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Improved Optimal Controller Designs for All Pole Systems and Standard Forms with One/Two Variable Zeroes

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***Abstract*—Use of the standard forms for controller design is known for a long time. Since first introduced in 1950s, many new contributions have been proposed in the literature. In these contributions, the standard forms are obtained for all poles and with no zero, one zero and two zeroes systems. In this study, for the first time, optimum values of standard form coefficients with two variable zeroes are obtained using only one constrained for the Integral Squared Time Error (ISTE) and Integral of the Squared Time Error (IST^2E) criteria. Again in this study, an improved generalized controller design approaches using standard forms with all pole and two variable zeroes have been given for n^{th} degree all pole systems. By improving the previously proposed approaches, stabilities of the overall transfer functions have been guaranteed. In the proposed approaches, a PI or PID controller in the feed forward path and a polynomial controller, which its degree changes according to system degree in the inner feedback path, have been used. Parameters of these controllers are obtained using the standard form coefficients and the proposed simple mathematical operations without the restrictions of the previously proposed approaches. Comparative examples for the use of the proposed approaches and the obtained standard forms together with the previously proposed methods are also given in the MATLAB.**

***Index Terms*—Standard forms, controller design, PI, PID, optimization.**

I. INTRODUCTION

Nowadays classical controllers are still popular in spite of many proposed modern control methods due to the robust performance and easiness in the design steps. One of these classical control methods is optimal controller design method based on the minimization of the error signal by adjusting the controller parameters with respect to the system transfer function. Many error criteria have been proposed to minimize the error signal in the literature. The integral squared error (ISE) criterion is one of the most popular criteria since these allowed solutions to be obtained in the s-domain by using Parseval's theorem [1] and given recursive formula in [2]. Others are integral absolute error (IAE) and integral time absolute error (ITAE). However, despite of the ISE criterion, to obtain the results of the IAE and ITAE criteria, too much computation time or simulation is needed. The minimization operation must be implemented for the system in every case in the classical approach of the optimal controller design method. Thus, these methods take too much time and needs an expert person. Therefore, it is not very practical.

A study about using IAE and ITAE criteria to obtain standard forms has been presented by Graham and Lathrop [3]. They have considered only all pole standard forms. Dorf and Bishop [4] proposed the ISE criterion; however the standard form coefficients have not been given in their study. They gave obtained standard form coefficients for the systems with one zero using the ITAE criteria for a ramp input. General transfer function of the standard form is given in (1)

$$\frac{C(s)}{R(s)}=\frac{c_k s^k+c_{k-1}s^{k-1}+\ldots+c_1 s+c_0}{s^n+d_{n-1}s^{n-1}+\ldots+d_1 s+d_0}. \quad (1)$$

Dorf and Bishop [4], [5] and some textbooks, which are devoting a separate section for the subject, propose and use $c_0 = d_0$ and $c_1 = d_1$ to get a zero steady-state error for a ramp input to obtain the closed loop transfer functions of n poles standard forms with one zero. However, this case restricts the independently chosen control parameters. Additionally, this does not mean that the obtained optimal coefficients are also optimal for the step input signal. In case of that optimal coefficients of the standard forms with a zero are required to be obtained for a step input, then $c_1 \neq d_1$ must be chosen [6], [7].

Otherwise, obtained standard forms may cause very oscillatory step responses for the same systems and there will be an increment on the overshoot of the responses. Atherton and Boz obtained coefficients of the standard forms with all pole and one zero for ISTE and IST^2E and presented results in Atherton and Boz [6] and Boz [7]. As for the use of standard forms in the controller design, many new design methods based on standard forms are cited in the

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literature [8]–[13]. Recently, Sari and Boz also presented a new approach to obtain the parameters of PI controller to be used in the feed forward path by using standard forms for all pole systems with a zero [14], [15]. Another study has been presented by Sari and Boz in 2011 about for the first time obtaining optimum values of standard form coefficients with five pole and two variable zeros for ISTE and IST^{2}E criteria [16]. In their study, a new simple generalized controller design approach for the new systems with all pole transfer function has been introduced to show the use of obtained standard form coefficients in the controller design. The design approach was based on using the standard forms with $c_1 \neq d_1$, and $c_2 \neq d_2$ optimized for the ISTE and IST^{2}E criteria. On the other hand, optimizations were carried out by constraining the c_1 and c_2. Thus; the optimal parameters of the standard forms were obtained only for c_2 equals to 1, 2, 3 and 4, while c_1 was changing 0.5 to 8. In the proposed approach, a proportional integral derivative (PID) controller that is in the feed forward and a polynomial controller, which is in the feedback path were used.

In this paper, the limitations of the previously designed controller by Sari and Boz [16] have been improved. By this improvement the coefficients in the system transfer function do not affect the stability unlike the previously proposed controller. Additionally, the proposed new controller structure has been compared with the previously proposed control systems by using in the first two examples. Furthermore, standard forms have been obtained for only constraining the c_2 value. In the previous studies, the c_2 and c_1 were both constrained. Optimal standard form coefficients have been calculated for c_2 values changing 1 to 50 and since the results are linear it became possible to be used for higher values of c_2 than 50. In the third and fourth examples, these standard forms have been used with the new controller scheme and successful results have been obtained. The simulation results of the method with some well-known design methods have been presented graphically for comparison.

