Robustness Stability Analysis of Higher-Order DPCM Prediction Filters

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***Abstract*—This paper considers the robustness of the differential pulse-code modulation system with higher-order predictors. Special attention is paid to the robust parametric stability of the prediction filters with respect to the predictor coefficients. A generalisation of robustness in the classical sense is performed, and appropriate relations for calculating the probability of robustness are derived using Kharitonov principle. The proposed robustness estimation method is used for the third- and fourth-order prediction filters on speech signals, where the application of traditional methods is too difficult. For this reason, the Monte Carlo method is used to solve complex probability integrals. Verification and error analysis are performed for the previously considered second-order predictor. Satisfactory predetermined accuracy is achieved by increasing the number of samples. The results obtained could be very useful to design a system with suitable values for the predictor coefficients.**

***Index Terms*—Digital filters; Linear feedback control systems; Monte Carlo methods; Probability density function.**

I. INTRODUCTION

Differential pulse-code modulation (DPCM) is a well-known coding technique that has been used for decades in various fields of science and technology, especially in telecommunications, speech [1]–[5], image and video coding [6]–[9], medicine [10]–[13], and also in combination with other techniques [14]. The idea for this paper, as well as for some previous papers, is that the DPCM system contains a negative feedback loop, so that the telecommunication system is indeed suitable for analysis from the aspect of control system theory [1], [15].

The predictive linear (recursive) filter is the most important part of any DPCM system. In general, the prediction and design of various prediction models have been of great importance for years, and it seems that they are gaining even more importance today [16], [17]. Linear prediction is very important in many scientific and engineering fields, e.g., economics, geophysics, system identification, adaptive filtering, spectral estimation, signal processing, etc. [18]–[21]. Their role in the speech signal is particularly important [18]. This is quite understandable considering that the current value of the signal (current sample) can be well modelled as a linear combination of the previous values [1], [22]. A historical overview of the development of linear prediction in speech signal coding can be found in [23]. The closed-loop predictor forms a kind of filter that is a good model of the human speech apparatus. The main task is to determine the predictor coefficients and study the properties of the filter.

The prediction gain increases slightly, i.e., it almost reaches saturation with an increase in the order of the predictor above four [1]. In this paper, therefore, a corresponding analysis will be performed up to the fourth order, although the same method could also be used effectively for higher orders.

It is very important to investigate the properties of the DPCM system and, in particular, its linear prediction filter as its integral part. The stability of the DPCM system for the first-order predictor has been studied in [24], for the second-order predictor in [25], and for higher-order predictors in [26]. We have explained why the stability analysis of the prediction filter is crucial for the stability and performances of the whole system (e.g., signal-to-noise ratio, gain prediction). The parametric sensitivity of the prediction filter is investigated in [27].

Robustness is another very important property of the system. The ability of the system to remain stable in the face of some disturbances or changes in certain parameters of the system is a very important characteristic in relation to the control system. There are different types of robustness, but in this paper, we consider parametric robustness. In contrast to sensitivity, here, we monitor the change in the system output in response to large parameter changes. The ability of

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a system to remain stable despite certain disturbances is very important for any system. In the case of parametric robustness, which is studied in this paper, the system is robust in particular if it remains stable despite the change of parameters by a certain value (within certain limits).

A robustness analysis of the predictors with respect to transmission error is carried out in [28]. Robustness for first- and second-order predictors has already been analysed in a previous paper [29]. In this paper, a generalisation of the ideal case is considered when the parameters are deterministic. Indeed, predictor coefficients are stochastic parameters, and we have performed a robust estimation, as we will do in this paper. Imperfections of any real system where we include probabilistic theory are discussed in many previous papers [29]–[33]. The aim is to know permanently for which values of the predictor coefficients the system is most robust, i.e., has the highest probability of robustness. The optimal choice of values for the predictor coefficients is of the utmost importance for better system performance.

As in the case of stability, it becomes very tedious to compute multidimensional integrals when calculating higher-order system robustness. Therefore, we will apply the well-known Monte Carlo method to compute the complex integrals. In this paper, we investigate the robust stability of third- and fourth-order prediction filters. Of course, the same proposed method can also be used for higher-order systems.

The problem is how best to define the limits of the parameters and what the values obtained later for the probability of robustness mean and how to use them to improve system performance. The idea of this paper is to complete the story of examining the properties of the DPCM prediction filter, which is crucial for the functioning of the entire DPCM system. This is a continuation of the previously conducted study on classical stability, stochastic stability, and sensitivity.

The remainder of the paper is organised as follows. The system for differential pulse-code modulation with special emphasis on the linear prediction filter is described in Section II. Section III deals with the Monte Carlo method and the possibilities of its application in solving complex integrals for our purpose. The robustness stability of the linear prediction filters is discussed in Section IV. First, we verified the Monte Carlo method in cases of first- and second-order predictors where this method is not necessary, but we already have experimental results in a previous paper for comparison. Then, we applied the proposed method for the robustness estimation of third- and fourth-order prediction filters. We analysed the results of the series of experiments and made concluding remarks in Section V. Finally, some possible further research directions are indicated in Section VI.

II. DPCM/ADPCM SYSTEM AND LINEAR PREDICTION

Differential pulse-code modulation (DPCM) is a method of converting an analogue signal into a digital signal, in which the analogue signal is sampled, and then the difference between the actual measured value and its estimated value is quantised, and finally coded by forming its digital value. The estimation or prediction of the current value of the input signal is based on the knowledge of its previous values. If the input signal is redundant, its previous values are correlated with the current values, so a good assessment and economical operation of the DPCM system are possible.

