Subharmonic Oscillations and Chaos in DC-DC Switching Converters

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***Abstract*—In this paper we have demonstrated the applicability of numerical path-following approach to the analysis of bifurcation behaviour and subharmonic oscillations in switch-mode DC-DC converters.**

The method developed in this research allows plotting the complete bifurcation diagrams on the parameter space that can provide useful practical information for engineers to determine the complex behaviour of SMPS.

In particular, we have provided specific complete bifurcation diagrams for the current controlled boost converter and discussed the key features of the interaction of smooth and non-smooth bifurcation phenomena.

***Index Terms*—Bifurcations, chaos, DC-DC converters, instabilities, subhramonics.**

I. INTRODUCTION

Power electronic circuits are designed to process electrical energy, in contrast to the function of processing signals in many other circuits used in various branches of electrical engineering. These circuits are used in almost all types of electrical equipment, ranging from multi-megawatt power system applications to mill watt battery management circuitry. The most familiar devices for providing voltage matching are DC–DC converters.

The majority of modern electronic devices are equipped with switch-mode power supplies (SMPS), which have great advantages over linear ones: smaller weight, size and much higher efficiency. Nearly all SMPS are designed to operate in a specific periodic regime. The periodicity is typically imposed by a clocked circuit or by the autonomous operation of the circuit itself. Because of the mentioned periodicity, the focus in this paper is on periodic steady state operating regimes of these circuits.

It has been shown in the several last decades that switching DC–DC converters, being time-varying nonlinear dynamical systems, are capable of exhibiting several periodic steady state responses as well as chaotic behaviour [1]. Several authors [2] highlight that next generation power management applications, requiring miniaturization and bandwidth extension, reducing the ratio of switching frequencies and natural converter dynamics, results in a fact that different fast-scale instabilities are more prone to be exhibited.

The chaotic operation of switching converters is generally undesirable. However, in some case, the robust chaotic behavior may be utilized to improve the performance of power converters, for example, to improve electromagnetic compatibility [3], [4] or reduce the electromagnetic interference(EMI), while the intermittency chaotic regions of operation should be avoided.

The proper design of the DC–DC converters assumes that all possible periodic (and aperiodic) solutions and their dependence on variation of different circuit parameters are known in advance. Thus it is possible to avoid the occurrence of some undesirable and even dangerous properties during operation of power supply- like increased output voltage ripples, audible noise or even the damage of power switching elements. Therefore a complete knowledge about domains of operation in the parameter space is of particular importance for power electronics engineers as it is necessary to choose the parameter values in order to obtain the desirable operating regime.

The study and control of the mentioned nonlinear phenomena requires the use of appropriate models. The most traditionally used averaged modelling approach was found to fail in predicting the wide variety of sub-harmonic and chaotic oscillations. This is due to the fact that this kind of models obeys the main non-linearity of the real system by averaging variables during the switching cycle. An alternative is the discrete-time modelling approach.

The general tool for nonlinear analysis based on the discrete-time modelling that shows the equilibrium solutions as a function of one or several parameters is a bifurcation diagram. The most common approach in the construction of these diagrams is so called Monte Carlo method [5]. This method has several noticeable disadvantages: it does not allow accurate detection of bifurcation points computes only stable orbits and skips narrow periodic windows and coexisting periodic regimes, which can be vital in the design of reliable DC-DC converters. This paper presents innovative approach based on numerical continuation of bifurcation branches, allowing the detection of stable and unstable periodic orbits as system parameters are varied.

The advantages of numerical path-following are demonstrated on one of the most widely used switching

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converters – boost converter under current mode control operating in continuous current mode.

II. THE MODEL OF THE BOOST CONVERTER

In the present study the dynamics of the current controlled boost converter is under consideration. The simplified schematic diagram of this DC-DC converter is shown in Fig. 1.

The inductor current is chosen as the programming variable which, by comparing with the reference level, generates the *ON-OFF* driving signal for the switch. Specifically, the switch is turned *ON* at the beginning of each cycle and the inductor current linearly increases.

As the value of inductor current reaches the value of I_{ref}, the switch is turned *OFF* and remains in this position until the next clock pulse.

