with the dispersion and multiple mode propagation. In many cases the investigation by the means of the guided waves are performed using only fundamental  $A_0$  and  $S_0$  wave modes. In recent years some works have been done in the investigation of these waves broadband excitation using piezoelectric wafer active sensors (PWAS) [4]–[9], evaluating such factors as the size of the element, selective excitation and mode control schemes which are basically used in SHM applications. The objective of this research was to investigate the use of piezoceramic ultrasonic transducers for the broadband generation of Lamb waves in different thickness isotropic plates and to determine how efficiency of the generated guided waves asymmetric and symmetric modes changes due the change of the object thickness.

## II. OBJECT OF THE INVESTIGATION

The Lamb waves refer just to one type of guided waves that can be excited in infinite plate. The theory of guided waves propagation is complicated due the dispersive nature of these waves and the fact that at least two modes – asymmetric  $A_0$  and symmetric  $S_0$  can propagate at any frequency. As the object for investigation the aluminium alloy 2024-T4 plates with the different thickness ( $d=1\div 3$  mm) were selected. The general view of the phase velocity  $c_{ph}$  dispersion curves of the fundamental asymmetric  $A_0$  and symmetric  $S_0$  Lamb waves modes for these plates are presented in Fig. 1. The theoretical dispersion curves were calculated using analytical method [10], assuming that the propagation velocity of the longitudinal waves is  $c_L = 6350$  m/s and the propagation velocity of the shear waves is  $c_T = 3100$  m/s.

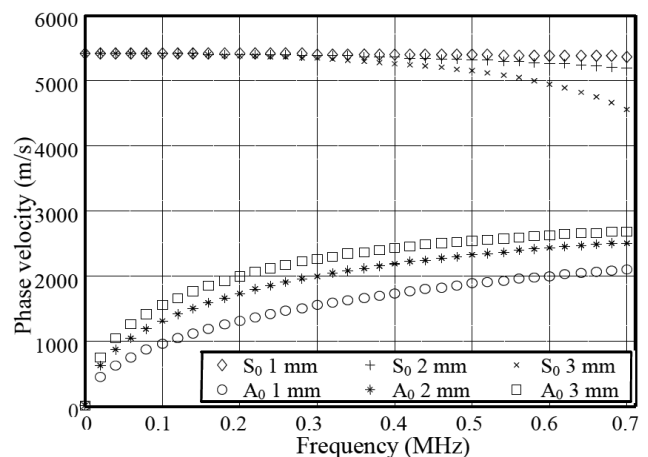


Fig. 1. The theoretical phase velocity  $c_{ph}$  dispersion curves of the asymmetric  $A_0$  and symmetric  $S_0$  Lamb waves modes propagating in  $1\div 3$  mm thickness aluminium alloy plates.

As it can be seen from presented graph, the asymmetric  $A_0$  mode has strong dispersion in all frequency ranges under analysis. The symmetric  $S_0$  wave mode in the same frequency bandwidth possesses very small dispersion which starts to increase from 400 kHz frequency and is more noticeable for 3 mm thickness plate. It can be noted that in the frequency ranges under investigation the phase velocity of  $A_0$  Lamb wave mode increases with frequency and thickness, and for  $S_0$  wave mode could be observed opposite

feature – decrease with a frequency and thickness change.

### III. THE SET-UP OF THE EXPERIMENT

The experimental investigation of the ultrasonic guided waves propagation in different thickness ( $d=1\div 3$  mm) aluminium alloy 2024-T4 plates were performed using experimental set-up presented in Fig. 2. The size of the 1 mm thickness aluminium plate was 600 mm  $\times$  600 mm, and the size of the 2 mm and 3 mm thickness plates were 1250 mm  $\times$  600 mm respectively.