For nonstationary input signals, optimal results are achieved with the DPCM method, in which the prediction parameters change synchronously with the change in the statistical characteristic of the signal. Such prediction and quantisation techniques are called "adaptive prediction and adaptive quantisation", and the corresponding modulation procedure is called "adaptive differential pulse-code modulation" (ADPCM) [1], [4], [16].

The ADPCM encoder (Fig. 1(a)) consists of a quantiser, an inverse quantiser, a predictor and additional buffer, and a predictor coefficient estimator for adaptive prediction. As we can see, a predictive (recursive) filter is located both at transmission (in the encoder, Fig. 1(a)) and at reception (in the decoder, Fig. 1(b)). These predictive filters are of particular interest for further analysis, as they have already been used for stability analysis [24], [26].

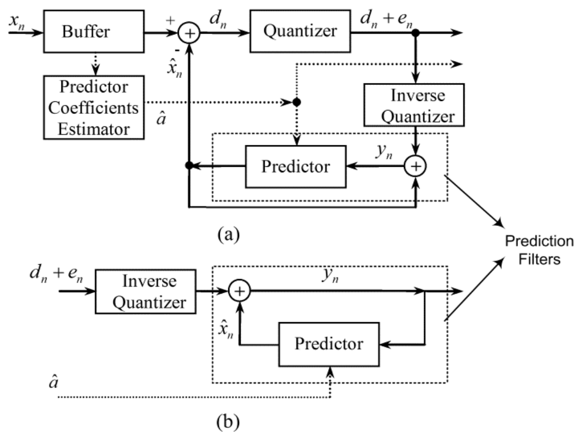


Fig. 1. Block scheme of the ADPCM system: (a) Encoder; (b) Decoder.

The value of the linear prediction of the sample x_n can be written for the N^{th} order of the predictor in the form of the following sum

$$\hat{x}_n = \sum_{i=1}^{k} a_i y_{n-i}, \tag{1}$$

where a_i are the predictor coefficients [26]. As mentioned already, these coefficients have a decisive influence on the quality of the transmission, i.e., on the error that occurs during the evaluation. The correct selection of these coefficients depends on the accuracy of the estimation of the input sample value (x_n), which in turn affects the quantisation error e_n and the prediction gain. A good estimate guarantees a lower amplitude dynamic of the difference signal d_n compared to x_n. This makes it possible to quantise the difference signal with a smaller number of amplitude levels, which means a saving in the bit rate, i.e., the prediction gain. On the other hand, if the prediction coefficients are poorly chosen, the difference signal can become larger than the input sample, resulting in a larger quantisation error and multiplying the total error due to the feedback loop, which can ultimately lead to system failure.

This is the reason why we consider the parametric robust stability of this system.

The k^{th}-order predictor model (1) in z-domain is

$$\hat{X}(z) = \left(\sum_{i=1}^{k} a_i z^{-i}\right) Y(z) = W_P(z) Y(z), \tag{2}$$

where the transfer function of the predictor is

$$W_P(z) = \sum_{i=1}^{k} a_i z^{-i}. \tag{3}$$

The transfer functions of the prediction filter in the encoder and decoder (Fig. 1) have the following forms, respectively:

$$W_R(z) = \frac{W_P(z)}{1 - W_P(z)} = \frac{\sum_{i=1}^{k} a_i z^{-i}}{1 - \sum_{i=1}^{k} a_i z^{-i}}, \tag{4}$$

$$W_R^D(z) = \frac{1}{1 - W_P(z)} = \frac{1}{1 - \sum_{i=1}^{k} a_i z^{-i}}. \tag{5}$$

Finally, the characteristic equation for both filters is given by

$$z^k - \sum_{i=1}^{k} a_i z^{k-i} = 0. \tag{6}$$

III. THE MONTE CARLO METHOD

A. Brief Survey of the Monte Carlo Method

The Monte Carlo method allows for the approximate numerical solution of a wide range of different problems by performing statistical experiments with the help of the computer. This method is particularly useful for solving mathematical problems that are too complex to be solved analytically [34], [35]. Incidentally, the term itself was created in the late 1940s at Los Alamos National Laboratory during the development of the atomic bomb [34]. The development of this method coincides with the advent of the first computers, which significantly accelerated iterative numerical calculations. This method is still popular and irreplaceable in various areas of research [36].

B. Application of the Monte Carlo Method in Robustness Analysis of High-Order Prediction Filters

In this paper, we will use the Monte Carlo method for the numerical approximation of multiple definite integrals whose values are very difficult to determine with classical integration, especially for larger dimensions. In some cases, it is very difficult to determine the integration limits and thus to solve the integrals, so that approximation methods are required. With Monte Carlo integration, we can obtain numerical solutions for the required integrals of arbitrary dimensions with the required accuracy. A similar method has already been used in [26] to calculate the probability of stability for a higher-order predictor.

In this paper, we will first use the Monte Carlo method to verify the results we have already obtained by classical integration. This verification was, of course, carried out in the form of a series of simulation experiments for second-order predictors. Then, the proposed and tested Monte Carlo method is applied to estimate the robustness stability of prediction filters of higher orders, where classical integration becomes practically impossible, especially considering that the parameters are stochastic variables in the general case. The random number generator is used to obtain the values of the predictor coefficients that are normally distributed around the nominal value. Experiments are performed for a different number of samples until a satisfactory accuracy is achieved. The method is very good because we avoid integration over the stability area [37], but only need the boundaries of that area. The robustness probability is calculated as the quotient between the number of favourable cases (samples belonging to the stability area) and the total number of samples.

