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Analytical Determination of Mechanical Characteristics of Asynchronous Motors by Varying the Electric Current Frequency

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Introduction

The electric drives are still one of the fastest developing areas of the science at the present time. The asynchronous electric drives are most widely used in the industry and agriculture [1, 2] and they contain asynchronous motors of various types and powers. With the introduction of the static frequency converters the application areas of asynchronous motors broadened because by using them it is possible to achieve such a high electric drive rotation speeds which are unachievable for the direct-current motors. The rotation speed of the asynchronous motors at the present time is most often controlled by frequency variation. In this way a very wide speed regulation range, high uniformity of the regulation under changing loads, high speed stability, high efficiency of the electric drive operation and also the reversibility of the speed regulation in respect of the indicated speed are obtained.

When only the frequency of the current f_1 is changed the speed of the rotation of the magnetic field of the asynchronous motor in the air gap changes in direct proportion, and its amplitude value Φ_m changes in the inverse proportion. In order to decrease the variation of the rotating magnetic flux when the frequency of the current is varied according to one or the other law the electric supply voltage of the asynchronous motor is also changed using the same frequency converter. The law according to which the supply voltage of the motor is to be changed is determined not only by the frequency of the current f_1, but also by the nature of the change of the static load of the motor shaft. In this way the sufficiently high compatibility of the mechanical characteristics of the working machine and asynchronous motor is achieved.

It is relatively simple to calculate the mechanical characteristic of asynchronous motor under the nominal supply voltage and the current frequency on the basis of the magnitudes provided in the catalogs and using the Kloss formula [3÷5]. However the analytical method for calculation of mechanical characteristics under various supply voltages of the motors and frequencies of the currents which depend on the selected regulation law of the speed of rotation could not be found in the scientific sources. It is very important to determine the mechanical characteristics of asynchronous motors theoretically when designing and investigating the frequency-based asynchronous drives.

Analytical making of the mechanical characteristic of asynchronous motor under the rated supply parameters

In the catalogs the following rated parameters of asynchronous motors are provided in most cases: the power P_n, the rotation speed of the rotor n_n, the phase current of the stator I_{1n}, the efficiency factor η_n, the power factor cos φ_n, the relative breakdown torque μ_c, the relative starting torque μ_p, etc. On the base of the indicated parameters and according to respective formulas the other magnitudes are calculated with the sufficient precision, which are required to determine the mechanical characteristics of the asynchronous motor.

At first the mutual limit point of the asynchronous motor operation and starting mechanical characteristics is determined. This point is defined by the critical slip of the rotor s_c

$$s_c = s_n \frac{\mu_c + \sqrt{\mu_c^2 - 1 + 2 s_n (\mu_c - 1)}}{1 - 2 s_n (\mu_c - 1)}; \qquad (1)$$

here $s_n = (n_1 - n_n)/n_1$ – the nominal slip of the rotor; $n_1 = 60 f_{1n} / p$ – the magnetic field rotation speed; f_{1n} – the nominal current frequency; p – the number of pole-pairs of the rotating magnetic field of the stator.

When the four particular rotor slip magnitudes are available, the relative magnitudes of the torques are known in advance, i.e. when the slip is $s = 0$, the torque $\mu = 0$, when slip is s_n, $\mu = 1$, when slip is s_c, torque is $\mu = \mu_c$, and when $s = 1$, torque is $\mu = \mu_p$.

When other values of the rotor slip are available in the operational range of the mechanical characteristic

($0 < s < s_c$), the relative magnitudes of the torques are calculated according to the following Kloss formula

$$\mu = \frac{2\,\mu_c}{\frac{k_1}{s^\alpha} + \frac{s^\alpha}{k_1}}; \qquad (2)$$

here $\alpha = \lg b / \lg(s_c / s_n)$ – the exponent index; $b = \mu_c + \sqrt{\mu_c^2 - 1}$ – the relative torque; $k_1 = s_n^\alpha\, b$ – the coefficient.

When other values of the slip are present in the mechanical characteristic range of the locked-rotor ($s_c < s < 1$), the relative magnitudes of the torques are calculated according to the following Kloss formula

$$\mu = \frac{2\,\mu_c}{\frac{k_2}{s^\beta} + \frac{s^\beta}{k_2}}; \qquad (3)$$

here $k_2 = (\mu_c / \mu_p) - \sqrt{(\mu_c / \mu_p)^2 - 1}$ – the coefficient; $\beta = \lg k_2 / \lg s_c$ – exponent index.