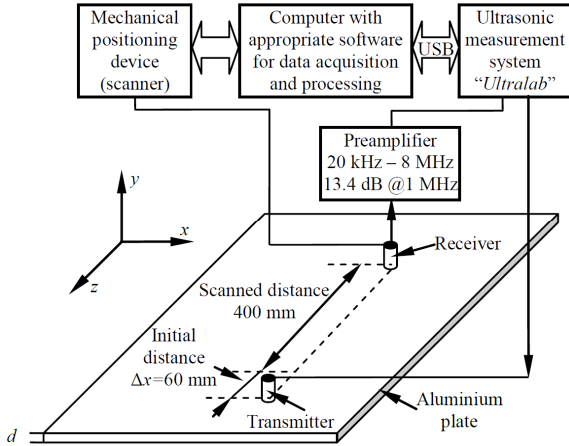


Fig. 2. Experimental set-up.

Measurements were performed using ultrasonic measurement system "Ultralab" which was developed at Kaunas University of Technology. Ultrasonic broadband

contact type transducers operating in thickness mode and having special convex form protector with diameter of the contact area of 0.1 mm were used [6]. Such point type protector with equivalent diameter of the contact area close to 0 mm is useful for generation and reception of ultrasonic guided waves, because it operates as point source of generated waves and on other hand enables to monitor propagating guided waves with minimal added distortions [6]. Positioning of the ultrasonic transducers has been performed by the precise mechanical scanning unit. The frequency of the ultrasonic transducers  $f=400$  kHz has been selected. The transmitter was driven by 1 period, 100 V amplitude ratio pulse. The total gain of the measurement system was 42 dB. Averaging of 8 received signals was used. The measurements were performed using pitch-catch method, because this type of inspection can be performed having access just from one side of the object under investigation. According to this technique the transmitter was attached at the fixed position and the receiver was scanned in one direction (Fig. 2) with the scanning step of 1 mm.

### IV. ANALYSIS OF THE SIGNALS

Using presented experimental set-up the signals at each position were recorded. The obtained data enabled to create B-scan image (dependence of received signals on the distance between transmitter and receiver) in which multiple propagating guided waves modes can be observed (Fig. 3(a), Fig. 4(a), Fig. 5(a)).

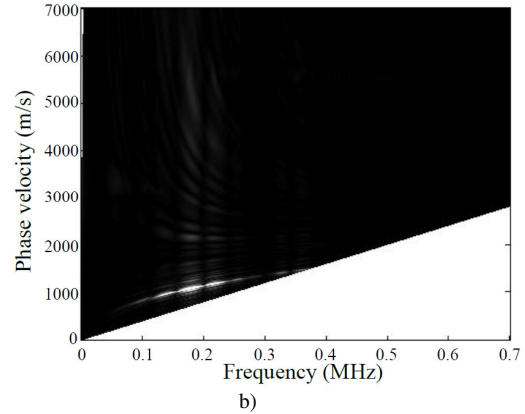
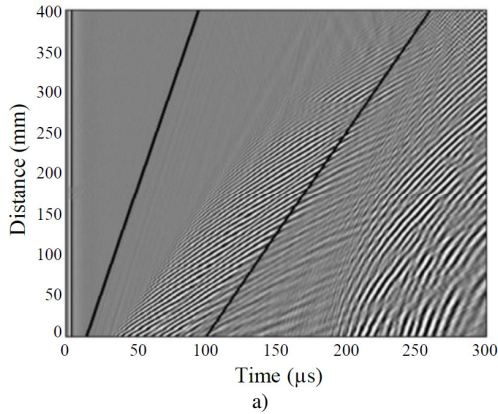


Fig. 3. a) The B-scan image of the Lamb wave signals measured on the  $d=1$  mm thickness aluminium alloy 2024-T4 plate with selected  $A_0$  and  $S_0$  modes and b) experimental dispersion curves obtained by 2D FFT.