In the general case of the k^{th}-order predictor ($k \geq 3$), the robustness stability can be determined by solving multidimensional probability integrals [32]. However, the calculation is very complex. The boundaries of the stability area are usually complex mathematical expressions and it becomes very difficult to determine the robustness because one has to integrate the stability area [38]. For this reason, we will evaluate the probability of robustness using the Monte Carlo method.

For this purpose, we will use the Schur-Cohn stability criterion [39]. We have adjusted the coefficients in the Schur-Cohn determinant [40] to the characteristic equation (6). The corresponding determinant now has the following form

$$\Delta_i = \left| \begin{array}{cccc|cccc} -a_k & 0 & \cdots & 0 & 1 & -a_1 & \cdots & -a_{i-1} \\ -a_{k-1} & -a_k & \cdots & 0 & 0 & 1 & \cdots & -a_{i-2} \\ -a_{k-2} & -a_{k-1} & \cdots & 0 & 0 & 0 & \cdots & -a_{i-3} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{k-(i-1)} & -a_{k-(i-2)} & \cdots & -a_k & 0 & 0 & \cdots & 1 \\ \hline 1 & 0 & \cdots & 0 & -a_k & -a_{k-1} & \cdots & -a_{k-(i-1)} \\ -a_1 & 1 & \cdots & 0 & 0 & -a_k & \cdots & -a_{k-(i-2)} \\ -a_2 & -a_1 & \cdots & 0 & 0 & 0 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{i-1} & -a_{i-2} & \cdots & 1 & 0 & 0 & \cdots & -a_k \end{array} \right|, \tag{7}$$

where i is the order of the determinant $i = 1, 2, 3, ..., k$. The system is stable if and only if $\Delta_i \leq 0$ for even values i, and $\Delta_i \geq 0$ for odd values i.

It should be noted that the stability area for the second order was determined by the Routh-Hurwitz criterion in earlier papers [29], [31]. In [38], the following system of equations defining the stability area was obtained using the same criterion, which is also adapted here to (6) and with negative signs in front of the coefficients as in the case of Schur-Cohn

$$a_1 + a_2 + a_3 < 1,\ a_1 - a_2 + a_3 > -1,\ a_1 a_3 + 1 > -a_2 + a_3^2. \tag{8}$$

IV. THE ROBUSTNESS STABILITY ANALYSIS OF THE DPCM PREDICTION FILTER

Regarding the DPCM system and the linear prediction filter it contains, an analysis of the robustness of the predictor to transmission error has already been discussed in [29], as we said before. As far as parametric robustness is considered, the most important one is related to system stability. A system is said to be robust (in the classical sense) to parameter changes if it remains stable when the parameters change within a certain range. In this paper, we deal with parametric robustness in terms of the predictor coefficients. As discussed in [24] and [41], the quantiser has a nonlinear characteristic (with saturation), so it stabilises the whole system and the prediction filter is crucial for stability and robustness. Robustness analysis for predictors of first and second order has already been performed in [29], where a classical integration was performed to calculate the probability of robustness. In this paper, we will consider the robust stability of the higher-order prediction filter. We use the well-known Kharitonov principle for parametric robustness estimation [42], [43] and the Monte Carlo method to compute complex integrals. We will perform a robustness analysis for the third- and fourth-order prediction filters and verify the Monte Carlo method for the lower-order prediction filter before that.

A. *Verification of the Monte Carlo Method for the Second-Order Prediction Filter*

As mentioned above, we will first verify the proposed Monte Carlo method for the robustness already computed using classical integration for the second-order predictors [29]. We will perform a series of experiments for a different number of samples (trials) in the Monte Carlo simulation. Specifically, we did this for 10,000, 100,000, 1,000,000, and 10,000,000 samples and repeated the experiment three times for each of the samples. The results are shown in Table I. It can be seen that, as expected, the robustness value obtained with an approximation method such as Monte Carlo approaches the value obtained with classical integration as the number of samples increases. Satisfactory accuracy to three decimal places by rounding has already been achieved for 1,000,000 samples. It is possible to perform a more detailed analysis of the error [37], as well as the time needed to perform the Monte Carlo simulation experiment by increasing the number of samples, but this is not necessary for our further analysis and considering the nature of our problem. In this regard, the given number of samples of 1,000,000 is quite sufficient, so further experiments for higher-order predictors will be performed with this number of trials.

TABLE I. THE ROBUSTNESS ESTIMATION FOR THE SECOND-ORDER PREDICTION FILTER USING CLASSICAL INTEGRATION AND MONTE CARLO METHOD.

Methods	**Trials**			
	10^4	10^5	10^6	10^7
Monte Carlo	0.9583	0.9612	0.9615	0.9617
	0.9621	0.9615	0.9615	0.9617
	0.9627	0.9611	0.9617	0.9617
Traditional	0.961687			

B. *The Robustness of the Third-Order Prediction Filter*

The characteristic equation (6) of the third-order prediction filter has the following form

$$z^3 - a_1 z^2 - a_2 z - a_3 = 0. \tag{9}$$

If we want to adapt the form to the original Kharitonov polynomials [43], (9) can be written as

$$z^3 + a_1^* z^2 + a_2^* z + a_3^* = 0, \tag{10}$$

where $a_1^* = -a_1$, $a_2^* = -a_2$, and $a_3^* = -a_3$ because now negative signs are included in the coefficients, which reverses the lower and upper bounds. However, it does not change the results and conclusions that we will show.