II. Integral Performance Criteria and Standard Forms

In general, the closed loop transfer function of a plant can be given as presented in (1). On the other hand, the steady state error of the system has been shown in (2)

$$e_{ss}=\left(c_0-d_0\right)r(t)+\left(c_1-d_1\right)\frac{dr(t)}{dt}+\left(c_2-d_2\right)\frac{d^2r(t)}{dt^2}+ \\ +\ldots+\left(c_{k-1}-d_{k-1}\right)\frac{d^{k-1}r(t)}{dt^{k-1}}+\left(c_{k-1}\right)\frac{d^kr(t)}{dt^k}. \quad (2)$$

In (2), r(t) is the form of input and it determines the size of the steady-state error. $c_0 = d_0$ condition is required to get zero steady-state error for a step function. There are many possibilities of C(s)/R(s) for which steady state error is zero with a step unit, because the order of the numerator of C(s)/R(s) can be less than or equal to the order of denominator. On the other hand, when a ramp function has been used as an input, the steady-state error only become zero for $c_0 = d_0$ and $c_1 = d_1$ conditions [17].

The performance definitions of a dynamical system are usually given by their transient response. On the other hand, to determine the transient response, the system output is measured in terms of rising time, overshoot and steady-state error, where a step or ramp signal is applied to the system input. All of these measurements must be zero ideally, since the system output must exactly follow the input signal. However, this is not the case practically; therefore the outputs are expected to follow the input as much as possible closer. A controller is usually used in a system to achieve the desired responses in case of lack of performance values. There are many controller design methods that are practically used currently [18], [19] and [20].

A controller normally works based on the minimization of the error signal that is the difference between reference input r(t), and controlled output signal c(t) as given in (3) below

$$e(t)\rightarrow 0, \quad (3)$$

where $t \geq 0$. Hence, a suitable criterion to characterize the optimal time response of a system is usually given as an integral function of the error, or its weighted products. An integral error criterion may be presented in a general form as presented in (4)

$$J=\int_0^\infty \emptyset[e(t),t]dt. \quad (4)$$

Therefore, an optimum dynamic performance can be taken as the time response that gives a minimum value of J. The integral performance criterion can be written in different forms as given in (5) and (6), so a control system is considered to be optimal if the selected performance index is minimized based on the variation on the controller parameters:

$$J_{ISE}=\int_0^\infty e^2(t)dt, \quad (5)$$

$$J_{IAE}=\int_0^\infty |e(t)|\,dt. \quad (6)$$

The time weighted versions of these two criteria have been presented for ISE and for IAE in Zhuang [21] and Graham and Lathrop [3], respectively. These criteria can be expressed more general as given in (7).

$$J_n(\theta)=\int_0^\infty t^n[e(\theta,t)]^2dt. \quad (7)$$

That is the general time weighted integral squared error criterion, and

$$J_n'=\int_0^\infty t^n|e(\theta,t)|\,dt, \quad (8)$$

that is the general time weighted integral absolute error criterion where θ refers to variable parameters which are chosen for the minimization of $J_n(\theta)$. According to (7), J_0, J_1 and J_2 are named as ISE, ISTE and IST^{2}E, respectively.

III. Standard Forms with Two Zeroes as a Function of C_2

In this study, new optimum values of standard form coefficients with four and five poles and two variable zeroes are obtained for the ISTE and IST^{2}E criteria by constraining only the c_2 value. The optimum values of these coefficients for the J_1 and J_2 criteria for $T_{24}(s)$ as a function of c_2, are given in Fig. 1 and Fig. 2, respectively. Similarly, the optimum values of the standard form coefficients for the J_1 and J_2 criteria for $T_{25}(s)$ are given in Fig. 3 and Fig. 4, respectively. J_1 and J_2 integral values for different values of c_2 for $T_{24}(s)$ and $T_{25}(s)$ are given in Fig. 5 and Fig. 6, respectively.

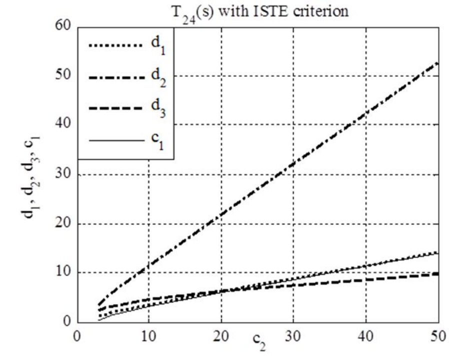

Fig. 1. Optimum values of d_1, d_2, d_3 and c_1 for $T_{24}(s)$ with ISTE criterion.

