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Improving Quality of Regulation of a Nonlinear MIMO Dynamic Plant

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***Abstract*—The paper presents a robust two-degrees of freedom control system for a multi-input, multi-output (MIMO) nonlinear dynamic process. There are discussed the advantages of the model following control (MFC) structure for MIMO systems, its stability and robustness properties and a method for synthesis of a multi-controller structure of the MFC model loop. The problems under study are exemplified by synthesis of a position and yaw angle control system for a 3DOF nonlinear mathematical model of a drillship, where the MFC structure is used to reduce effects of the impact of the sea current and wind forces on the ship's hull.**

***Index* Terms—Nonlinear systems, modal controllers, multi-controller structure, MFC, MIMO.**

I. Introduction

The nonlinear plants are commonly encountered in many different areas of science and technology. However, despite the great progress in analysis and synthesis of nonlinear control made over the past years the majority of proposed solutions is still based on linear controllers. Such an approach consists in designing either robust [1], [2] or adaptive controllers with varying parameters systematically tuned up with changing plant operating conditions [3], [4]. These may be supplemented by different control structures [5]–[7] and auxiliary algorithms utilizing e.g. recalculations of the control systems after system change or failure [4], [8], [9].

In the paper an adaptive modal MIMO controller with (stepwise) varying parameters in the process of operation is studied. The modal controllers making up the considered adaptive control system are designed for all possible operating points of the nonlinear MIMO plant. The appropriate set of parameter values of the tuned controller is selected during system operation on the basis of auxiliary measured signals, on which the operating points of the nonlinear plant are dependent.

To make the robustness of the controller higher and the compensation of disturbances more effective we suggest incorporating it into the Model Following Control (MFC) structure. The robust MFC system is known for its outstanding robustness to plant parameter and/or structure perturbations, as well as great disturbances dumping at the input and at the output of the plant to be controlled [10]–[12]. In the paper we fully utilize this theory and adopt the MFC advantages to synthesize a robust control system of improved quality for a class of nonlinear MIMO systems.

The problems under study are exemplified by synthesis of a position and yaw angle control system for a drillship described by a 3DOF nonlinear mathematical model of low-frequency motions over the drilling point. The effectiveness of the proposed MFC structure is shown by its use to compensate the environmental disturbances, the impact of the sea current and wind forces on the ship's hull.

The paper is organized as follows. A mathematical description of the adopted nonlinear control plant is brought in Section 2. Sections 3 and 4 present the structure of the proposed control system, its properties and a method of synthesis. An example, which demonstrates the disturbance damping property, is presented in Section 5. Finally the paper is concluded with Section 6.

II. Nonlinear Model of a Drill Ship

A nonlinear mathematical model of ship's low-frequency motions in 3DOF has been developed on the basis of tests carried out on a physical 1/20-scale model of the "Wimpey Sealab" drilling vessel [13]. It may be presented in the form of nonlinear state-space equations:

$$\begin{cases} \dot{x}_1 = x_4 \cos x_3 - x_5 \sin x_3 + V_c \cos \Psi_c, \\ \dot{x}_2 = x_4 \sin x_3 + x_5 \cos x_3 + V_c \sin \Psi_c, \\ \dot{x}_3 = x_6, \\ \dot{x}_4 = 0.088 x_5^2 - 0.132 x_4 V_s + 0.958 x_5 x_6 + 0.958 u_1, \\ \dot{x}_5 = -1.4 x_5 V_s - 0.978 x_5^3 / V_s - 0.543 x_4 x_6 + \\ \quad + 0.037 x_6 |x_6| + 0.544 u_2, \\ \dot{x}_6 = (-0.764 x_4 x_5 + 0.258 x_5 V_s - 0.162 x_6 |x_6| + u_3)/a, \\ y_1 = x_1,\ y_2 = x_2,\ y_3 = x_3, \end{cases} \tag{1}$$

where $V_s = \sqrt{x_4^2(t) + x_5^2(t)}$, $a = k_{zz}^2 + 0.0431$, k_{zz}^2 is the square of the relative inertia radius with respect to the ship's length L_{pp}. V_c and Ψ_c are the velocity and direction of the sea current as indicated in Fig. 1. All the signals appearing in (1) are dimensionless, i.e. they are related to the ship's

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dimensions and displacement, with the dimensionless time $t = t_r / \sqrt{L_{pp} / g} \approx 0.32\ t_r$.