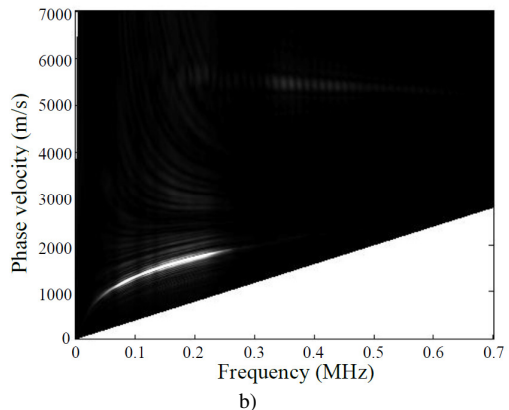
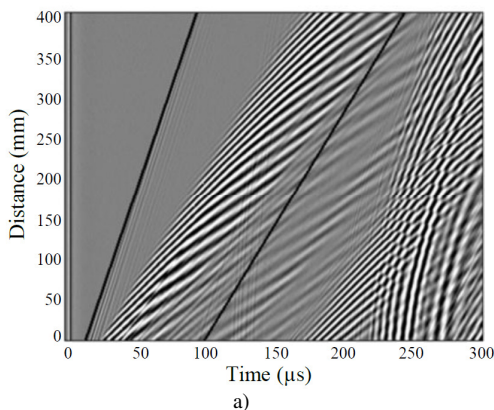


Fig. 4. a) The B-scan image of the Lamb wave signal measured on the  $d=2$  mm thickness aluminium alloy 2024-T4 plate with selected  $A_0$  and  $S_0$  wave modes and b) experimental dispersion curves obtained by 2D FFT.

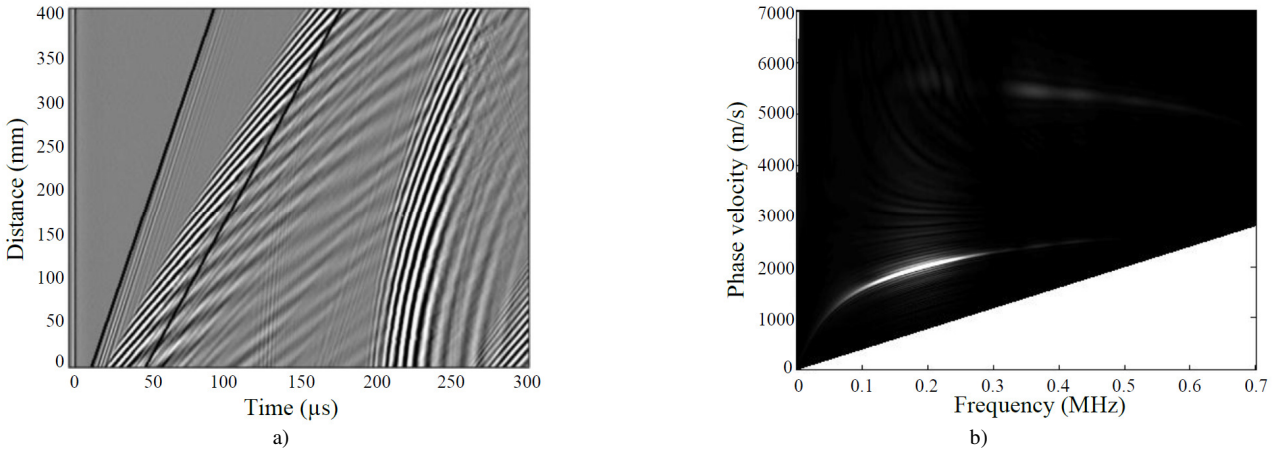


Fig. 5. a) The B-scan image of the Lamb wave signals measured on the  $d=3$  mm thickness aluminium alloy 2024-T4 plate with selected  $A_0$  and  $S_0$  wave modes and b) experimental dispersion curves obtained by 2D FFT.