The set (family) of four characteristic polynomials according to Kharitonov's Theorem [43] is now (in our case, all coefficients are real numbers, so the original system of eight polynomials is reduced to four, leaving minus signs to preserve the idea of physical predictor coefficients):

$$R_1(z) = z^3 - \overline{a}_1 z^2 - \underline{a}_2 z - \underline{a}_3, \tag{11}$$

$$R_2(z) = z^3 - \overline{a}_1 z^2 - \overline{a}_2 z - \underline{a}_3, \tag{12}$$

$$R_3(z) = z^3 - \underline{a}_1 z^2 - \overline{a}_2 z - \overline{a}_3, \tag{13}$$

$$R_4(z) = z^3 - \underline{a}_1 z^2 - \underline{a}_2 z - \overline{a}_3. \tag{14}$$

If we omit the negative signs,

$$R_1^*(z) = z^3 + \overline{a}_1^* z^2 + \underline{a}_2^* z + \underline{a}_3^*, \tag{15}$$

$$R_2^*(z) = z^3 + \overline{a}_1^* z^2 + \overline{a}_2^* z + \underline{a}_3^*, \tag{16}$$

$$R_3^*(z) = z^3 + \underline{a}_1^* z^2 + \overline{a}_2^* z + \overline{a}_3^*, \tag{17}$$

$$R_4^*(z) = z^3 + \underline{a}_1^* z^2 + \underline{a}_2^* z + \overline{a}_3^*. \tag{18}$$

It can be stated that $R_1^*(z) = R_3(z)$, $R_2^*(z) = R_4(z)$, $R_3^*(z) = R_1(z)$, and $R_4^*(z) = R_2(z)$. Although Kharitonov's polynomials do not cover all cases (four polynomials, and there are eight combinations), the aforementioned negative sign will not affect the overall probability of robustness due to symmetry.

The stability area, S_3 for the third-order predictor in the parametric plane (a_1, a_2, a_3) is determined by the conditions of (8) or directly from (7) for $k = 3$.

A sufficient condition for robust stability is that the sets of coefficients in all four polynomials (11)–(14) fulfill (7), (8). However, the predictor coefficients are stochastic variables. This is a much more stringent requirement than that the nominal values of the predictor coefficients or values around them satisfy these inequalities, as was required for ordinary stability.

In the experiment, to obtain predictor coefficients for the third- and fourth-order ADPCM systems, a recorded speech signal with a length of 12000 samples and a sampling frequency of 8 KHz was fed to the input of this system. During processing, the signal is divided into frames of length *M* and the predictor coefficients (a_1, a_2, a_3, ...) are calculated for each frame. The predictor coefficients are determined from the correlation coefficients using the Levinson-Durbin algorithm [2]. After calculating all predictor coefficients for all signal frames, their means a_{1m}, a_{2m}, a_{3m}, ... and standard deviation σ_1, σ_2, σ_3, ... were calculated. Therefore, the ADPCM method was used to form a set of values of the predictor coefficients (a_1, a_2, a_3, ...) on the basis of which their means were obtained, which are further used in the analysis as predictor coefficients of the DPCM system. The experiment was repeated for different frame lengths M, and these values are listed in Tables II and III for the third- and fourth-order predictors, respectively. The analysed values of the parameter M are the most commonly used values in practice. Indeed, a decrease in the value of the M parameter means an increase in the total secondary information to be transmitted to the receiver of the system. With extremely small values of M, the amount of secondary information becomes comparable to the basic information and significantly affects the performance of the system. An increase in the value of M means an increase in the probability that samples with different statistical characteristics (both small- and large-amplitude dynamics) will be included in the frame, so that a good adaptation of the system parameters (such as the quantiser range) cannot be achieved. As a result, the quality of signal processing decreases. Furthermore, an increase in the value of the parameter M means an increase in the delay of the signal if the signal is processed and transmitted in real time.

The values obtained for the mean and standard deviation of the third-order predictor coefficients are shown in Table II.

TABLE II. THE MEANS AND STANDARD DEVIATIONS OF THE THIRD-ORDER PREDICTOR COEFFICIENTS FOR DIFFERENT VALUES OF *M*.

***M* [sample]**	**10**	**20**	**50**	**100**	**150**	**200**
a_{1m}	0.980	1.133	1.320	1.452	1.510	1.529
σ_1	0.240	0.251	0.252	0.243	0.260	0.250
a_{2m}	-0.190	-0.287	-0.464	-0.640	-0.718	-0.763
σ_2	0.276	0.338	0.384	0.390	0.410	0.378
a_{3m}	-0.036	-0.001	0.039	0.101	0.126	0.152
σ_3	0.157	0.182	0.190	0.199	0.205	0.192

From Table II, we can conclude that the predictor coefficient has a maximum value for $M = 200$ and a minimum value for $M = 10$ (for the coefficient a_1^*, the conclusion is reversed). Similarly, the coefficient a_2 has a maximum value for $M = 10$ and a minimum value for $M = 200$. Finally, the coefficient a_3 has a maximum value for $M = 200$ and a minimum value for $M = 10$. This means that, in accordance with the introduced notation, $\overline{a_{1m}} = -\underline{a_{1m}}^* = 1.529$, $\underline{a_{1m}} = -\overline{a_{1m}}^* = 0.980$, $\overline{a_{2m}} = -\underline{a_{2m}}^* = -0.190$, $\underline{a_{2m}} = -\overline{a_{2m}}^* = -0.763$, $\overline{a_{3m}} = -\underline{a_{3m}}^* = 0.152$, and $\underline{a_{3m}} = -\overline{a_{3m}}^* = -0.036$.