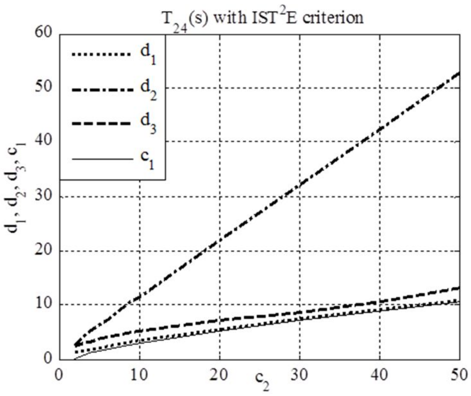

Fig. 2. Optimum values of d_1, d_2, d_3 and c_1 for T_{24} (s) with IST^{2}E criterion.

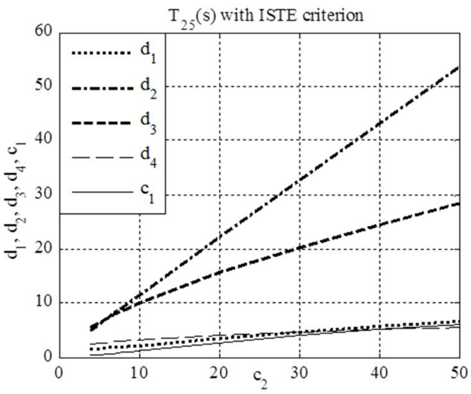

Fig. 3. Optimum values of d_1, d_2, d_3, d_4 and c_1 for $T_{25}(s)$ with ISTE criterion.

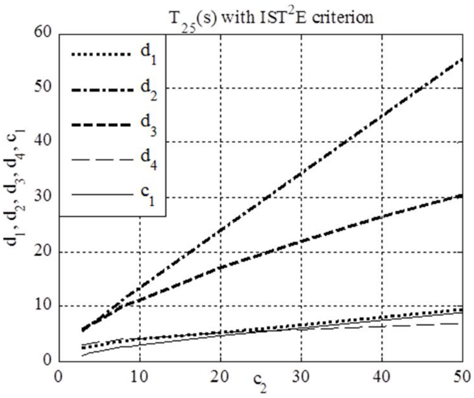

Fig. 4. Optimum values of d_1, d_2, d_3, d_4 and c_1 for $T_{25}(s)$ with IST^{2}E criterion.

Step responses of the obtained standard forms for different values of c_2 are also given for $T_{24}(s)$ and $T_{25}(s)$ in Fig. 7 to Fig. 10. It can be seen from the Fig. 5 and Fig. 6 that the integral error values are decreasing dramatically while c_2 value is increasing.

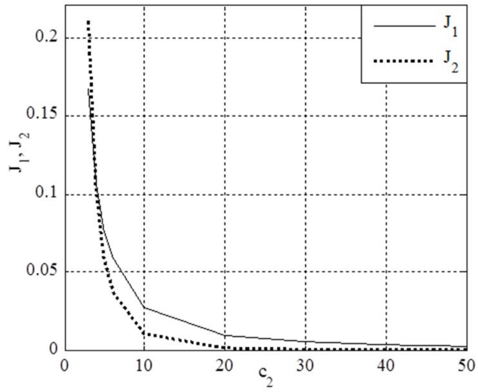

Fig. 5. J_1 and J_2 integral values for different values of c_2 for $T_{24}(s)$.

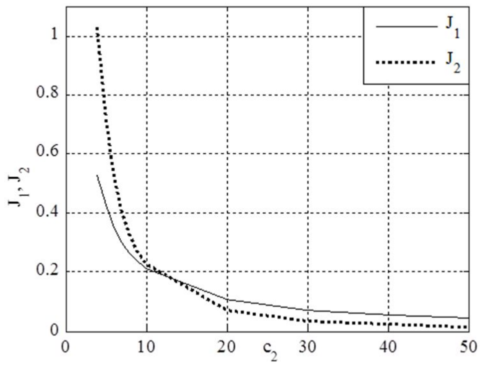

Fig. 6. J_1 and J_2 integral values for different values of c_2 for $T_{25}(s)$.

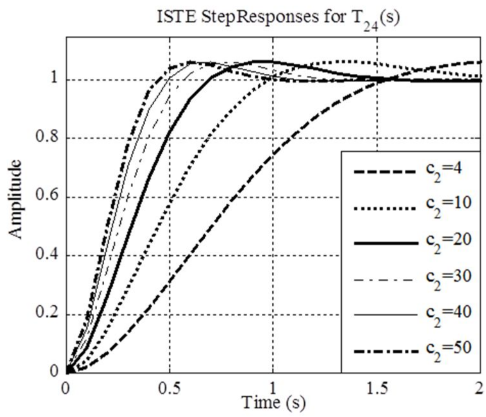

Fig. 7. ISTE step responses for different values of c_2 for $T_{24}(s)$.

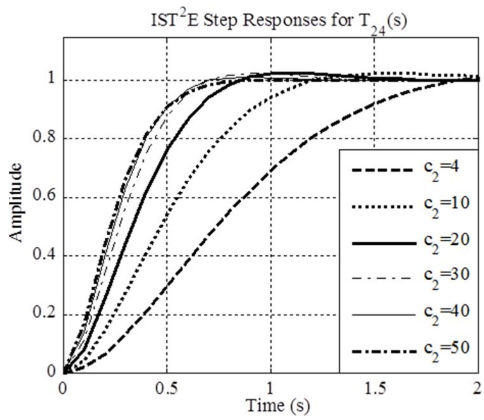

Fig. 8. IST^{2}E step responses for different values of c_2 for $T_{24}(s)$.