On the basis of the preliminary and calculated magnitudes of the relative torques the mechanical characteristic of the asynchronous motor is estimated when $U_1 = U_{1n}$ and $f_1 = f_{1n}$.

Analytical making of the family of mechanical characteristics of asynchronous motor by varying the current frequency

When the shaft load of the asynchronous motor M_s is not dependent on the rotation speed ($M_s = \text{const}$), its supply voltage when varying the current frequency f_1 is changed according to the following law: $U_1 / f_1 = \text{const}$. If the shaft load of the motor is directly proportional to the square of rotation speed ($M_s \equiv n^2$), the motor voltage, when varying the current frequency f_1, is changed according to such law: $U_1 / f_1^2 = \text{const}$.

In order to calculate the relative magnitudes of the asynchronous motor torques under different rotor slips, under current frequency $f_{1\,i} \neq f_{1n}$ and supply voltage $U_{1\,i} \neq U_{1\,n}$, the i-th speed of the rotating magnetic field is determined at first

$$n_{1i} = 60\, f_{1i} / p. \qquad (4)$$

The i-th relative magnitude of the rotation speed of the magnetic field is claculated

$$\nu_i = n_{1i} / n_1. \qquad (5)$$

The i-th nominal slip of the asynchronous motor is determined

$$s_{ni} = s_n / \nu_i. \qquad (6)$$

The i-th critical slip of the asynchronous motor is calculated

$$s_{ci} = s_c / \nu_i. \qquad (7)$$

The constant of the k-th speed regulation law of the frequency and supply voltage is determined:

$$U_{1n} / f_{1n} = C_1 \qquad (8)$$

or

$$U_{1n} / f_{1n}^2 = C_2. \qquad (9)$$

The i-th value of the supply voltage of the k-th speed regulation law is calculated:

$$U_{11i} = f_{1i}\, C_1 \qquad (10)$$

or

$$U_{12i} = f_{1i}^2\, C_2. \qquad (11)$$

The i-th relative magnitude of the supply voltage of the k-th speed regulation law is determined

$$u_{ki} = U_{1ki} / U_{1n}. \qquad (12)$$

The i-th relative breakdown torque and the starting torque are calculated:

$$\mu_{ci} = \mu_c\, u_{ki}^2; \qquad (13)$$

$$\mu_{pi} = \mu_p\, u_{ki}^2. \qquad (14)$$

The i-th relative magnitude of the torque is determined

$$b_i = \mu_{ci} + \sqrt{\mu_{ci}^2 - 1}. \qquad (15)$$

The i-th exponent index is calculated

$$\alpha_i = \frac{\lg b_i}{\lg(s_{ci} / s_{ni})}. \qquad (16)$$

The i-th coefficient is determined

$$k_{1i} = s_{ni}^{\alpha_i}\, b_i. \qquad (17)$$

By using the (2) Kloss formula for the slip range ($0 < s < s_{c\,i}$) the relative magnitudes of the torques are calculated

$$\mu = \frac{2\,\mu_{ci}}{(k_{1i} / s^{\alpha_i}) + (s^{\alpha_i} / k_{1i})}. \qquad (18)$$

The i-th coefficient is determined

$$k_{2i} = (\mu_{ci} / \mu_{pi}) - \sqrt{(\mu_{ci} / \mu_{pi})^2 - 1}. \qquad (19)$$

The i-th exponent index is determined

$$\beta_i = \lg k_{2i} / \lg s_{ci}. \qquad (20)$$

By using the (3) Kloss formula for the slip range ($s_{c\,i} < s \leq 1$) the relative magnitudes of the torques are calculated

$$\mu = \frac{2\,\mu_{ci}}{(k_{2i} / s^{\beta_i}) + (s^{\beta_{ii}} / k_{2i})}. \qquad (21)$$

On the base of the calculated magnitudes of the relative torques under different current frequencies and aupply voltages the family of mechanical characteristics of the asynchronous motor is created.

The results of the analytical research

Assume that the in the frequency-based electric drive the squirrel-cage asynchronous motor is used the nominal power of which is P_n = 4 kW, the rotation speed n_n = 1430 min^{-1}, the current I_{1n} = 8,6 A, the repeatability of the breakdown torque μ_c = 2,2 and the repeatability of the starting torque μ_p = 2,0. The angular rotation speed of this motor ω_n = 0,1047 n_n = 0,1047 · 1430 = 149,7 rad/s and the torque $M_n = P_n / \omega_n$ = 4000 / 149,7 = 26,7 Nm. According to expressions (4)÷(17) and also (19) and (20), the magnitudes required to determine the relative torques are calculated under the selected current frequencies. They are given in Tables 1 and 2.