The corresponding probability density function (PDF) for the polynomial R_1 (R_3^*) is

$$f(\overline{a_1}, \underline{a_2}, \underline{a_3}) = \frac{1}{\overline{\sigma_1}\sqrt{2\pi}} \frac{1}{\underline{\sigma_2}\sqrt{2\pi}} \frac{1}{\underline{\sigma_3}\sqrt{2\pi}} \times$$

$$\times \exp\left[-\frac{1}{2}\left(\left(\frac{\overline{a_1} - \overline{a_{1m}}}{\overline{\sigma_1}}\right)^2 + \left(\frac{\underline{a_2} - \underline{a_{2m}}}{\underline{\sigma_2}}\right)^2 + \left(\frac{\underline{a_3} - \underline{a_{3m}}}{\underline{\sigma_3}}\right)^2\right)\right], \quad (19)$$

where $\overline{a_{im}}$ ($i = 1,2,3$) and $\overline{\sigma_i}$ ($i = 1,2,3$) are the means and standard deviations, respectively, for the maximum value of the coefficients a_i ($\overline{a_i}$) ($i = 1,2,3$), while $\underline{a_{im}}$ ($i = 1,2,3$) and $\underline{\sigma_i}$ ($i = 1,2,3$) are the means and standard deviations for the minimum value of the coefficients a_i ($\underline{a_i}$), ($i = 1,2,3$). The probability density functions for the other three polynomials $R_2 : f\left(\overline{a_1}, \overline{a_2}, \underline{a_3}\right)$, $R_3 : f\left(\underline{a_1}, \overline{a_2}, \overline{a_3}\right)$, and $R_4 : f\left(\underline{a_1}, \underline{a_2}, \overline{a_3}\right)$ can be derived in the same way.

We can now perform robust stability estimation. For the polynomial R_1, we obtain

$$P_1 = \frac{\iiint_{S_3} f(\overline{a_1}, \underline{a_2}, \underline{a_3}) d\overline{a_1} d\underline{a_2} d\underline{a_3}}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\overline{a_1}, \underline{a_2}, \underline{a_3}) d\overline{a_1}^* d\underline{a_2}^* d\underline{a_3}^*} \quad (20)$$

$$P_1 = \iiint_{S_3} f(\overline{a_1}, \underline{a_2}, \underline{a_3}) d\overline{a_1} d\underline{a_2} d\underline{a_3}.$$

Remark 1: The integral in the denominators (20) represents the total probability and its value is 1.

Accordingly, the relations for the robustness probability of the other three characteristic polynomials have the following forms:

$$P_2 = \iiint_{S_3} f\left(\overline{a_1}, \overline{a_2}, \underline{a_3}\right) d\overline{a_1} d\overline{a_2} d\underline{a_3}, \quad (21)$$

$$P_3 = \iiint_{S_3} f\left(\underline{a_1}, \overline{a_2}, \overline{a_3}\right) d\underline{a_1} d\overline{a_2} d\overline{a_3}, \quad (22)$$

$$P_4 = \iiint_{S_3} f\left(\underline{a_1}, \underline{a_2}, \overline{a_3}\right) d\underline{a_1} d\underline{a_2} d\overline{a_3}, \quad (23)$$

where S_3 is defined by (8) or by the general conditions (7) for $k = 3$.

Remark 2: It should be emphasized that the probability P_1 actually refers to the polynomial R_3 and not to R_1^* since the integration areas as well as the Schur-Cohn criterion are adapted to the characteristic equation (9), i.e., to negative coefficients. Following the same logic, P_2 corresponds to the polynomial R_4, P_3 to R_1, and P_4 to R_2.

Since, as mentioned, it is difficult to calculate integrals (20)–(23) already for the third order, we use the Monte Carlo method with a given number of samples of 1,000,000. In this way, we can calculate the corresponding values for the robustness probability: $P_1 = 0.362$, $P_2 = 0.201$, $P_3 = 0.552$, and $P_4 = 0.693$.

The total robustness for the third-order prediction filter is

$$P = \prod_{i=1}^{4} P_i = 0.028. \quad (24)$$

We can see that the probability of robustness is quite low (about 3 %). In the classical sense, this third-order prediction filter is not robustly stable. Indeed, three of the four polynomials are stable in the classical sense, but one of the Kharitonov polynomials is not, i.e., it does not meet the conditions (7), (8) (the previously computed probability for this polynomial is $P_2 = 0.201$). Therefore, the polynomial R_1 would be stable in the deterministic case, even if only with a probability of 0.362.

The robustness analysis presented here is a generalisation of the classical robustness approach. In the deterministic case ($\sigma = 0$), we obtain robustness probability values equal to 1 (100 %) (stable in terms of robustness) or 0 (unstable). In the case of the third-order predictor, the total robustness is 0, since the values of the predictor coefficients in the polynomial R_2 do not meet the conditions (7), (8), so the corresponding probability is small (24).

C. *The Robustness of the Fourth-Order Prediction Filter*

The characteristic equation (6) for the fourth-order prediction filter has the following form

$$z^4 - a_1 z^3 - a_2 z^2 - a_3 z - a_4 = 0. \quad (25)$$

In agreement with the discussion for the third-order predictor, (25) can be written as follows

$$z^4 + a_1^* z^3 + a_2^* z^2 + a_3^* z + a_4 = 0, \quad (26)$$

where $a_1^* = -a_1$, $a_2^* = -a_2$, $a_3^* = -a_3$, and $a_4^* = -a_4$.