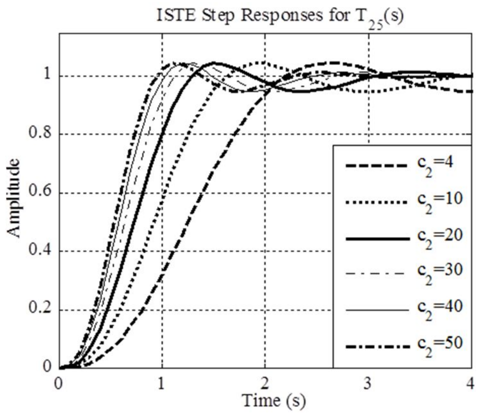

Fig. 9. ISTE step responses for different values of c_2 for $T_{25}(s)$.

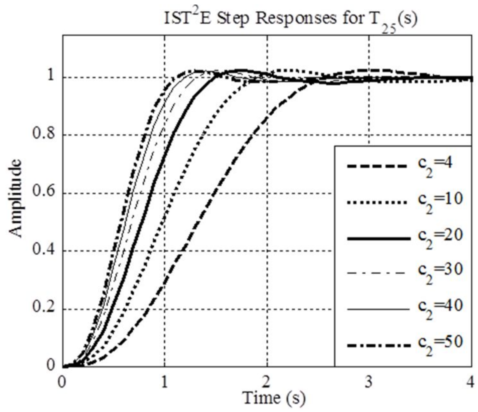

Fig. 10. IST^{2}E step responses for different values of c_2 for $T_{25}(s)$.

IV. Generalized Optimal Controller Design Method With No Limitations for N$^{\text{TH}}$ Degree All Pole Systems

PI and PID are well known controllers since long time. On the other hand they are still popular in use because of their limited number of parameters, which make them easy to tune and their relatively robust performances [22], –[31]. In this section an improved PI and PID controller tuning methods using the standard forms and additional polynomial feedback controller will be given.

n^{th} degree all pole system's transfer function can be represented by

$$G(s)=\frac{a_0}{b_n s^n+b_{n-1}s^{n-1}+\ldots+b_2 s^2+b_1 s+b_0}. \tag{9}$$

This system had been controlled using a polynomial controller in the inner feedback path and a PI controller in the feed forward path by Boz and Sari [15], and a PID controller in the feed forward path by Sari and Boz [16]. Transfer function of the system which use PI controller in the feed forward path had been given as follows

$$T(s)=\frac{l_1 a_0 s+l_0 a_0}{b_n s^{n+1}+b_{n-1}s^n+\left(b_{n-2}+a_0 k_{n-2}\right)s^{n-1}+\ldots+(b_1+a_0k_1)s^2+(a_0k_0+a_0l_1+b_0)s+a_0l_0}. \tag{10}$$

Again, transfer function of the system which use PID controller in the feed forward path had been given as follows

$$T(s)=\frac{l_2 a_0 s^2+l_1 a_0 s+l_0 a_0}{b_n s^{n+1}+b_{n-1}s^n+\left(b_{n-2}+a_0 k_{n-2}\right)s^{n-1}+\ldots+(b_1+a_0k_1+a_0l_2)s^2+(a_0k_0+a_0l_1+b_0)s+a_0l_0}. \tag{11}$$

In the both transfer functions, the coefficient belonging s^n is equal to b_{n-1} which is a coefficient of the system, and it is independent from the controller parameters. This means that the standard form coefficients are directly related to b_{n-1} and they have not been chosen independently. As a result of this, the error on the system response curves will increase when the coefficients of b_{n-1} decrease. This might also drive the controlled system into the instability. In this study, this unwanted case for both control systems has been removed by increasing the polynomial controller by one degree. Thus, now all of the coefficients of the closed loop control system parameters can be independently adjusted according to the standard form coefficients. This also eliminates the instability of the system and guarantees the stability. Furthermore, b_n coefficient has been taken equal to 1 for the simplification. The realization steps and formulation of both improved methods have been given below.

A. Design by Using A PI Controller in the Feed Forward Path

The system at (9) can be controlled using a PI controller in the feed forward path and a polynomial controller in the inner feedback path as shown in Fig. 11.