The yaw angle and the ship's position are defined in an Earth-based reference system the origin of which is located over the drilling point on the seabed. In contrast, force and speed components with respect to water are determined in a moving system related with the ship's body and the axes directed to the front and the starboard of the ship with the origin placed in its gravity center. These are shown in Fig. 1.

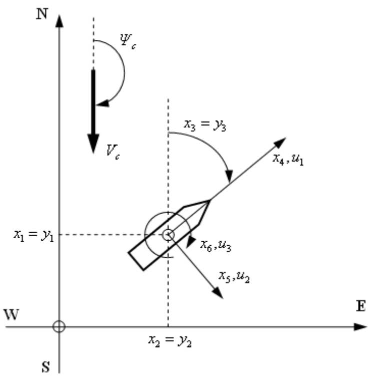

Fig. 1. Ship's co-ordinate systems.

Wind disturbances are one of the most important environmental disturbances occurring in the dynamic positioning of vessels. The wind forces and the moment for each degree of freedom are usually defined as follows:

$$\begin{cases} F_x = 0.5C_x(\gamma_p)\rho_a V_p{}^2 S_x, \\ F_y = 0.5C_y(\gamma_p)\rho_a V_p{}^2 S_y, \\ M_z = 0.5C_z(\gamma_p)\rho_a V_p{}^2 S_y L_{pp}, \end{cases} \tag{2}$$

where C_x, C_y and C_z are the force and moment coefficients, ρ_a is the density of air, S_x and S_y are the transverse and lateral projected areas, while V_p and γ_p are the relative wind speed and directions [14]. The velocity components included in the mathematical description of the wind are shown in Fig. 2.

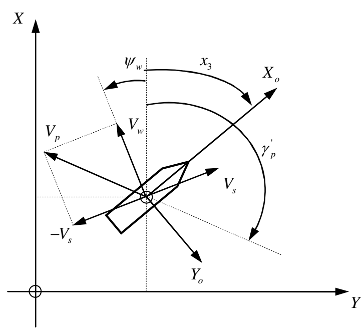

Fig. 2. Wind and ship velocity components.

The coefficients C_x, C_y and C_z in (2) are usually determined on the basis of the ship's hull and superstructure model tests in wind tunnels. The results of model tests carried out in a wind tunnel on a physical model of "Wimpey Sealab" drill-ship for the nominal relative wind speed $\bar{V}_p = 25kn$ in the range of angles $\gamma_p \in (-90 \div 90)\deg$ [13] can be approximated within this range and extrapolated beyond this range by the following function:

$$\begin{cases} F_x = \left[44\sin\left(1.3abs\left(\gamma_p\right)\text{-}2\right)+4\right]\cdot 10^3 kN, \\ F_y = \left[87\sin\left(\gamma_p\right)+11\sin\left(2\gamma_p\right)\right]\cdot 10^3 kN, \\ M_z = \left[540\sin\left(2\gamma_p\right)+210\sin\left(\gamma_p\right)\right]\cdot 10^3 kNM, \end{cases} \tag{3}$$

where $\gamma_p = x_3 - \gamma_p'$ is the value of the direction of the relative wind γ_p' converted to a reference system $\{X_o, Y_o\}$ associated with the moving ship. The detailed description of the wind disturbances modeled for the "Wimpey Sealab" model may be found in [15]. These forces are converted into dimensionless values and introduced into (1). Furthermore, it is assumed that the wind above the water surface V_w is turbulent and consists of two components: the slowly-varying component $\bar{V}_w$ (average wind speed) and the high-frequency component ΔV_w describing the sudden gusts of wind. Thus, $V_w = \bar{V}_w + \Delta V_w$, where ΔV_w is assumed to be a Gaussian noise with zero-mean and variance $\left(0.2\bar{V}_w\right)^2$.