The set of four characteristic polynomials assumed for the fourth-order prediction filter according to Kharitonov's theorem is now

$$R_1(z) = z^4 - \overline{a}_1 z^3 - \overline{a}_2 z^2 - \underline{a}_3 z - \underline{a}_4, \quad (27)$$

$$R_2(z) = z^4 - \underline{a}_1 z^3 - \overline{a}_2 z^2 - \overline{a}_3 z - \underline{a}_4, \quad (28)$$

$$R_3(z) = z^4 - \underline{a}_1 z^3 - \underline{a}_2 z^2 - \overline{a}_3 z - \overline{a}_4, \quad (29)$$

$$R_4(z) = z^4 - \overline{a}_1 z^3 - \underline{a}_2 z^2 - \underline{a}_3 z - \overline{a}_4. \quad (30)$$

For the case of the third-order predictor, we have already explained what form the corresponding Kharitonov polynomials would take if the set of coefficients $\left(a_1^*, a_2^*, a_3^*, a_4^*\right)$ were used. A sufficient condition for robust stability is that the sets of coefficients in all four polynomials (27)–(30) meet the Schur-Cohn criterion (7). Experiments were performed with the same speech signal sample and repeated for the same frame lengths as for the third-order predictor. The values obtained for the mean and standard deviation of the predictor coefficients are listed in Table III.

Following the same principle as for the third-order predictors, we obtain the following: $\overline{a_{1m}} = 1.572$, $\underline{a_{1m}} = 0.977$, $\overline{a_{2m}} = -0.941$, $\underline{a_{2m}} = -0.202$, $\overline{a_{3m}} = 0.493$, $\underline{a_{3m}} = 0.037$, $\overline{a_{4m}} = -0.075$, and $\underline{a_{4m}} = -0.227$.

The corresponding probability density function for the polynomial R_1 is

$$f(\overline{a}_1, \overline{a}_2, \underline{a}_3, \underline{a}_4) = \frac{1}{\overline{\sigma}_1\sqrt{2\pi}} \frac{1}{\overline{\sigma}_2\sqrt{2\pi}} \frac{1}{\underline{\sigma}_3\sqrt{2\pi}} \frac{1}{\underline{\sigma}_4\sqrt{2\pi}} \times$$
$$\times \exp\left[-\frac{1}{2}\left(\left(\frac{\overline{a}_1 - \overline{a_{1m}}}{\overline{\sigma}_1}\right)^2 + \left(\frac{\overline{a}_2 - \overline{a_{2m}}}{\overline{\sigma}_2}\right)^2 + \left(\frac{\underline{a}_3 - \underline{a_{3m}}}{\underline{\sigma}_3}\right)^2 + \left(\frac{\underline{a}_4 - \underline{a_{4m}}}{\underline{\sigma}_4}\right)^2\right)\right], \quad (31)$$

where $\overline{a_{im}}$ ($i = 1,2,3,4$) and $\overline{\sigma_i}$ ($i = 1,2,3,4$) are the means and the standard deviations for the maximum value of the coefficients a_i($\overline{a_i}$) ($i = 1,2,3,4$), while $\underline{a_{im}}$ ($i = 1,2,3,4$) and $\underline{\sigma_i}$ ($i = 1,2,3,4$) are the means and the standard deviations for the minimum value of the coefficients a_i($\underline{a_i}$), respectively. We can derive the probability density functions for the other three polynomials, $R_2 : f\left(\underline{a}_1, \overline{a}_2, \overline{a}_3, \underline{a}_4\right)$, $R_3 : f\left(\underline{a}_1, \underline{a}_2, \overline{a}_3, \overline{a}_4\right)$, and $R_4 : f\left(\overline{a}_1, \underline{a}_2, \underline{a}_3, \overline{a}_4\right)$, in the same way.

TABLE III. THE MEANS AND STANDARD DEVIATIONS OF THE FOURTH-ORDER PREDICTOR COEFFICIENTS FOR DIFFERENT VALUES OF *M*.

M **[sample]**	**10**	**20**	**50**	**100**	**150**	**200**
a_{1m}	0.977	1.136	1.335	1.479	1.540	1.572
σ_1	0.248	0.260	0.280	0.264	0.275	0.266
a_{2m}	-0.202	-0.316	-0.546	-0.770	-0.860	-0.941
σ_2	0.312	0.395	0.507	0.499	0.507	0.493
a_{3m}	0.037	0.108	0.231	0.374	0.430	0.493
σ_3	0.214	0.274	0.373	0.364	0.388	0.372
a_{4m}	-0.075	-0.096	-0.140	-0.190	-0.210	-0.227
σ_4	0.133	0.142	0.172	0.168	0.193	0.176

Now it would be even more difficult to calculate the following probability of robust stability for the characteristic polynomials R_1 to R_4 by classical integration:

$$P_1 = \int\iiint_{S_4} f(\overline{a}_1, \overline{a}_2, \underline{a}_3, \underline{a}_4) d\overline{a}_1 d\overline{a}_2 d\underline{a}_3 d\underline{a}_4, \quad (32)$$

$$P_2 = \int\iiint_{S_4} f\left(\underline{a}_1, \overline{a}_2, \overline{a}_3, \underline{a}_4\right) d\underline{a}_1 d\overline{a}_2 d\overline{a}_3 d\underline{a}_4, \quad (33)$$

$$P_3 = \int\iiint_{S_4} f\left(\underline{a}_1, \underline{a}_2, \overline{a}_3, \overline{a}_4\right) d\underline{a}_1 d\underline{a}_2 d\overline{a}_3 d\overline{a}_4, \quad (34)$$

$$P_4 = \int\iiint_{S_4} f\left(\overline{a}_1, \underline{a}_2, \underline{a}_3, \overline{a}_4\right) d\overline{a}_1 d\underline{a}_2 d\underline{a}_3 d\overline{a}_4, \quad (35)$$

where S_4 is the stability area for the fourth-order prediction filter described by (7) for $k = 4$.