For reconstruction of the dispersion curves patterns a two dimensional Fourier transform (2D FFT) method was applied. According to this method the data from time to distance domain  $u(t, x)$  are transformed into frequency – wavenumber domain  $U(f, k)$  [11], [12]

$$U(f, k) = \iint u(t, x) e^{-j(kx - \omega t)} dt dx. \quad (1)$$

The data from frequency – wave number domain is transformed into the frequency – phase velocity domain  $U(f, c_{ph})$  assuming

$$c_{ph} = \frac{f}{k}. \quad (2)$$

The measured signals in the form of B-scan image and obtained experimental dispersion curves aluminium alloy 2024-T4 plates with different thickness are presented in Fig. 3–Fig. 5 respectively. In order to get more reliable view of generated guided waves dispersion curves patterns and to eliminate the impact of the waves generated by the edge of the plate, filtering using mowing time window was performed. The boundaries of this window are shown in Fig. 3–Fig. 5(a)) by a solid line.

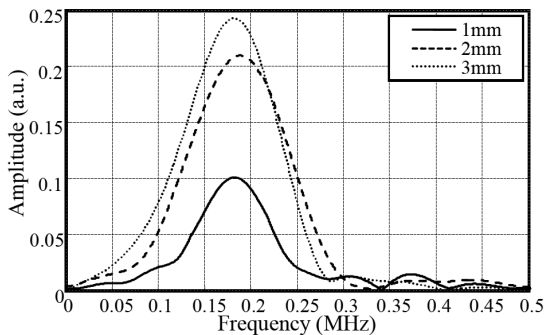


Fig. 6. Amplitude spectra of the  $A_0$  wave mode signal measured in 130 mm distance for 1–3 mm thickness aluminium alloy 2024-T4 plates.

As it can be seen from presented dispersion curves, the most efficient excitation was performed of asymmetric  $A_0$  Lamb wave mode, which was generated in frequency range from 50 kHz up to 400 kHz. Meanwhile, only weak trail of symmetric  $S_0$  mode could be observed in frequency range of

300 – 400 kHz. Additionally the amplitude of the generated  $S_0$  mode reduces with reduction of the thickness. It can be seen also that  $A_0$  and  $S_0$  wave modes are generated in slightly different frequency ranges.

For more accurate determination of the signals frequencies and the efficiency of generation the amplitude spectrum of  $A_0$  and  $S_0$  mode signals measured at the distance of 130 mm were calculated. The signals corresponding to the propagating  $A_0$  and  $S_0$  Lamb wave modes were selected using tapered cosine window.

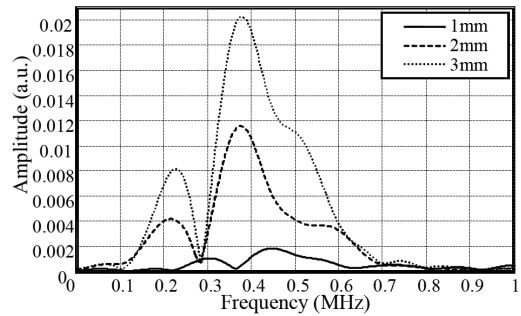


Fig. 7. Amplitude spectra of the  $S_0$  wave mode signal measured in 130 mm distance for 1–3 mm thickness aluminium alloy 2024-T4 plates.

The analysis showed that the central frequency of the asymmetric  $A_0$  wave mode generated in different thickness aluminium alloy 2024-T4 plates is equal to 185 kHz, meanwhile the frequency of the generated  $S_0$  mode is 390 kHz. The graphs of the generated asymmetric  $A_0$  and symmetric  $S_0$  wave modes amplitude spectra are presented in Fig. 6 and Fig. 7 respectively.

## V. ANALYSIS OF WAVE STRUCTURE

It was assumed that the main reason of so different amplitudes of the generated  $A_0$  and  $S_0$  modes and observed regularities is different distribution of particle displacement in the cross-section of propagating guided wave. Of course most important is component of particle displacement of velocity the normal to surface of the plate as in the experiment the longitudinal wave transducer was used. In each mode, the displacement distribution of these waves changes with frequency and thickness, while the stress-free boundary conditions are maintained constant at upper and lower plate surface. The calculation of the displacement

distribution across the plate of the  $A_0$  wave mode at 185 kHz frequency and  $S_0$  wave mode at 390 kHz frequency were performed using analytical method, described in [10]. The coordinate system used in calculation of the elastic perturbations in the Lamb waves propagating in a solid plate is shown in Fig. 8 [10].