III. Model Following Control for MIMO Processes

In the Model Following Control structure (Fig. 3), described for the first time in [10], the basic control task is performed by the main controller matched in the most optimal way to the process model. The corrective control signal, generated by the auxiliary controller, depends on the difference between the outputs of the adopted model and the actual process. Thus, the effect produced by the process-model mismatch, caused e.g. by disturbances and by possible process perturbations, can be neutralized.

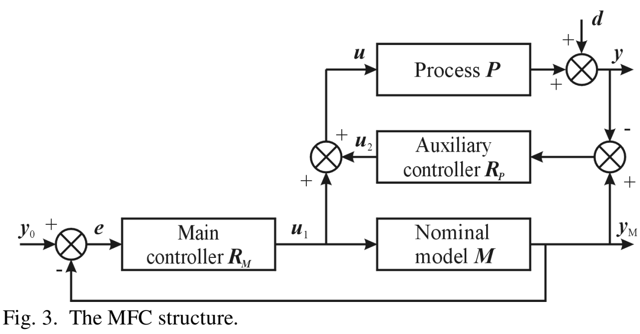

Fig. 3. The MFC structure.

Let the system components be described by continuous transfer function matrices of appropriate dimensions: model $\boldsymbol{M}(s)$, process $\boldsymbol{P}(s)$, main $\boldsymbol{R}_M(s)$ and auxiliary $\boldsymbol{R}_P(s)$ controller. For the MFC system with a perturbed process the disturbance sensitivity function is defined in the frequency domain $s = j\omega$, for $\omega \in (0, \infty)$ as

$$\boldsymbol{S}_{d-MFC}(s) = \left(\boldsymbol{I} + \boldsymbol{P}(s)\boldsymbol{R}_P(s)\right)^{-1}, \tag{4}$$

whence it follows that the quality with which the ideal output is reproduced may be shaped independently of $\boldsymbol{R}_M(s)$ by $\boldsymbol{R}_P(s)$ controller to suit requirements on suppression of disturbances. The sensitivity function (4) is then used to determine conditions to be met in order to achieve the required control performance. The effect of disturbances on the output of the perturbed process is seen to be low if the inequality

$$\bar{\sigma}\left(\boldsymbol{S}_{d-MFC}(s)\right) \leq \gamma , \tag{5}$$

where $\bar{\sigma}(\boldsymbol{A})$ is the greatest singular value of the matrix $\boldsymbol{A}$, holds true over the frequency range of interest to us.

As it was shown in [14], in view of properties exhibited by singular values of matrices, the MFC system is more stable and robust to disturbances in relation to the classic single-loop feedback system if the following inequality

$$\underline{\sigma}\left(\boldsymbol{I}+\boldsymbol{M}(s)\boldsymbol{R}_M(s)\right) < \bar{\sigma}\left(\boldsymbol{I}+\boldsymbol{M}(s)\boldsymbol{R}_P(s)\right) \tag{6}$$

is satisfied. Condition (6) may then be used to synthesize the MFC controllers that improve robustness of the system over the working frequency range.

IV. Description of the Proposed Control System Structure

In a typical approach the plant model $\boldsymbol{M}(s)$ adopted in the MFC structure is taken as a linear dynamic system, which models in the best way the real plant $\boldsymbol{P}(s)$. However, the nonlinearities of the process may be too strong to model the plant by a linear time invariant system, even in a robust MFC structure. That is why, in such a case we propose to synthesize the model control loop with an adaptive, gain scheduling controller and a nonlinear model of the process.