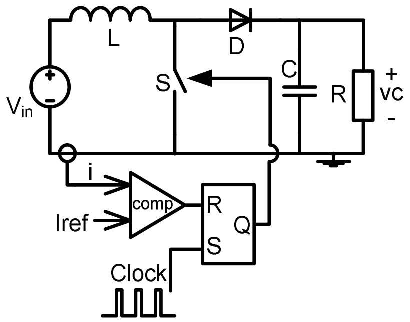

Fig. 1. Simplified schematic diagram of the peak current programmed boost converter.

The typical waveforms of inductor current *i*, *CLOCK* pulses and switching signal are shown in the Fig. 2.

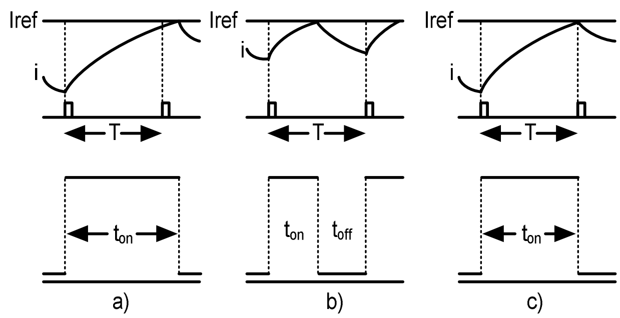

Fig. 2. Evolutions of the inductor current between two clock instants and the corresponding control signals.

Nevertheless the operation of the boost converter in two mentioned operation phases is described by two sets of linear differential equations; it is now the common practice to adopt the sampled data modelling in the form of stroboscopic maps, where the state variables are defined at clock instances [5]. For example, S. Benerjee in his book [1] provides rather simple and precise model of the boost converter under peak current mode control that could be used without any significant practical restrictions.

Let the state variables at clock instant be v_n and i_n and those in the next clock instant are correspondingly v_{n+1} and i_{n+1}. As it could be seen from the Fig. 2.(a,b) there are two ways in which the state can move from one clock instant to the next. A clock pulse may arrive before the inductor current reaches the I_{ref} (Fig. 2, a). In that case, the obtained discrete time model is the following:

$$\begin{cases} i_{n+1} = I_{ref}\rho T_0 / 2\varsigma + i_n, \\ v_{n+1} = v_n e^{-\gamma}, \end{cases} \tag{1}$$

where the variables are normalized:

$$\varsigma = \sqrt{L/C} / 2R, \tag{2}$$

$$T_0 = T / \sqrt{LC}, \tag{3}$$

$$\rho = V_{in} / I_{ref} R, \tag{4}$$

$$\omega = \sqrt{1 - \varsigma^2}, \tag{5}$$

$$\gamma = T / RC. \tag{6}$$

On the other hand, if the inductor current reaches I_{ref} before the arrival of the next clock pulse (Fig. 2(b)), the map for the second case will be the following:

$$\begin{cases} i_{n+1} = (e^{-\varsigma\tau'_n}(C_1\cos(\omega\tau'_n) + C_2\sin(\omega\tau'_n)) + \rho) I_{ref}, \\ v_{n+1} = (\rho - e^{-\varsigma\tau'_n}(K_1\cos(\omega\tau'_n) + K_2\sin(\omega\tau'_n))) I_{ref} R, \end{cases} \tag{7}$$

where;

$$\tau_n = (2\varsigma/\rho)(1 - i_n / I_{ref}), \tag{8}$$

$$V_f = (v_n / I_{ref} R) e^{-2\varsigma\tau_n}, \tag{9}$$

$$\tau'_n = T_0 - \tau_n, \tag{10}$$

$$C_1 = 1 - \rho, \tag{11}$$

$$C_2 = (\rho - V_f)/2\varsigma\omega + C_1\varsigma/\omega, \tag{12}$$

$$K_1 = \rho - V_f, \tag{13}$$

$$K_2 = (2\varsigma/\omega)\big((V_f - \rho)/2 - C_1\big). \tag{14}$$

The borderline between these two modes is given by the value I_{border} for which the inductor current reaches the reference value exactly at the arrival of the next clock pulse

$$I_{border} = I_{ref}\left(1 - \rho T_0 / 2\varsigma\right). \tag{15}$$

The proposed model will be used in the next section of this paper in process of the construction of complete bifurcation diagrams and investigation of different types of smooth and non-smooth nonlinear phenomena.