Closed loop transfer function of the inner feedback controller and the system can be represented as

$$G'(s)=\frac{a_0}{s^n+\left(b_{n-1}+a_0k_{n-1}\right)s^{n-1}+\left(b_{n-2}+a_0k_{n-2}\right)s^{n-2}+\ldots+(b_2+a_0k_2)s^2+(b_1+a_0k_1)s+a_0k_0+b_0}, \tag{12}$$

and the resulting closed loop transfer function of $G'(s)$, the PI controller and the unity feedback is given by

$$T(s)=\frac{l_1 a_0 s+l_0 a_0}{s^{n+1}+(b_{n-1}+a_0k_{n-1})s^n+\left(b_{n-2}+a_0k_{n-2}\right)s^{n-1}+\ldots+(b_1+a_0k_1)s^2+(a_0k_0+a_0l_1+b_0)s+a_0l_0}. \tag{13}$$

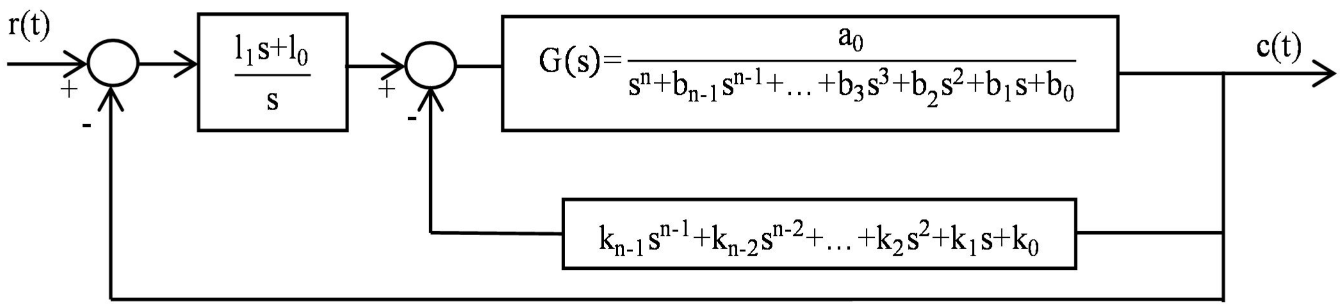

Fig. 11. The use of PI controller in the feed forward path for n^{th} degree all pole systems.

To simplify the analysis, numerator and denominator coefficients of the system's closed loop transfer function can be arranged as:

$$d_n=b_{n-1}+a_0k_{n-1}, \tag{14}$$
$$d_{n-1}=b_{n-2}+a_0k_{n-2}, \tag{15}$$
$$d_{n-2}=b_{n-3}+a_0k_{n-3}, \tag{16}$$
$$d_3=b_2+a_0k_2, \tag{17}$$
$$d_2=b_1+a_0k_1, \tag{18}$$
$$d_1=b_0+a_0k_0+a_0l_1, \tag{19}$$
$$d_0=a_0l_0=1, \tag{20}$$
$$c_1=a_0l_1. \tag{21}$$

Substituting these values into the (13) gives the new transfer function of the system, to be

$$T(s)=\frac{c_1s+1}{s^{n+1}+d_ns^n+d_{n-1}s^{n-1}+\ldots+d_2s^2+d_1s+1}, \tag{22}$$

where n+1 degree standard form with a variable zero can be represented as in (22). Using (14) to (21) with the transfer function given in (22) results in the controller parameters as:

$$l_0=\frac{1}{a_0}, \tag{23}$$
$$l_1=\frac{c_1}{a_0}, \tag{24}$$
$$k_0=\frac{d_1-c_1-b_0}{a_0}, \tag{25}$$
$$k_1=\frac{d_2-b_1}{a_0}, \tag{26}$$
$$k_2=\frac{d_3-b_2}{a_0}, \tag{27}$$
$$k_3=\frac{d_4-b_3}{a_0}, \tag{28}$$
$$k_{n-2}=\frac{d_{n-1}-b_{n-2}}{a_0}, \tag{29}$$
$$k_{n-1}=\frac{d_n-b_{n-1}}{a_0} \tag{30}$$

or generalizing the formula for k=0, 1, 2, 3, 4, …. n-1

$$\sum_{i=0}^{n-1}k_i=\frac{d_1-c_1-b_0}{a_0}+\sum_{i=1}^{n-1}\frac{d_{i+1}-b_i}{a_0}, \tag{31}$$

and

$$\sum_{i=0}^{1}l_i=\sum_{i=0}^{1}\frac{c_i}{a_0} \tag{32}$$

equations can be obtained.

B. Design by Using a PID Controller in the Feed Forward Path

The system at (9) can be controlled using a PID controller in the feed forward path and a polynomial controller in the inner feedback path as shown in Fig. 12.

Closed loop transfer function of the inner feedback controller and the system can be represented as

$$G'(s)=\frac{a_0}{s^n+\left(b_{n-1}+a_0k_{n-1}\right)s^{n-1}+\left(b_{n-2}+a_0k_{n-2}\right)s^{n-2}+\ldots+(b_2+a_0k_2)s^2+(b_1+a_0k_1)s+a_0k_0+b_0}, \tag{33}$$

and the resulting closed loop transfer function of G'(s), the PID controller and the unity feedback is given by

$$T(s)=\frac{l_2a_0s^2+l_1a_0s+l_0a_0}{s^{n+1}+(b_{n-1}+a_0k_{n-1})s^n+\left(b_{n-2}+a_0k_{n-2}\right)s^{n-1}+\ldots+(b_1+a_0k_1+a_0l_2)s^2+(a_0k_0+a_0l_1+b_0)s+a_0l_0}, \tag{34}$$