According to the adopted linear approach, we linearize the process model (1) in a typical location over the drilling point and obtain a set of local linear models on basis of which a set of linear modal controllers is calculated and a gain scheduling controller composed. The matrix transfer functions of the linearized plant model in the $s \in C$ domain, can be presented in the form of a relatively right prime polynomial matrix fraction description

$$\boldsymbol{M}(s) = \boldsymbol{B}_1(s)\boldsymbol{A}_1^{-1}(s). \tag{7}$$

Then, the linear modal controllers defined in time domain by:

$$\begin{cases} \dot{\boldsymbol{x}}_r(t) = \boldsymbol{A}_r\boldsymbol{x}_r(t) + \boldsymbol{B}_r\boldsymbol{e}(t), \\ \tilde{\boldsymbol{u}}(t) = \boldsymbol{C}_r\boldsymbol{x}_r(t) + \boldsymbol{D}_r\boldsymbol{e}(t), \end{cases} \tag{8}$$

may be synthesized by the pole placement technique. To design modal controllers in s-domain we employ a polynomial procedure, where the controller transfer function matrix $\boldsymbol{T}_r(s) = \boldsymbol{M}_2^{-1}(s)\boldsymbol{N}_2(s)$ is obtained by solving the Diophantine polynomial matrix equation

$$\boldsymbol{M}_2(s)\boldsymbol{A}_1(s) + \boldsymbol{N}_2(s)\boldsymbol{B}_1(s) = \Delta(s), \tag{9}$$

where $\boldsymbol{A}_1(s)$ and $\boldsymbol{B}_1(s)$ are known polynomial matrices describing the control plant (7), and $\boldsymbol{M}_2(s)$ and $\boldsymbol{N}_2(s)$ are unknown polynomial matrices for which (9) is solved. In the case of MIMO systems minimal solutions of (9) (of minimal degree with respect to the matrix $\boldsymbol{N}_2(s)$), which should satisfy the conditions $\deg_{rj}\boldsymbol{N}_2(s) \leq \deg_{rj}\boldsymbol{M}_2(s)$, $j = 1,2,...,m$, are sought. The detailed description of different pole-placement methods for the MIMO dynamical systems as well as its impact on the controller synthesis and performance of the closed loop control system may be found in [16].

Utilizing the above synthesis method for velocities in the range $V_s \in [-4.9 \div 4.9]$ [*knots*], with the resolution of 0.2 [*knot*] over the entire range of round angle $\Psi_c - x_{30} \in [0 \div 360^0]$, with the resolution of 5^0 has yielded a set of 3650 modal controllers. As they exhibit a time lag affected PD behavior during the system operation the incremental values $\tilde{\boldsymbol{u}}(t)$ generated by the tuned controller are added to the nominal values of the control signals $\boldsymbol{u}_0$ in steady states, which are calculated during the plant linearization. In order to limit the effect of excessive forces and moments produced by the controller there are introduced constraints imposed on maximal values of control signals.

If the values of $\boldsymbol{u}_0$ are known and the modal controllers are properly designed (for the given operating points), there exists a theoretical possibility that the steady-state error will tend to zero $\boldsymbol{e}(t) \to \boldsymbol{0}$ at $\tilde{\boldsymbol{u}}(t) \to \boldsymbol{0}$. Unfortunately, the model loop controller cannot compensate steady-state errors caused by unmeasurable disturbances from wind and waves acting on the ship. Thus to eliminate the deviations of the ship's course and position in steady state an auxiliary controller in the MFC control structure is proposed to be used.

According to the result of analysis presented in Section 3 to synthesize the component controllers operating in the discussed system we propose to:

- design the main (model) loop with the use of an above described algorithm and
- choose the auxiliary controller $\boldsymbol{R}_P(s)$ in order to extend the range of allowable process perturbations (6), with due regard for stability conditions.

The block diagram of the proposed control system for ship's course and position over the drilling point is depicted in Fig. 4. As in the classic MFC structure, the essential component of the plant input signal is generated in the main control system here containing the nominal model of the plant (1) and its controller designed as a single adaptive modal controller with stepwise variable parameter values calculated according to the above described method.