Finally, we can compute the corresponding robustness probability values using the Monte Carlo method for the same number of samples: $P_1 = 0.119$, $P_2 = 0.416$, $P_3 = 0.449$, and $P_4 = 0.220$.

The total robustness for the fourth-order prediction filter is

$$P = \prod_{i=1}^{4} P_i = 0.005. \quad (36)$$

We can see that the total robustness is even lower than in the case of the third-order predictor (about 0.5 %). As in the previous case, this fourth-order prediction filter is not robustly stable in the classical sense. Now even three of the four polynomials are not stable in the classical sense, only one of the Kharitonov polynomials is stable ($P_3 = 0.449$), so the system would also be robustly unstable in the classical sense in this way. It is interesting to note that for the fourth order, the probability of 0.449 corresponds to a stable system in the ideal case ($\sigma = 0$) and already to an unstable one of 0.416. For the third-order predictor, a probability of 0.362 was sufficient for stability under the same hypothetical conditions. Therefore, one should not be surprised at the small probabilities obtained for total robustness. And if all four polynomials are stable in the classical sense, the total robustness can often only be of the order of 1 %. We note that similar conclusions have already been obtained in a study of second-order predictors [29]. There, one of the four polynomials was unstable in the classical sense, and the total probability was also quite low.

D. A Different Approach to Robustness Analysis - The Case Study of the Third-Order Prediction Filter for the Fixed Frame Length

From all this, it can be concluded that this was a too stretchy and crude definition of parametric robustness estimation because the differences between the lower and upper bounds for the same predictor coefficients are enormous, considering that the analysis was performed with a wide range of frame lengths. Each set value of the predictor coefficient satisfies (7) and the probabilities of stability are high, which was also shown in [26]. For example, for M = 100 in the case of the third-order predictor, we have examined classical stability for each frame and the set of three coefficients a_1, a_2, a_3 always belongs to the area S_3, i.e., to the system of inequalities (7), (8). However, if Kharitonov's Theorem was applied, all polynomials would be unstable in the classical sense, again due to the large differences in coefficients for a given frames (Fig. 2). This is a key point of robust stability. The stable system hardly remains stable if the system parameters are changed to their limits.

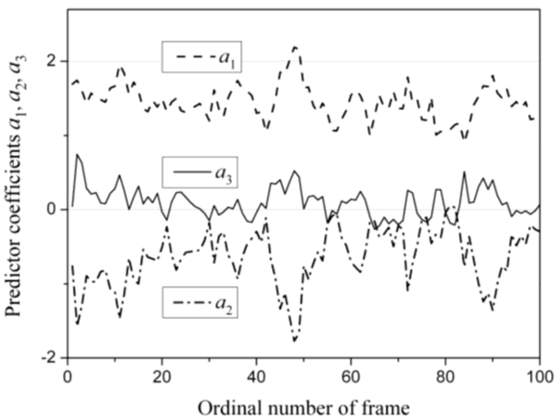
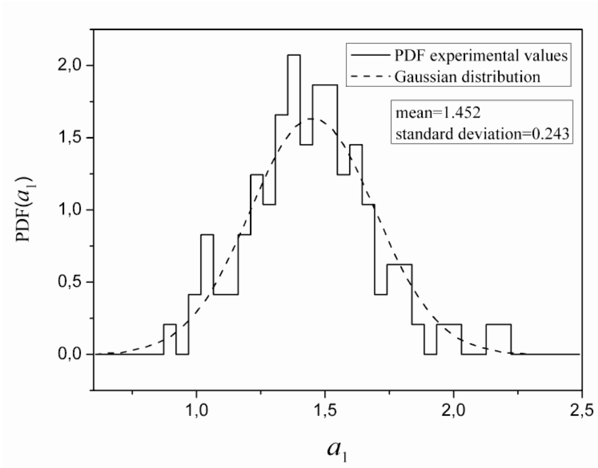


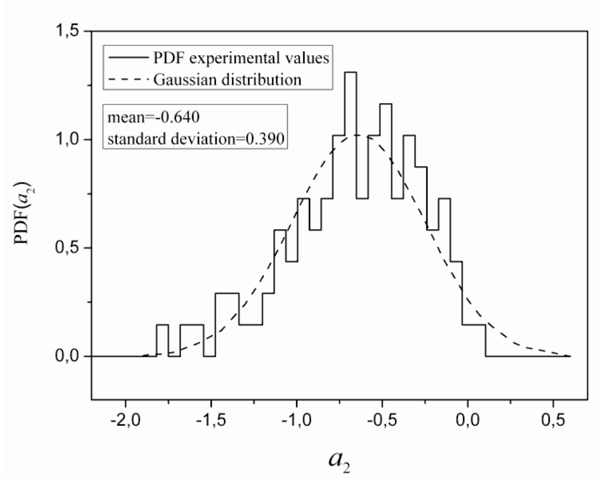
Fig. 2. Values of predictor coefficients a_1, a_2, and a_3 for each frame from 1 to 100.

Statistic was generated (the column for M = 100 in Table II) based on these values, as well as for all other values from Tables II and III. The corresponding distributions of the predictor coefficients and the normal (Gaussian) distribution with the same means and standard deviations are shown in Fig. 3 for illustration purpose.