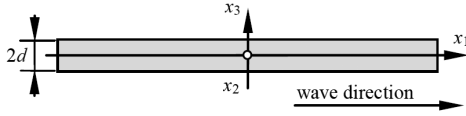


Fig. 8. The coordinate system.

According to this method, the scalar and vector potentials for asymmetric Lamb wave mode are [10]:

$$\phi = A_a \sinh(\kappa_d x_3) e^{ikx_1}, \quad (3)$$

$$\psi = A_a \frac{2ik\kappa_d x_3 \cosh(\kappa_d d)}{(k^2 + \kappa_s^2) \cosh(\kappa_s d)} \cosh(\kappa_s x_3) e^{ikx_1}. \quad (4)$$

Similarly, for an symmetric Lamb wave mode, the scalar and vector potentials are [10]:

$$\phi = A_s \cosh(\kappa_d x_3) e^{ikx_1}, \quad (5)$$

$$\psi = A_s \frac{2ik\kappa_d x_3 \sinh(\kappa_d d)}{(k^2 + \kappa_s^2) \sinh(\kappa_s d)} \sinh(\kappa_s x_3) e^{ikx_1}, \quad (6)$$

$$\kappa_d = \frac{\sqrt{k_d^2 - k_1^2}}{i}, \quad (7)$$

$$\kappa_s = \frac{\sqrt{k_s^2 - k_1^2}}{i}, \quad (8)$$

where  $A_a$ ,  $A_s$  are the boundary conditions for asymmetric and symmetric modes respectively,  $k$  is the wavenumber,  $d$  is the half thickness.

Finally, the displacements in-plane ( $u_1$ ) and out-of-plane ( $u_3$ ) components can be readily calculated from the following relationship:

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad (9)$$

$$u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}. \quad (10)$$

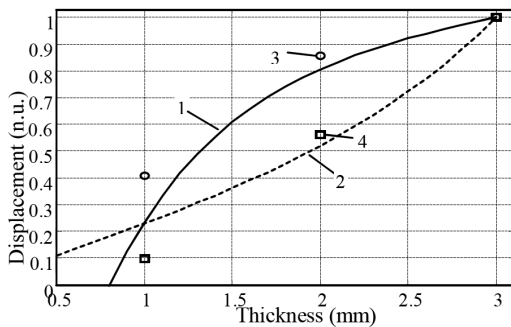


Fig. 9. The normalized out-of-plane displacement distribution of the  $A_0$  and  $S_0$  wave modes for different thickness aluminium alloy 2024-T4 plates: 1 – theoretical displacement for  $A_0$  mode; 2 – theoretical displacement for  $S_0$  mode; 3 – normalized amplitude of the  $A_0$  mode from experiment data; 4 – normalized amplitude of the  $S_0$  mode from experiment data.

The calculated normalized out-of-plane ( $u_3$  component) displacement distribution dependence from the thickness of the aluminium alloy 2024-T4 plate for asymmetric  $A_0$  wave mode at 185 kHz frequency and symmetric  $S_0$  wave mode at 390 kHz frequency are presented in Fig. 9. As it can be seen from presented graph, the character of the out-of-plane displacement distribution on the plate surface is similar with obtained experimentally measured amplitude changes regularities for investigated Lamb wave modes.

## VI. CONCLUSIONS

The experimental investigation of the Lamb waves asymmetric  $A_0$  and symmetric  $S_0$  modes generation in three different thickness aluminium alloy 2024-T4 plates using ultrasonic broadband transducers were carried out. The experimental investigations have demonstrated that guided waves in these plates were generated in slightly different frequency ranges and the amplitude of generated modes reduces with the reduction of object thickness. The calculation of out-of-plane displacement has demonstrated that the character of the displacement distribution coincides well with experimental results.

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