III. THE CONSTRUCTION OF COMPLETE BIFURCATION DIAGRAMS

On the basis of the provided model, the complete bifurcation diagrams by means of numerical path-following were constructed, including stable and unstable branches. The first step, according to the methodology described in [6], was the detection of all possible stable and unstable periodic orbits (up to specific order) for defined parameter values – the construction of periodic skeleton (PS). The fixed points defining different periodic regimes in the PS were used as starting values for the computing of sequence of points at small intervals along the solution curve. The method used to compute these sequences was one of the modifications of Newton- Raphson method. This approach allowed tracking the periodic solutions of the system defining the stability of the fixed points at every step.

The obtained bifurcation diagrams are shown in Fig. 3–Fig. 6. For the sake of simplicity the bifurcation diagrams depict several periodic regimes of the main period doubling route to chaos and exemplary periodic window of period-6. Reference current (defining in this case the value of output voltage and the duty ratio) is chosen as the bifurcation parameter and the inductor current is normalized with respect to I_{ref}.

Fig. 3 clearly shows that as we begin with relatively small values of I_{ref} the system demonstrates a stable period-1 operation which is the only practically acceptable regime for the switching DC-DC converters. When I_{ref} is further increased (at $I_{ref}\approx 0.42(A)$), the period of operation doubles itself). It should be noted at this point, that the first period-doubling in the current-mode control converters is well known for the power electronics engineers as “subharmonic oscillations” for duty ratio greater than 0.5 [1]. Further increase of I_{ref} leads to the development of period-doubling cascade and the subsequent chaos.

The described path to chaos is rather common in many types of smooth systems. However, as it could be seen from the obtained diagrams, this circuit also exhibits several abrupt changes of bifurcation branches (at $I_{ref}\approx 0.61(A)$). This phenomenon is known as border collision bifurcation (BCB) [5]. The BCB occurs as the inductor’s current reaches the reference value exactly at the arrival of the next clock pulse. The condition for the appearance of this type of bifurcations could be interpreted in terms of the complete bifurcation diagram – the BCB are observed when the branches of bifurcation diagram (stable or unstable) cross the I_{border} defined in (15).

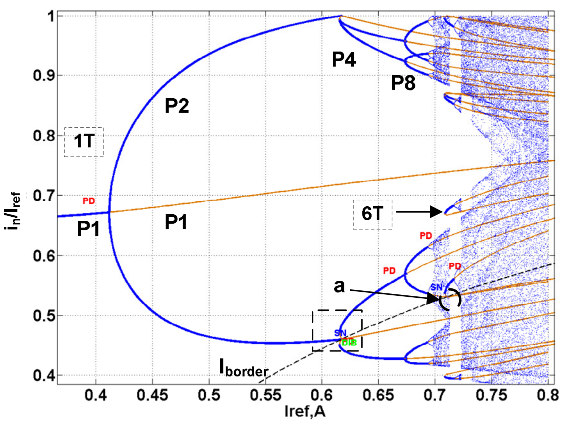

Fig. 3. Bifurcation diagram for normalized inductor current; depicting the period doubling route to chaos as well as period-6 window Parameters of the boost converter: R=40(Ω); L=1.5(mH); C=2(μF); T=100(μS); Vin=5(V); Iref=var. Bold solid lines represent stable periodic orbits, light lines – unstable orbits and dashed black line represents the Iborder (see (15)). Shaded area represents the chaotic mode of operation of the converter.

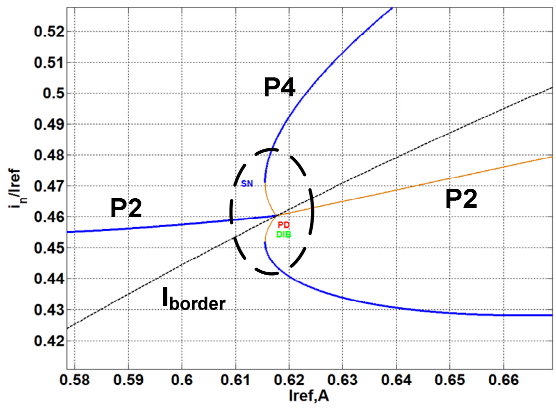

Fig. 4. The magnified fragment of the bifurcation diagram (see. Fig. 3.). Dashed circle highlights the area where the border collision leads to the subcritical period – doubling bifurcation.

It should be mentioned that the chaotic operation at the end of period-doubling cascade is not robust in this case, as some stable periodic windows appear within the chaotic area (see 6T periodic regime in Fig. 3).

