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Robust Observer Design with Pole Placement Constraints for Induction Motor Control

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***Abstract*—This paper presents an observation and control design methodology for a three-phase induction machine using a robust control technique. The state estimation and parameter identification of electrical motor are the main problem of the high-performance industrial application of electric drives. We propose a robust solution for real-time estimation of the rotor flux as well as the rotor speed. Indeed, to achieve a robust performance for the rotor flux observer, a H_∞ synthesis technique with pole placement will be used. The observer gain matrix is calculated by solving a convex optimization problem. The proposed solution has been tested and validated by an experimental set-up on a 3 KW squirrel age induction motor.**

***Index Terms*—Full-order observer, LMI, induction machine, robust control, sensorless control, vector control.**

I. INTRODUCTION

This paper aims at the study of the simplest and most economical sensorless speed control strategy for induction motor drives. In the literature, two high-performance control schemas are proposed and successfully implemented for the induction machine: direct torque control (DTC) [1] and field-oriented control (FOC) [2], [3]. Both control strategies aim to overcome the coupled structure between the dynamics of the electromagnetic torque and flux, in order to provide the same performance and flexibility of a DC machine [4]. Classical FOC is highly sensitive to parameter disturbances. To overcome these drawbacks, many researchers are trying to propose different control schemes over the last three decades. In this paper, we propose a solution based on linear matrix inequality (LMI) framework, to provide a robust rotor flux observer of the induction motor. Furthermore, based on the notion of quadratic H_∞ performance, a robust self-gain scheduling observer design method results from the constraint satisfaction stability and takes into account certain performance specifications for all admissible trajectories of the rotor speed.

II. MATHEMATIC MODELING OF INDUCTION MOTOR

In this section, we present the mathematical model of the induction motor in a (α–β) reference frame fixed to the stator. This model is developed in [5] under the assumption that the motor has a linear magnetic circuit. The nominal parameters of the induction motor used are given in Table I.

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The per-phase equivalent circuit of the machine is shown in Fig 1, and its dynamic model is given by the following mathematical equation:

$$\begin{cases} \dot{x} = \begin{bmatrix} \left(\frac{R_r}{L_r}\frac{1-\sigma}{\sigma} + \frac{R_s}{\sigma L_s}\right) I & \frac{R_r}{\beta L_r} I - \frac{p\omega}{\beta} J \\ \frac{M R_r}{L_r} I & -\frac{R_r}{L_r} I + p\omega J \end{bmatrix} x + B v_s, \\ i_s = Cx, \end{cases} \quad (1)$$

where $x = \begin{bmatrix} i_{s\alpha} & i_{s\beta} & \varphi_{r\alpha} & \varphi_{r\beta} \end{bmatrix}^T$,

$B = \begin{bmatrix} \left(1/\left(\sigma L_s\right)\right) I & 0_{2\times 2} \end{bmatrix}^T$ and $C = \begin{bmatrix} I & 0_{2\times 2} \end{bmatrix}$.

The relationship between the developed torque (C_e), load torque (C_r) and the dynamics of the rotating speed (ω) is defined as follows

$$\dot{\omega} = \left(p / Jm\right)\left(C_e - C_r\right). \quad (2)$$

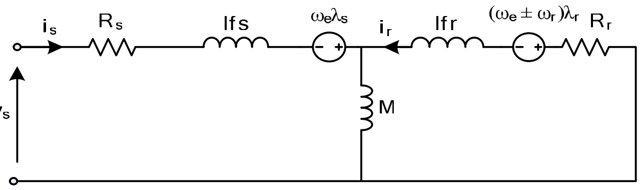

Fig. 1. Per phase steady-state equivalent circuit of induction motor in arbitrary rotating reference frame.