To simplify the analysis, numerator and denominator coefficients of the system's closed loop transfer function can be arranged as:

$$d_n=b_{n-1}+a_0k_{n-1}, \tag{35}$$
$$d_{n-1}=b_{n-2}+a_0k_{n-2}, \tag{36}$$
$$d_{n-2}=b_{n-3}+a_0k_{n-3}, \tag{37}$$
$$d_3=b_2+a_0k_2, \tag{38}$$
$$d_2=b_1+a_0k_1+a_0l_2, \tag{39}$$
$$d_1=b_0+a_0k_0+a_0l_1, \tag{40}$$
$$d_0=a_0l_0=1, \tag{41}$$
$$c_1=a_0l_1, \tag{42}$$
$$c_2=a_0l_2. \tag{43}$$

Substituting these values into the (34) gives the new transfer function of the system, to be

$$T(s)=\frac{c_2s^2+c_1s+1}{s^{n+1}+d_ns^n+d_{n-1}s^{n-1}+\ldots+d_2s^2+d_1s+1}, \tag{44}$$

n+1 degree standard form with two variable zeros can be represented as in (44). Using (35) to (43) with the transfer function given in (44) results in the controller parameters as:

$$l_0=\frac{1}{a_0}, \tag{45}$$
$$l_1=\frac{c_1}{a_0}, \tag{46}$$
$$l_2=\frac{c_2}{a_0}, \tag{47}$$
$$k_0=\frac{d_1-c_1-b_0}{a_0}, \tag{48}$$
$$k_1=\frac{d_2-c_2-b_1}{a_0}, \tag{49}$$
$$k_2=\frac{d_3-b_2}{a_0}, \tag{50}$$
$$k_3=\frac{d_4-b_3}{a_0}, \tag{51}$$
$$k_{n-2}=\frac{d_{n-1}-b_{n-2}}{a_0}. \tag{52}$$

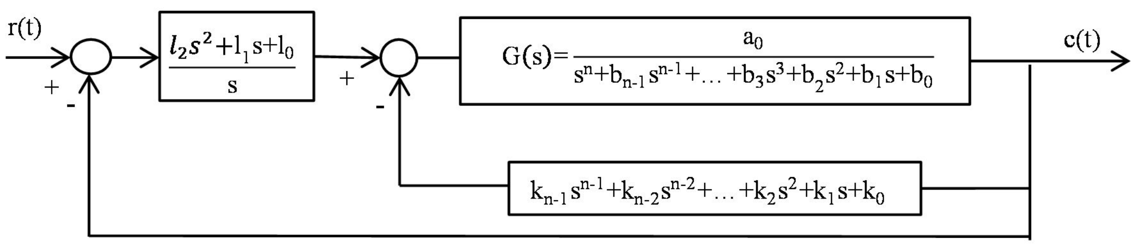

Fig. 12. The use of PID controller in the feed forward path for n^{th} degree all pole systems.

$$k_{n-1}=\frac{d_n-b_{n-1}}{a_0} \tag{53}$$

or generalizing the formula for k=0, 1, 2, 3, 4, …. n-1

$$\sum_{i=0}^{n-1} k_i=\sum_{i=0}^{1}\left(\frac{d_{i+1}-c_{i+1}-b_i}{a_0}\right)+\sum_{i=2}^{n-1}\left(\frac{d_{i+1}-b_i}{a_0}\right), \tag{54}$$

and

$$\sum_{i=0}^{2} l_i=\sum_{i=0}^{2}\frac{c_i}{a_0} \tag{55}$$

equations can be obtained.

V. EXAMPLES

In this section, four examples have been given for the proposed method. The first two examples are given with a system using PI controller, and other two examples are implemented in a system with PID controller. The third and fourth examples have been implemented by using new standard forms obtained in this study. In the examples, for the method developed by Boz and Sari [15]; B-S method, and for the method developed by Sari and Boz [16]; S-B method have been used. On the other hand, for the method developed in this study; S-K-B method expression has been used. For the same systems, results of some well-known PID controller design methods are also obtained. These are Astrom Hagglund (A-H) [18], Gain-phase (G-P) [20] and Refined Ziegler-Nichols (R. Z-N) [19], controller design methods.

A. *Example 1*

Consider the third order all pole transfer function

$$G(s)=\frac{2}{s^3+s^2+6s+3}, \tag{56}$$

Comparing the transfer function of this system with that of (9), gives the following values, n = 3, a_0 = 2, b_3 = 1, b_2 = 1, b_1 = 6 and b_0 = 3. Then choosing c_1 = 6 for IST^{2}E criteria from [32] and using them in the generalized formulae, which are given in (31) and (32), result in the controller parameters and these data are summarized in Table I with B-S method results and summary of the results obtained from A-H, G-P and R. Z-N methods. Finally, step responses of all design methods together with that of the suggested design method, S-K-B, are plotted in the same figure for comparison (Fig. 13).

TABLE I. RESULTS OF EXAMPLE 1.