Table 1. Magnitudes claculated under the selected current frequencies, required to determine the relative torques, using the speed regulation law U_{1i} / f_{1i} = const

Magnitude	Current frequency $f_{1\,i}$, Hz			
	20	35	50	65
$n_{1\,i}$, min^{-1}	600	1050	1500	1950
$\nu_{1\,i}$	0,4	0,7	1	1,3
$s_{n\,i}$	0,1167	0,0667	0,0467	0,0359
$s_{c\,i}$	0,550	0,314	0,220	0,1693
C_1	4,6	4,6	4,6	4,6
$U_{1\,i}$, V	92	161	230	299
$u_{1\,i}$	0,4	0,7	1	1,3
$\mu_{c\,i}$	0.352	1,078	2,2	3,72
$\mu_{p\,i}$	0,320	0,980	2,0	3,38
b_i	1,288	1,481	4,16	7,30
α_i	0,1633	0,2535	0,920	1,282
$k_{1\,i}$	0,907	0,746	0,248	0,1026
$k_{2\,i}$	0,642	0,642	0,642	0,642
β_i	0,741	0,383	0,293	0,2495

Table 2. Magnitudes claculated under the selected current frequencies, required to determine the relative torques, using the speed regulation law U_{1i} / f_{1i}^2 = const

Magnitude	Current frequency $f_{1\,i}$, Hz			
	20	35	50	65
$n_{1\,i}$, min^{-1}	600	1050	1500	1950
$\nu_{1\,i}$	0,4	0,7	1	1,3
$s_{n\,i}$	0,1167	0,0667	0,0467	0,0359
$s_{c\,i}$	0,550	0,314	0,220	0,1693
C_2	0,092	0,092	0,092	0,092
$U_{1\,i}$, V	36,8	112,7	230	389
$u_{1\,i}$	0,16	0,49	1	1,691
$\mu_{c\,i}$	0.0563	0,528	2,2	6,29
$\mu_{p\,i}$	0,0512	0,480	2,0	5,72
b_i	1,0547	1,377	4,16	12,5
α_i	0,03435	0,2065	0,920	1,6285
$k_{1\,i}$	0,9797	0,787	0,248	0,0554
$k_{2\,i}$	0,642	0,642	0,642	0,642
β_i	0,741	0,383	0,293	0,2495

According to expressions (18) and (21) all the relative torques are calculated under the selected current frequencies and slips. They are given in Tables 3 and 4.

Table 3. Results of calculation of the relative torques obtained under the selected current frequencies and slips and using the speed regulation law U_{1i} / f_{1i} = const

Slip	Current frequency $f_{1\,ii}$, Hz			
	20	35	50	65
0	0	0	0	0
$s_{n\,i}$	0,341	1,0	1,0	1,0
$s_{c\,i}$	0,352	1,078	2,2	3,72
0,2	0,347	1,071	2,192	3,717
0,4	0,3515	1,0734	2,167	3,636
0,6	0,351	1,0458	2,1085	3,542
0,8	0,3388	1,0125	2,052	3,457
1	0.320	0,98	2,0	3,38

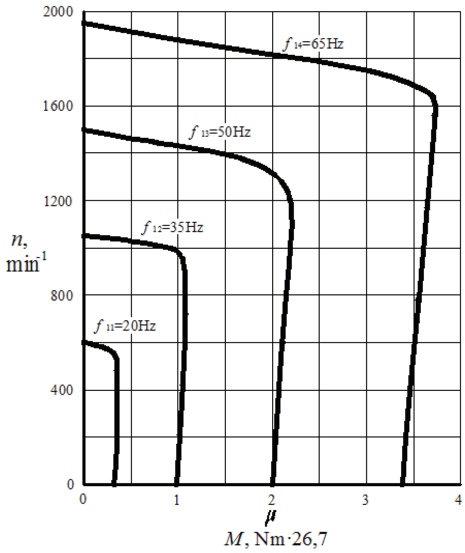

Fig. 1. The family of mechanical characteristics of the asynchronous motor created using speed regulation law U_{1i} / f_{1i} = const