The electromagnetic torque C_e can be expressed according to the rotor flux and stator current as follows

$$C_e = p\left(3M / 2L_r\right)\left(\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha}\right). \quad (3)$$

III. FIELD-ORIENTED CONTROL WITH INDUCTION MACHINE LOSSES MINIMIZATION

In d–q reference frame locked to the rotor flux vector rotating at the stator frequency ($\omega_e = \omega_s$), the electromagnetic torque (3) can be re-expressed as follows

$$C_e = 3p\left(M / \left(2L_r\right)\right)\left(\varphi_{rd} i_{rq} - \varphi_{rq} i_{rd}\right). \quad (4)$$

When the direct axe of d–q frame is aligned with the rotor flux vector ($\lambda_r = \lambda_{rd}$), the developed torque is defined by

$$C_e = 3Mp\left(1/\left(2L_r\right)\right)\lambda_{rd} i_{rq}. \quad (5)$$

The dynamics of the rotor flux are expressed by the first order differential equation given by

$$\lambda_{rd} = Mi_{Ld} - \frac{L_r}{R_r}\frac{d\lambda_{rd}}{dt}. \quad (6)$$

The synchronous angular speed can be calculated as follows

$$\mu_s \; \mathrm{Z} \; MR_r / \left(L_r \lambda_{\mathrm{rd}}\right) i_{Lq} + p\mu. \quad (7)$$

In [6] an optimization algorithm of electrical energy dissipation of the drive system has been proposed. Below is a reminder of the expression of the optimal flux magnitude

$$\mathrm{a}_{ropt} = \sqrt{\left(\left(2L_r\right)/\left(3p\right)\right)\sqrt{\left(R_q / R_d\right)}Ce}, \quad (8)$$

where the constants R_d and R_q are:

$$R_d = R_s + \left(\left(M\mu_s\right)^2\right)/\left(R_r + R_c\right), \quad (9)$$

$$R_q = R_s + \left(R_s R_c\right)\left(R_r + R_c\right). \quad (10)$$

IV. Robust Adaptive Rotor Flux Observer

In this section, we present the design of a robust observer for a three-phase squirrel cage motor. The rotor speed estimator will be equally presented.

A. Adaptive Scheme for Speed Identification

According to (1), we can redefine the state space of induction motor as follows:

$$\begin{cases} \dot{x} = Ax + \mu A_\mu x + Bv_s, \\ i_s = Cx, \end{cases} \quad (11)$$

where $A = \begin{bmatrix} \left(\frac{R_r}{L_r}\frac{1-\pm}{\pm} + \frac{R_s}{\pm L_s}\right)I & \left(\frac{R_r}{£ L_r}\right)I \\ \left(MR_r / L_r\right)I & -\left(R_r / L_r\right)I \end{bmatrix}$ and $A_\mu = \begin{bmatrix} 0 & -\left(\frac{p}{£}\right)J \\ 0 & \left(p\right)J \end{bmatrix}$.

From the state equation given above (11), we adopt a Luenberger state observer by the following equation:

$$\begin{cases} \dot{\hat{x}} = \left(\hat{A} + \hat{\mu}A_\mu\right)\hat{x} + \hat{B}v_s + H\left(\hat{i}_s - Cx\right), \\ \hat{i}_s = \hat{C}\hat{x}. \end{cases} \quad (12)$$

In the remainder of this paper, the uncertain matrices Â and $\hat{B}$ are split into two parts; the first one corresponds to nominal parameters and the second one to unknown behavior and marked Δ (*). According to (11) and (12) the dynamics of state estimation error $\left(e \; \mathrm{Z} \; \hat{x} - x\right)$ under the assumption $\hat{B} \; \mathrm{Z} \; B$, can be described as follows

$$\dot{e} = \left(A + HC + \mu A_\mu\right)e + \left(\Delta A + \Delta\mu A_\mu\right)\hat{x}, \quad (13)$$

where

$$\Delta A = \begin{bmatrix} \left(\left(\left(1-\pm\right)/\pm L_r\right)\Delta R_r + \left(1/\pm L_s\right)\Delta R_s\right)I & \left(\left(1/£ L_r\right)\Delta R_r\right)I \\ \left(\left(M/L_r\right)\Delta R_r\right)I & -\left(\left(1/L_r\right)\Delta R_r\right)I \end{bmatrix},$$

$\hat{\mu} = \mu + \Delta\mu$.