The goal of the auxiliary controller is to dump the effects produced by the process-model mismatch caused by wind disturbances acting on the ship and possible ship model perturbations.

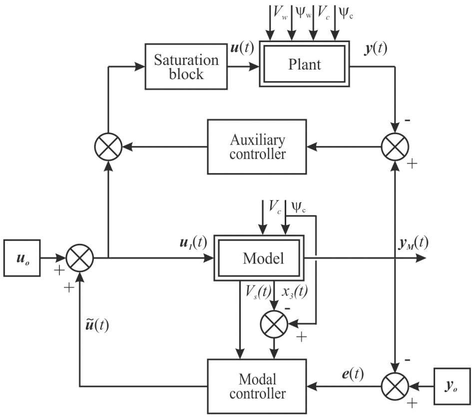

Fig. 4. Block diagram of the proposed control system structure.

V. Results of Simulation Tests

Fig. 5 and Fig. 6 present results of simulations of bringing the ship (1) to the new drilling point situated about 100m on the right above the old one ($y_{10} = 0.96$, $y_{20} = 0.26$) with the new course angle $y_{30} = 60^0$. All simulation tests have been carried out in the presence of sea current of $V_c = 2$ [$knots$] at $\Psi_c = 180^0$ and wind $V_w = 25$ [$knots$] at $\Psi_c = 90^0$ with the use of the nonlinear model (1). When the classic control loop, with the controller synthesized by the pole-placement method, is used, the non-measurable forces and moment derived from the wind cause the steady-state positioning errors (Fig. 5).

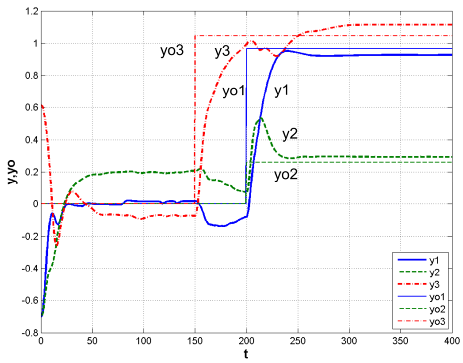

Fig. 5. Ship's position and yaw angle, steady-state errors caused by wind.

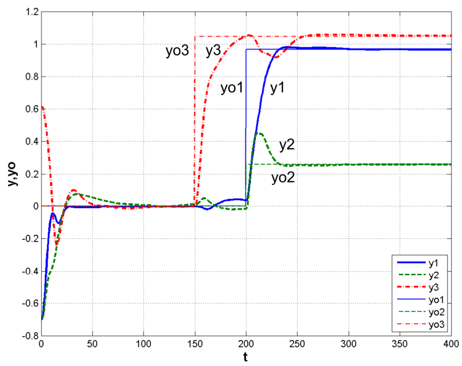

Fig. 6. Ship's position and yaw angle in a MFC control system.

The use of a MIMO MFC control structure brought the ship to the drilling point and assumed preset course angle without steady-state errors (Fig. 6). The chosen here auxiliary controller $\boldsymbol{R}_P(s)$ contained the same modal controller as in the model loop together with three PI controllers with parameters chosen as following: $k = 1$, and $T_i = 0.05$. The integration blocks bring the steady state errors to zero, thus the control goal has been met and the proposed MFC control structure has proved its ability to dump disturbances.

VI. Conclusions

In the paper an application of the MFC control structure to control of a nonlinear MIMO plant has been discussed. A method for synthesis of an adaptive gain scheduling modal controller in the model loop as well as conditions the auxiliary controller has to satisfy have been given. The presented example of a positioning control system for a drilling vessel, with the wind acting on the ship, shows efficiency of the method and the appropriateness of its use to control strongly nonlinear MIMO plants under the influence of non-measurable disturbances. The method makes it also possible to implement easily such a control system in typical off-the-shelf controllers.

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