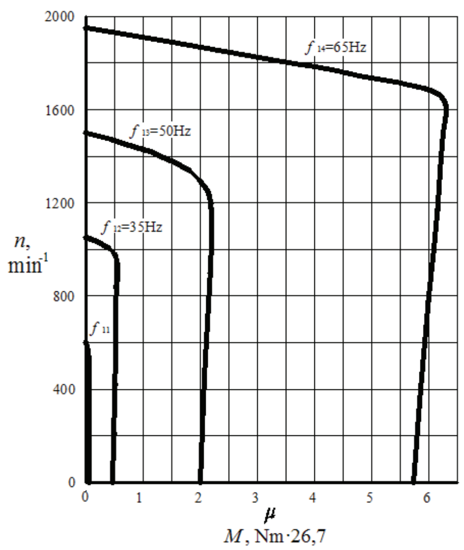

Fig. 2. The family of mechanical characteristics of the asynchronous motor created using speed regulation law U_{1i} / f_{1i}^2 = const

Table 4. Results of calculation of the relative torques obtained under the selected current frequencies and slips and using the speed regulation law $U_{1i} / f_{1i}^2 = \text{const}$

Slip	Current frequency $f_{1i\,i}$, Hz			
	20	35	50	65
0	0	0	0	0
$s_{n\,i}$	0,0562	0,502	1,0	1,0
$s_{c\,i}$	0,0563	0,528	2,2	6,29
0,2	0,0563	0,526	2,192	6,28
0,4	0,0563	0,526	2,167	6,15
0,6	0,0562	0,512	2,1085	5,99
0,8	0,0542	0,496	2,052	5,85
1	0.0512	0,480	2,0	5,72

Conclusions

1. By using the Kloss formulas it is possible to determine the mechanical characteristics of the asynchronous motors analytically when any rotation speed regulation law based on the current frequency manipulation is used.

2. When changing the current frequency not only the speed of rotation of the magnetic field changes but also the critical slips indicated in the mechanical characteristics. The voltage change intensity depends on the selected speed regulation law.

3. Due to the variation of the current frequency, when the voltage is changed the relative magnitudes of the critical torques change at first, and also the exponent indexes of the Kloss formulas and coefficients of the stable (operation) part of the mechanical characteristic.

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The analytical method for the determination of the mechanical characteristics of the asynchronous motors by varying the electric current frequency is analyzed. Not only the current frequency but also the supply voltage of the motor is changed in the frequency converters used to regulate the rotation speed of the asynchronous electric drives by using particular speed regulation law. The presented analytical method for the determination of mechanical characteristics is based on the same Kloss formulas which are dedicated to calculate the relative magnitudes of the torques under respective slips in the starting and operational parts of the mechanical characteristics and under nominal supply voltage parameters of the asynchronous motors. All the magnitudes included in these Kloss formulas (coefficients, exponent indexes, relative magnitudes of the starting and breakdown torques) are recalculated according to the modified parameters of the supply voltage and after that according to them the new relative magnitudes of the torques are determined under the selected slips. Using these relative magnitudes the families of the speed regulation mechanical characteristics of the asynchronous motors are determined. Ill. 2, bibl. 5, tabl. 4 (in English; abstracts in English and Lithuanian).

J. Bukšnaitis. Analizinis asinchroninių variklių mechaninių charakteristikų sudarymas keičiant srovės dažnį // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 6(112). – P. 3–6.

Nagrinėjamas analizinis metodas asinchroninių variklių mechaninėms charakteristikoms sudaryti keičiant srovės dažnį. Asinchroninių elektros pavarų sukimosi greičio reguliavimo dažnio keitikliuose, naudojant vieną ar kitą greičio reguliavimo dėsnį, keičiamas ne tik srovės dažnis, bet ir variklio maitinimo įtampa. Teikiamas analizinis mechaninių charakteristikų sudarymo metodas remiasi tomis pačiomis Klioso formulėmis, skirtomis momentų santykiniams dydžiams apskaičiuoti, esant atitinkamiems slydimams paleidimo ir darbo mechaninės charakteristikos dalyse ir nurodytiesiems asinchroninių variklių maitinimo įtampos parametrams. Visi šiose Klioso formulėse esantys dydžiai (koeficientai, laipsnio rodikliai, paleidimo ir kritinio momentų santykiniai dydžiai) perskaičiuojami pagal pakeistus maitinimo įtampos parametrus, o paskui, esant pasirinktiems slydimams, pagal juos nustatomi nauji momentų santykiniai dydžiai. Naudojantis šiais santykiniais dydžiais sudaromos asinchroninių variklių greičio reguliavimo mechaninių charakteristikų šeimos. Il. 2, bibl. 5, lent. 4 (anglų kalba; santraukos anglų ir lietuvių k.).