ΔR_s and ΔR_r are the mismatch part of the stator and rotor resistances respectively. To guarantee the stability of the proposed observer (12), we impose a convergence sufficiently fast and robust of the estimation error to zero. In fact, we impose a gain for the transfer function from the disturbances to the estimation error maximized by a positive L_2*–gain*. Therefore, for a positive γ we suppose that

$$sup\frac{\|e\|_2}{\|w\|_2} = ¥, \quad (14)$$

where $B_w w = \Delta A\hat{x}$ is the disturbance signal and B_w being the input error matrix with appropriate dimensions.

To simplify the design of the speed adaptive mechanism, we neglect the influence of dynamical matrix uncertainty on the dynamics of the estimation error in this section.

Based on the Lyapunov theory [7], [8], the sufficient condition for the asymptotic stability of (13) is to find a Lyapunov candidate function *V*, where *V* is continuous, positive definite and differentiable. And where the time derivative of *V* is defined as negative.

The positive definite function is defined below

$$V\left(e, \Delta\mu\right) = e^T e + \left(\Delta\mu^2 / «\right). \quad (15)$$

Under the assumption that the induction motor is considered a slow time-varying system, the time derivative of *V* is expressed by the following equation

$$\dot{V}\left(e, \Delta\mu\right) = e^T \mathcal{P} e + V_1, \quad (16)$$

where $\mathcal{P} = \left(A + \mu A_\mu + HC\right) + \left(A + \mu A_\mu + HC\right)^T$ and $V_1 = \Delta\mu\left(\hat{x}^T A_\mu^T Pe + e^T PA_\mu \hat{x}\right) + 2\frac{\Delta\mu}{«_1}\frac{d\Delta\mu}{dt}$.

By the Lyapunov stability theory, the system is asymptotically stable if the time derivative of *V* is negative definite

$$\dot{V}\left(e, \Delta\mu\right) = e^T \mathcal{P} e + V_1. \quad (17)$$

The first condition of Lyapunov is

$$\Delta\mu\left(\hat{x}^T A_\mu^T e + e^T A_\mu \hat{x}\right) + 2\frac{\Delta\mu}{«}\frac{d\Delta\mu}{dt} = 0. \quad (18)$$

The time derivative of the rotor speed is given as follows

$$\frac{d\mu}{dt} = \text{-«} e^T A_\mu \hat{x}. \tag{19}$$

To estimate the rotor speed, we adopt the adaptive law given by the following equation

$$\hat{\mu} = K_{pv}\left(e^T A_\mu \hat{x}\right) + K_{iv}\int\left(e^T A_\mu \hat{x}\right)dt, \tag{20}$$

where K_{pv} and K_{iv} are the arbitrary positive gains.

The second condition of Lyapunov theorem is

$$\left(A + \mu A_\mu + HC\right)^T + \left(A + \mu A_\mu + HC\right) < 0. \tag{21}$$

The equation (21) is considered as a convex optimization problem.

B. Observer Gain Calculation

The previous subsection shows the existence of an estimator of the mechanical speed of the motor. This existence is based on the search of a gain matrix H which meets the second condition of Lyapunov (21). The gain matrix H has two purposes; the first objective is to guarantee the stability of the mechanical speed estimator. Based on its definition, the second objective of the matrix H is to ensure the overall stability of the observer defined by (12) on the one hand, and on the other hand to ensure some dynamic and static performances of the proposed observer. Indeed the calculation of observer gain H is deduced from some constraints on the dynamics of the observation error; these constraints are transformed into a convex optimization problem. The first constraint is given by (14), so we impose an L_2*–gain* performance on the dynamics of the observation error. This can be interpreted as an extension of H_∞ control problem [9], [10]. Indeed, for the extended form of the estimation error given in (22), the design problem is to find a stabilizing matrix H and ensure the disturbance attenuation performance requirement for the sub-system $G(s)$, where $G(s)$ is the transfer function (23) of the canal $w \rightarrow e$:

$$\tilde{e} = \left(A + HC + \mu A_\mu\right)\tilde{e} + B_w w, \tag{22}$$

$$G(s) = \left(sI - A - HC - \mu A_\mu\right)^{-1} B_w. \tag{23}$$

According to [9], the attenuation requirement (24) holds for some positive finite γ, if the dynamic system (24) has a state feedback H_∞ controller H, if and only if there exists $X = X^T > 0$ to make the satisfaction of the following LMI

$$\begin{bmatrix} (A+HC)^T X + X(A+HC) & XB_w & I \\ B_w^T X & -¥I & 0 \\ I & 0 & -¥I \end{bmatrix} < 0. \tag{24}$$