Results of suggested controller designs (S-K-B) and B-S methods			Results obtained from A-H, G-P and R. Z-N methods			
	B-S	**S-K-B**		**A-H**	**G-P**	**R. Z-N**
c_1	6	6	K_c	1,514	1,514	1,514
d_1	7,18	7,18	ω_c	2,45	2,45	2,45
d_2	7,67	7,67	T_c	2,56	2,56	2,56
d_3	3,48	3,48	~	4		
J	0,53	0,53	¤$_m$	45		
						1,04
l_1	0,0712	3	K_p	1,07	0,77	0,91
l_0	0,0034	0,5	T_i	1,97	0,82	1,28
k_2		1,24	T_d	0,493	0,33	0,32
k_1	-2,683	0,83				
k_0	-1,486	-0,91				

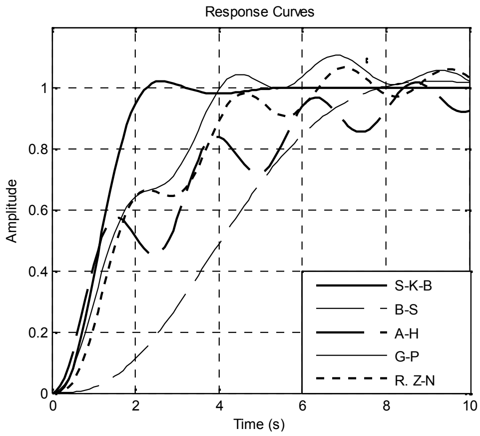

Fig. 13. Step responses for Example 1.

B. *Example 2*

In this case, consider the fourth order all pole transfer function

$$G(s)=\frac{6}{s^4+s^3+12s^2+5s+3}. \tag{57}$$

Coefficients of the transfer function are n = 4, a_0 = 6, b_4 = 1, b_3 = 1, b_2 = 12, b_1 = 5 and b_0 = 3. Again choosing c_1 = 6 for IST^{2}E criteria from [33] and using them in the generalized formulae, which are given in (31) and (32), result in the controller parameters and these data are summarized in Table II with B-S method results and summary of the results obtained from A-H, G-P and R. Z-N methods. Finally, step responses of all design methods and suggested design method, S-K-B, are also given in Fig. 14.

TABLE II. RESULTS OF EXAMPLE 2.

Results of suggested controller designs (S-K-B) and B-S methods			Results obtained from A-H, G-P and R. Z-N methods			
	B-S	**S-K-B**		**Å-H**	**G-P**	**R. Z-N**
c_1	6	6	K_c	5,333	5,333	5,333
d_1	7,676	7,676	ω_c	2,236	2,236	2,236
d_2	11,376	11,376	T_c	2,81	2,81	2,81
d_3	8,5894	8,5894	~	4		
d_4	3,2725	3,2725	¤$_m$	45		
J	2,9	2,9				0,4754
			K_p	3,77	2,71	3,2
l_1	0,0087	1	T_i	2,16	2,635	1,405
l_0	0,0004	0,167	T_d	0,54	0,343	0,35
k_3		0,379				
k_2	-1,866	-0,57				
k_1	-0,779	-1,06				
k_0	-0,498	-0,22				

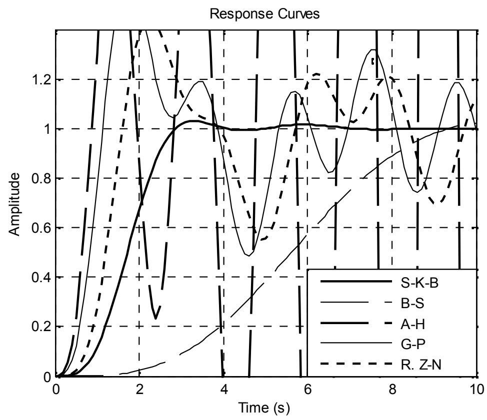

Fig. 14. Step responses for Example 2.

C. Example 3

Consider the third order all pole transfer function

$$G(s)=\frac{5}{s^3+s^2+11s+2}, \quad (58)$$

Comparing the transfer function of this system with that of (9), gives the following values, n = 3, $a_0 = 5$, $b_3 = 1$, $b_2 = 1$, $b_1 = 11$ and $b_0 = 2$. Then choosing $c_2 = 20$ for IST^{2}E criteria from Fig. 2 and using them in the generalized formulae, which are given in (54) and (55), result in the controller parameters and these data are summarized in Table III with S-B method results and summary of the results obtained from A-H, G-P and R. Z-N methods. Finally, step responses of all design methods and suggested design method, S-K-B, are plotted in the same figure for comparison (Fig. 15).

TABLE III. RESULTS OF EXAMPLE 3.