For the further explanation of the effect of BCB on the dynamics of converter, magnified fragment of the bifurcation diagram is shown in Fig. 4. It could be seen, that in switch mode converters the collision with the switching border may lead to unpredictable changes in bifurcation pattern. The dashed circle in the Fig. 4 highlights the area where the BCB leads to the merging of stable *P2* and *P4* regimes by means of unstable periodic branch. The region of the coexistence of stable *P2* and *P4* regimes is also observed for the definite range of I_{ref} values. This leads to the fact that in the mentioned range the behaviour of the system will skip between coexisting regimes if any source of sufficient external noise is present in the converter [7], [8].

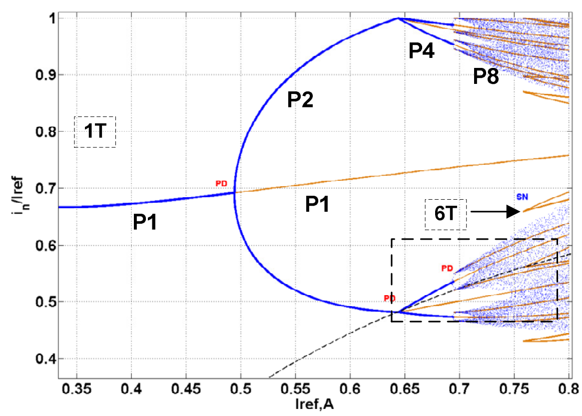

Fig. 5. Bifurcation diagram for normalized inductor current; depicting the period doubling route to chaos as well as unstable period-6 regime. Parameters of the boost converter: R=40(Ω); L=1.5(mH); C=20(μF); T=100(μS); Vin=5(V); Iref=var. Bold solid lines represent stable periodic orbits, light lines – unstable orbits and dashed black line represents the Iborder.

The dynamics of the system qualitatively changes as the second bifurcation parameter – capacitance is varied. Fig. 5 represents the bifurcation diagram for the same parameters of the system with the exception of the C.

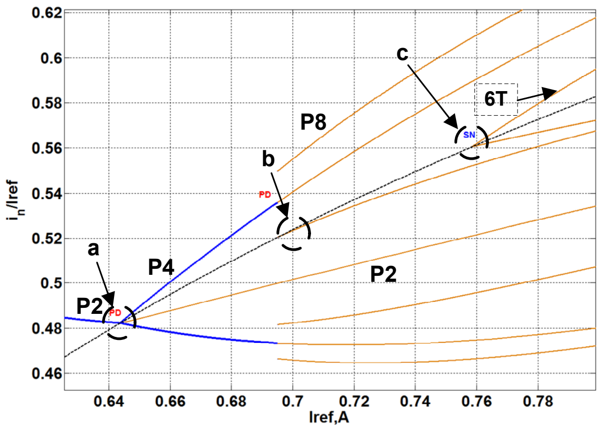

Fig. 6. The magnified fragment of the bifurcation diagram (see. Fig. 5.). Dashed circles highlight the areas where border collision occurs, leading to the sudden change in the dynamics of the boost converter.

In the case of the increased value of C the period-doubling route to chaos is abruptly interrupted by BCB and after the second period-doubling bifurcation the system exhibits chaotic mode of operation. The magnified fragment of the bifurcation diagram (see Fig. 6) shows that at the collision with I_{border} may lead to non-smooth period doubling (see areas a and b) or non-smooth saddle-node (see point c) bifurcations. In the latter case no stable periodic orbits occur. One of the most significant features of the bifurcation diagrams shown in Fig. 5 and Fig. 6 is the presence of robust chaotic operation – the absence of periodic windows and stable coexisting periodic regimes. Thus after I_{ref}=0.7(A) the converter will robustly operate in the chaotic mode and no external noise could lead to the occurrence of any other regimes of operation.

IV. CONCLUSIONS

The investigation provides the study of nonlinear phenomena in DC-DC switching converters. The main objectives of this paper are to explore different types of complex behaviour, such as subharmonic oscillations and chaos, in the converters as system parameters are changed. The feasibility of using numerical path-following technique in the process of the construction of complete bifurcation diagrams is demonstrated. The boost converter under current mode control is used as an example circuit and the corresponding bifurcation diagrams, based on the discrete time model are presented. The appearance of non-smooth border collisions, leading to abrupt behaviour and chaos, is demonstrated and the corresponding analysis of the influence of mentioned phenomena on branches of bifurcation diagrams is provided.

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