The bilinear matrix inequality (24) with unknown matrix H and X, can be transformed on a linear matrix inequality by using the variable change $W = XHC$. The inequality (24) is re-written as follows

$$\begin{bmatrix} A^T X + W^T + W + XA & XB_w & I \\ B_w^T X & -¥I & 0 \\ I & 0 & -¥I \end{bmatrix} < 0. \tag{25}$$

To improve the dynamic and static performances of the proposed observer, we impose some constraints on its eigen values. We define the pole placement region $\mathfrak{D}(d, r, ¯)$ by the intersection of three LMI regions. The first region is the disk centered at the point (0, 0) with radius r. The second region is the conic sector on the left-half complex plane with the apex at the point (0, 0) and inner angle 2θ, where $0 < ¯ < ® / 2$. The last region, is the left-half complex plane such as $\mathfrak{Re}(z) < -d$ where d is a positive real constant. For a complexe pole $s = -¶\mu_n \pm j\mu_d$ of our observer, where $0 \leq ¶ \leq 1$ is the damping ratio, $\mu_n = |s|$ is the natural frequency and $\mu_d = \sqrt{1 - ¶^2}$ is the damped frequency. The first region maximizes the natural frequency of the complexe pole "s" by $\mu_{n\max} = r$. So this constraint impose a homogenous dynamics for all observer poles. If the pole "s" is enclosed in the second region, then it's minimum damping ratio is $¶ = \sin(¯)$. This constraint minimizes they settling and rise times. The third region guarantees a minimum decay rate "d" for the observer. Made in the choice of pole placement region enhances the performance of uncertain system.

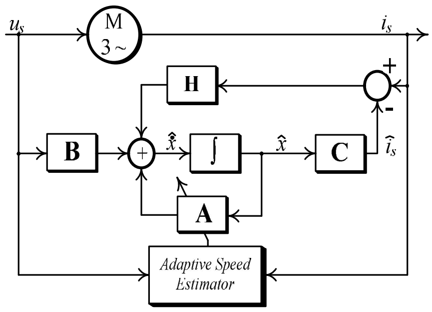

Fig. 2. State observer of induction motor.

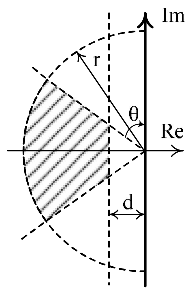

Fig. 3. LMI region.

According to [9], if we are using the variable change $W = XHC$, the poles of the system $G(s)$ enclosed in the disk $\mathfrak{D}(d, r, ¯)$ are equivalent to the existence of a solution $X = X^T > 0$ as:

$$A^T X + XA + W^T + X + W + 2d < 0, \tag{26}$$

$$\begin{bmatrix} -rX & A^T X + W^T \\ XA + W & -rX \end{bmatrix} < 0, \quad (27)$$

$$\begin{bmatrix} \sin(\bar{\ })\left(XA + A^T X + W + W^T\right) & \cos(\bar{\ })\left(XA - A^T X + W - W^T\right) \\ \cos(\bar{\ })\left(A^T X - XA - W + W^T\right) & \sin(\bar{\ })\left(XA + A^T X + W + W^T\right) \end{bmatrix} < 0. \quad (28)$$

By using the Matlab LMI Toolbox, we can solve the LMIs described in (25)–(28). The observer gain matrix is deduced by the following equation

$$H = X^{-1} W C^{-1}. \quad (1)$$

V. EXPERIMENTAL SETUP AND RESULTS

To validate the performances of the proposed solution, an experimental test is performed by using the Matlab/Simulink framework and the DS1104 board (Fig. 4). The used induction motor is a 3.1 KW, three-phase squirrel cage machine. The machine parameters are given in Table I. The experimental platform scheme is shown in Fig. 4. The drive system is fed by a Semikron IGBT voltage source inverter. To achieve the energy optimization, the motor is driven by DFOC strategy. The overall diagram of the vector control is shown in Fig. 5.