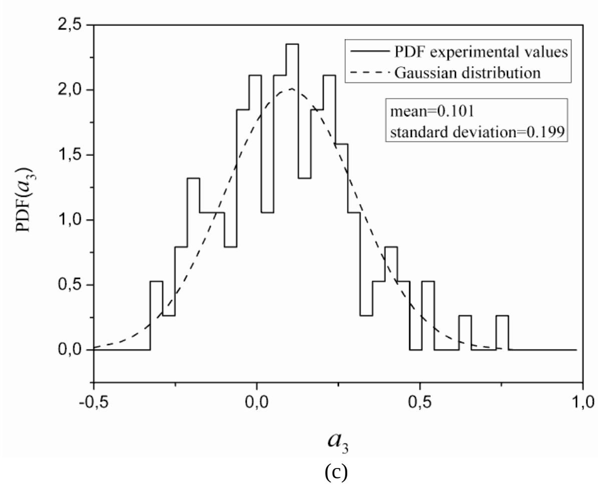
According to the proposed approach, we can therefore perform experiments and later appropriate robustness analysis for each frame length (10, 20, 50, 100, 150, 200) and for the third- and fourth-order predictors, but due to the scope of the work, we will focus on the length frame M = 100 already considered and give corresponding conclusions that are also valid for other M and predictor orders.



(a)



(b)



(c)

Fig. 3. PDF of the predictor coefficients (a) a_1, (b) a_2, and (c) a_3, respectively (for M = 100).

Then a series of repeated experiments was performed for the same predictor (in this case study, third order) and the same frame lengths (M = 100 here). Thus, the limits for the coefficients were determined in a completely different way.

Since we have a large agreement, the difference between the same coefficients is very small (on average at the third decimal place). In this way, there are now small differences between the lower and upper bounds of the same coefficient in Kharitonov's polynomials, which was not the case in the previous analysis when we chose the bounds based on the length of the frame. This is very close to the stability analysis performed in [26], i.e., we have similar probabilities of robustness for each individual polynomial. In the classical sense, they are all stable, and the probabilities are high as before. However, considering that for the total robustness the probabilities for R_1 to R_4 are multiplied, the values are not large. As an example, again we have only presented the values for M = 100 for the third-order predictor already mentioned (Table IV).

TABLE IV. THE MEANS AND STANDARD DEVIATIONS OF THE THIRD-ORDER PREDICTOR COEFFICIENTS FOR THE FIXED FRAME LENGTH (M = 100).

No. experiments	1	2	3	4	5
a_{1m}	1.452	1.444	1.447	1.453	1.443
σ_1	0.243	0.254	0.248	0.241	0.256
a_{2m}	-0.640	-0.635	-0.638	-0.641	-0.634
σ_2	0.390	0.392	0.391	0.388	0.395
a_{3m}	0.101	0.103	0.102	0.100	0.104
σ_3	0.199	0.199	0.198	0.197	0.200

Using the same principle as for robustness over the entire range of frame lengths, we obtain the following: $\overline{a_{1m}} = 1.453$, $\underline{a_{1m}} = 1.443$, $\overline{a_{2m}} = -0.634$, $\underline{a_{2m}} = -0.641$, $\overline{a_{3m}} = 0.104$, and $\underline{a_{3m}} = 0.100$. Using the same method and with the same number of trials in the Monte Carlo simulation, we obtain the following results for the individual robustness probabilities of Kharitonov's polynomials: P_1 = 0.411, P_2 = 0.405, P_3 = 0.410, P_4 = 0.416, and finally the total robustness: P = 0.028. Each of the above probabilities is roughly equivalent to those calculated in [26] for M = 100, but the total probability is again extremely small, as in the study for variable frame length, as we have assumed. In the classical sense, all four polynomials are stable, but in stochastic analysis, the combination of predictor coefficient values that is polynomial in R_4 gives the best results. Of course, this is a fine-tuning, as the differences are not as drastic as in the method that includes all values of the frame length in the same estimation.

This concludes the study on the stability, probability of stability, and robustness of the DPCM prediction filter, which was conducted in the previous research period.

V. CONCLUSIONS

In this paper, the robust stability of higher-order DPCM prediction filters is analysed. The experiments were performed for a given speech signal and experimentally determined values for the predictor coefficients for the third and fourth orders of the system. Due to the complexity of the classical calculation of integrals, the Monte Carlo method was used to solve complex integrals. It is possible to apply the proposed method very efficiently for higher-order predictors. The error problem in Monte Carlo integration is solved by increasing the number of samples until the desired accuracy is achieved. In contrast to the analysis for the lower-order predictors and the same one for the higher-order predictors here, this paper also proposed a different approach to determine the limit values of the predictor coefficients in a much finer way. The obtained values for the probability of robustness are important for the system design, i.e., the choice of the optimal values of the predictor coefficients. With larger values for the probability, the system has a greater chance of remaining stable and better performances, and the speech coding is much more efficient. Finally, there is the possibility of fine-tuning the values of the predictor coefficients, keeping in mind that the parametric sensitivity of the same has been studied in previous papers.

VI. FUTURE WORK

It is also possible to generalise this analysis to the entire DPCM system, whereby the derived conclusions regarding robust stability can be transferred to this system in many respects. The influence of the quantiser has already been explained in some previous work, but can certainly be included in the analysis.

This method can be applied in the same way for predictors higher than order four. One can go deeper into analysis of the Monte Carlo simulation error as well as the duration of the simulation, and thus find an optimum between these two conflicting parameters.

A correlation can be established between the probability of robustness and some parameters important for the performance of the system, such as the signal-to-noise ratio or the prediction gain, and a more detailed analysis is possible, as in stability-related analyses [25].

This paper is a basis for further analysis of the whole DPCM system, but also for further research on the application of this theory in the field of machine learning, artificial intelligence, and deep neural networks, where the main goal is prediction.

CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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