Results of suggested controller designs (S-K-B) and B-S methods			Results obtained from A-H, G-P and R. Z-N methods			
	S-B	S-K-B		Å-H	G-P	R. Z-N
c_1	5,238	5,238	K_c	1,8	1,8	1,8
c_2	20	20	ω_c	3,32	3,32	3,32
d_1	5,582	5,582	T_c	1,89	1,89	1,89
d_2	21,85	21,85	~	4		
d_3	7,1355	7,1355	¤$_m$	45		
J	0,0018	0,0018				1
			K_p	1,273	0,91	1,08
l_2	0,07856	4	T_i	1,456	0,68	0,947
l_1	0,00288	1,0476	T_d	0,364	0,24	0,237
l_0	0,00008	0,2				
k_2		1,2271				
k_1	-2,1927	-1,83				
k_0	-0,3998	-0,3313				

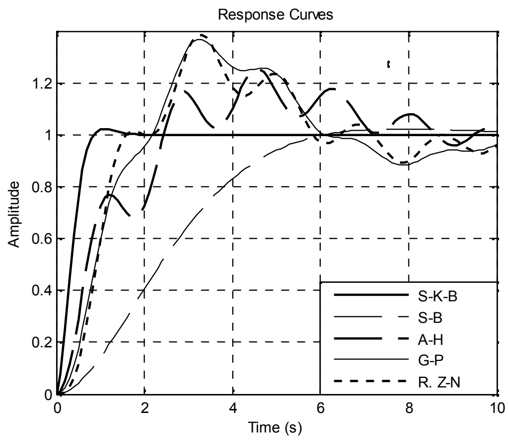

Fig. 15. Step responses for Example 3.

D. Example 4

Consider the fourth order all pole transfer function

$$G(s)=\frac{7}{s^4+s^3+9s^2+7s+3}, \quad (59)$$

Coefficients of the transfer function are n = 4, $a_0 = 7$, $b_4 = 1$, $b_3 = 1$, $b_2 = 9$, $b_1 = 7$ and $b_0 = 3$. Again choosing $c_2 = 20$ for IST^{2}E criteria from Fig. 4 and using them in the generalized formulae, which are given in (54) and (55), result in the controller parameters and these data are summarized in Table IV with S-B method results.

Summary of the results obtained from A-H, G-P and R. Z-N methods are given in Table IV. Finally, step responses of all design methods and suggested design method, S-K-B, are also given in Fig. 16.

TABLE IV. RESULTS OF EXAMPLE 4.

Results of suggested controller designs (S-K-B) and B-S methods			Results obtained from A-H, G-P and R. Z-N methods			
	S-B	S-K-B		Å-H	G-P	R. Z-N
c_1	4,531	4,531	K_c	1,572	1,572	1,572
c_2	20	20	ω_c	2,646	2,646	2,646
d_1	5,3245	5,3245	T_c	2,375	2,375	2,375
d_2	23,8405	23,8405	~	4		
d_3	17,0328	17,0328	¤$_m$	45		
d_4	5,1292	5,1292				1,033
J	0,071	0,071	K_p	1,111	0,795	0,943
			T_i	1,825	0,777	1,187
l_2	0,021173	2,8571	T_d	0,456	0,304	0,297
l_1	0,000935	0,6473				
l_0	0,00004	0,1429				
k_3		0,82584				
k_2	-1,6705	1,60656				
k_1	-2,9943	-2,2319				
k_0	-1,3998	-1,2413				

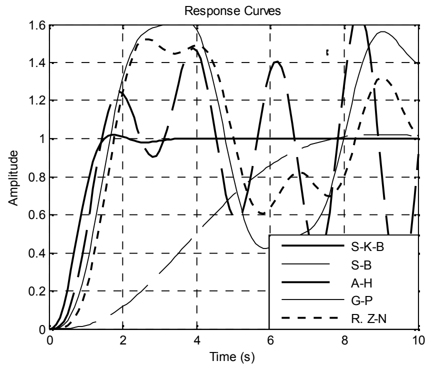

Fig. 16. Step responses for Example 4.

VI. CONCLUSIONS

In this study, previously proposed controller systems by Boz and Sari [15] and Sari and Boz [16] have been improved by increasing the polynomial controller, which is used in the feedback path, one degree and thus the limitations in the previous controllers have been removed. By this improvement the coefficients in the system transfer functions do not affect the stability of the overall system. Thus, the stability is guaranteed. The proposed new controller structures have been compared and the advantages of its performances over the previous proposed system together with some well-known design methods are shown in the first two examples. Additionally, in this study standard forms with two variable zeros have been obtained for only constraining the c_2 value. In the previous studies, both c_2 and c_1 values were constrained and the standard form coefficients were obtained only for c_2, which equal to 1 to 4. However, in the proposed model, the standard forms have been calculated for c_2, which is equal to 1 to 50 and since the results are linear it became possible to be used for higher values of c_2 than 50. Also, since the obtained standard form values are almost linear, it is possible them to be expressed mathematically and they can be used in a microprocessor based control structure very easily. Thus there is no need to optimize the system every time. In the third and fourth examples, these standard forms have been used with the new controller scheme and successful results have been given together with some well-known design methods. As it can be seen from the example results, the proposed method gives better performance values. Use of standard forms in the proposed design methods directly targets the step response of the system; therefore it is very advantageous to use it. Another application area of the method is the state feedback design since the polynomial feedback controller uses feedback from the derivatives of the output, which can be stated as the system states in case of the system is represented in controllable canonical form. Since any controllable system can be put in this form by a state transformation, the design approach can be applied to state feedback design for any controllable system as given in [12]. If all state coordinates are not directly available, then a state observer may be used as a solution of this problem.

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