TABLE I. INDUCTION MOTOR NOMINAL PARAMETERS.

Symbol	Description	Quantity	UOM
R_s	Stator resistance	2.3	Ω
R_r	Rotor resistance	1.8	Ω
L_s	Stator inductance	261	mH
L_r	Rotor inductance	261	mH
M	Mutual inductance	245	mH
σ	Leakage factor	0.134	–
Jm	Moment of inertia	0.22	Kgm²
f	Friction coefficient	0.001	–
p	Number of pole pairs	2	–

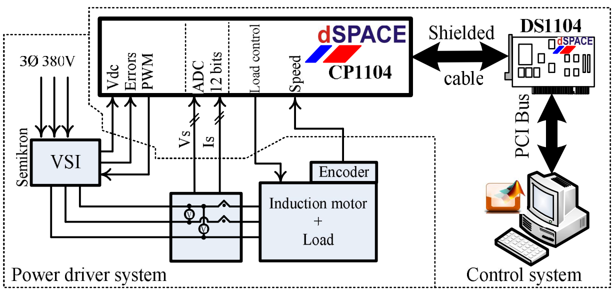

Fig. 2. A schematic platform for rapid prototyping.

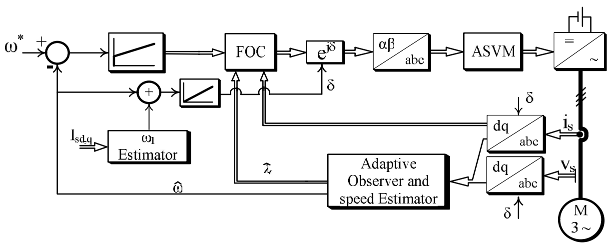

Fig. 3. Direct field-oriented control of three-phase AC induction motor.

Based on the command signals generated by the field-oriented control "FOC" block, the Adaptive Space Vector Modulation (ASVM) [6] block drives the three phase inverter and optimizes its electric energy dissipation. The block "adaptive observer" ensures at the same time the motor state as well as speed estimation. The block 'ω_l estimator' provides the slip frequency. Figure 6 illustrates the simulation results of the real and estimated rotor speed under a load torque C_r = *10 Nm*. Figure 7 and Fig. 8 show the actual and estimated variation of α and β components of the rotor flux respectively. The proposed robust observer can accurately estimate the rotor flux.

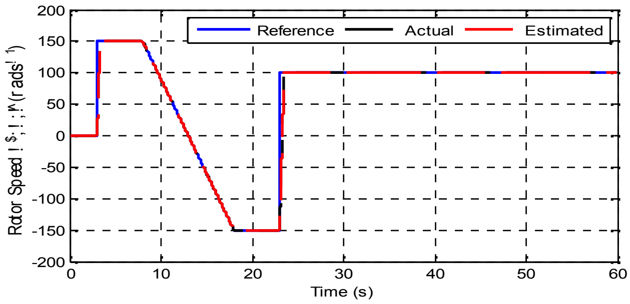

Fig. 4. Rotor speed tracking performance using the proposed solution.

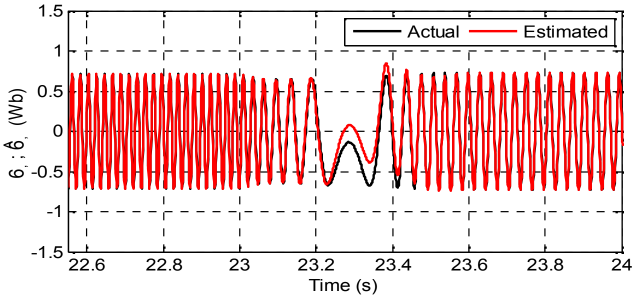

Fig. 5. Estimated and actual rotor flux λ_α.

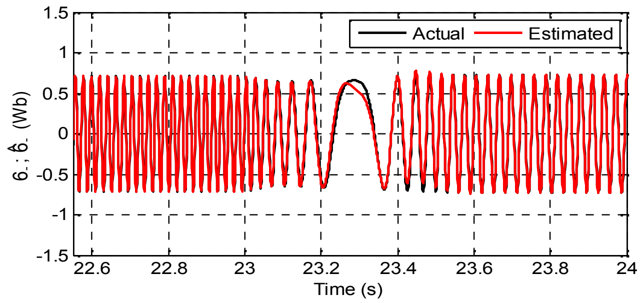

Fig. 6. Estimated and actual rotor flux λ_β.

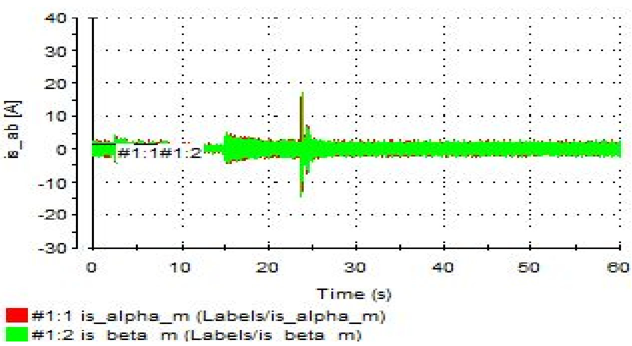

Fig. 7. Stator current in the α–β reference frame.

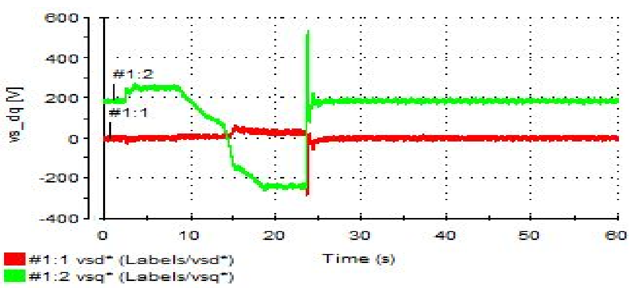

Fig. 8. Stator voltage in d–q frame.

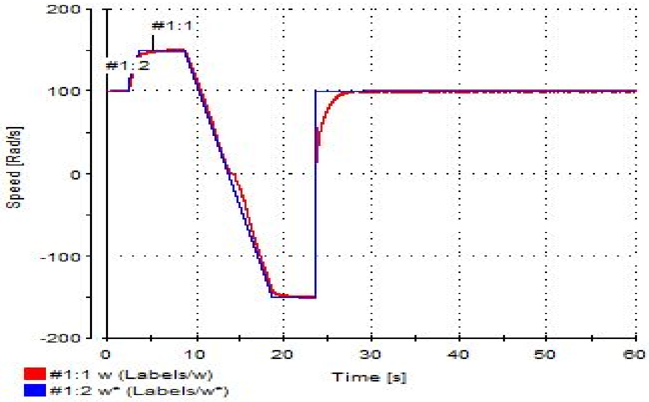

Fig. 9. Rotor speed tracking.

The last three figures show the experimental results. Figure 9 gives the wave form of the stator currents in the α–β reference frame. The stator current takes appropriate form and value. The stator voltage in the d–q reference frame is shown in Fig. 10. Figure 11 presents a perfect tracking of the mechanical speed.

VI. CONCLUSIONS

The aim of this paper is to design a sensorless vector control for a three-phase induction motor based on a robust state estimation. The proposed solution attempts to minimize the effect of parameter uncertainty and unmodeled dynamics of induction machine on the performance of the vector control algorithm. The observer gain matrix calculation is provided by solving an LMI deduced from the H_∞ control theory and closed-loop pole clustering constraints. The speed adaptive mechanism is used to provide a real-time tracking of real rotor speed. The proposed solution is verified by numeric simulation and experimental investigation. The stability and robustness of the proposed method are guaranteed.

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