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Comparison of Orthogonal Transforms for OFDM Communication System

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Introduction

Many modern communications systems are based on orthogonal frequency multiplexing (OFDM) technology. Such well-known applications as wireless LANs and asymmetric digital subscriber lines (ADSL) proved efficiency and usability of this modulation scheme. OFDM technology has been discovered more than 50 years ago, but become popular just few decades back due achievements in DSP hardware design [1].

Classical OFDM is based on frequency division and sinusoidal base functions. Therefore heart of every OFDM communication system is discrete Fourier transform (DFT). However, discoveries of new discrete orthogonal transforms [2, 3] and availability of their hardware implementations raised a question of building non-FFT multi-carrier systems with orthogonal modulation. Moreover, in many modern communication applications (for example, Bluetooth) non-orthogonal pseudo-noise (PN) sequence based modulation schemes (multicarrier CDMA) are utilized. Hybrid schemes also exist, for example, V.Jain and A.Myers in [4] propose to use combination of orthogonal wavelet division multiplexing (OWDM) and spread spectrum (SS) concepts. However, there is no clear how to exploit these new orthogonal transforms in multi-carrier communication system in a right manner.

The goal of our publication is describe results of an experiment where we replaced DFT in OFDM communication system model by several other pure orthogonal transforms. We conducted series of simulations of communication systems with different transforms and equalization methods. Our objectives were to examine following aspects:

- An impact of choice of orthogonal transform on BER of communication system;
- Impact of transform on efficiency of equalization;
- Impact of equalization method on BER of communication system;
- An impact of orthogonal transform on peak to average power ratio (PAPR) of the transmitted signal.

Results of our experiments allow better understand the impact and importance of orthogonal transformations in communication systems and outlines major problems in the system design.

The structure of paper is as follows: Section “Theory and definitions” gives mathematical background about channel types and orthogonal transforms used. Section “Results and discussion” contains actual simulation results and their information about simulation details. In “Conclusions” section we attempt to explain the obtained results.

Theory and definitions

OFDM is modulation scheme employing parallel transmission of digital information using multiple orthogonal subcarriers. As name of this technique states, these subcarriers are sinusoids with different frequency. Modulation process can be described with equation

$$s(k) = \frac{1}{\sqrt{N}} \sum d(n)\varphi(n,k), \quad k = 0,1,\ldots,N, \tag{1}$$

where k is sample index in time domain, n is sample index in the frequency domain and $\varphi(n,k)$ is DFT. In practice N must be limited, and this leads to block-wise transmission. Applications where large delay is acceptable (for instance, broadcasting) N can reach 10^3. In real-time applications, such as ADSL or wireless LANs, block size is limited to several hundreds. Demodulation process transforms received time domain signal $r(k)$ back into frequency domain

$$\hat{d}(n) = \frac{1}{\sqrt{N}} \sum r(k)\gamma(n,k), \quad n = 0,1,\ldots,N. \tag{2}$$

It is possible to perform modulation and demodulation with aid of filter bank [4]. However, most OFDM implementations utilize inverse fast Fourier transform (IFFT) for modulation and fast Fourier transform (FFT) for the demodulation.

Every OFDM communication system consists of units responsible for signal transformation, channel estimation and equalization and form synchronization devices. Block diagram of typical communication system model is depicted in Fig. 1. OFDM signal structure consists of frames. Beginning of each frame contains service information, synchronization symbols and pilot symbols. Each OFDM symbol (including pilot and sync symbols is prefixed by cyclic prefix (CP), which consists from samples from the end of the symbol.

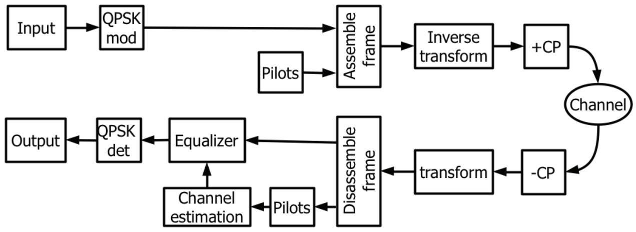

Fig. 1. Block diagram of the communication system

We modeled communication systems with four different orthogonal transformations:

- Identity transform;
- Discrete Fourier transform;
- Hadamard transform;
- Generalized orthogonal nonsinusoidal (Phi) transform.

Identity transform maps any vector to itself. In fact, identity transformation means absence of transformation at all. We have chosen this transform because after it signal sent in communication channel has zero PAPR.

Well known discrete Fourier transform (DFT) is derived from continuous time counterpart. Classical OFDM employs this transform. Base functions are defined as follows

$$\begin{cases} \varphi(n,k) = e^{j2\pi k\frac{n}{N}}, \\ \varphi^{-1}(n,k) = e^{-j2\pi k\frac{n}{N}}. \end{cases} \tag{3}$$

Hadamard transform base functions $\varphi(n,k)$ are columns of Hadamard matrix. Inverse base functions can be obtained from the rows of the same matrix:

$$H = \begin{bmatrix} H & H \\ H & -H \end{bmatrix}, \tag{4}$$

$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{5}$$

Phi transform is a rotation based transform being developed during latest decade. This transform rotates signal vector in arbitrary direction. In fact, using rotation we can obtain all three previously mentioned transforms. This transform facilitates rotation by real and complex angles [2]. Base functions of this transform are columns of the factorized matrix:

$$Y = B(\phi). \tag{6}$$

Which in turn is consists from Stairs-like Orthonormal Generalized Rotation (SOGRM) matrices

$$B = \begin{bmatrix} \tau & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \tau & \dots & 0 & 0 \\ \dots & \dots & & \ddots & \dots & \\ 0 & 0 & 0 & 0 & \dots & \tau \\ \tau & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \tau & \dots & 0 & 0 \\ \dots & \dots & & \ddots & \dots & \\ 0 & 0 & 0 & 0 & \dots & \tau \end{bmatrix}. \tag{7}$$

Which contains unitary four-element rotation matrices

$$T_{qp} = \begin{bmatrix} \tau_{q,p}^{1} \\ \tau_{q,p}^{2} \end{bmatrix} = \begin{bmatrix} \cos\phi_{r,p} e^{j\gamma_{q,p}} & \mp\sin\phi_{r,p} e^{j\gamma_{q,p}} \\ \pm\sin\phi_{r,p} e^{j\gamma_{q,p}} & \cos\phi_{r,p} e^{j\gamma_{q,p}} \end{bmatrix}. \tag{8}$$

If $\gamma=0$, then rotation by real angles is performed.

If communication channel is frequency selective, equalizer is required to achieve viable BER. Actually each transform requires its own transform-domain equalization method. Thus in identity transform based system we utilized adaptive Least mean-square (LMS) algorithm - based time-domain equalizer (TDE). This equalizer performs deconvolution between estimated channel impulse response and transmitted samples. Notice, that CP in this scheme is not used efficiently.

In case of DFT based OFDM, equalization is based on relatively simple frequency domain channel estimation and equalization. Each OFDM symbol (group of N samples) is prefixed by CP. Cyclic nature of OFDM symbols provides very efficient way for inter-carrier interference (ICI) and inter-symbol interference (ISI) mitigation. Firstly CP serves as guard interval, secondly Fourier transform decomposes circulant channel matrix into series of independent channels [5]. Therefore channel estimation and equalization is reduced to series of scalar divisions and multiplications of complex values. Channel frequency response estimate is ratio between received $d_p(n)$ and transmitted $p_p(n)$ pilot samples

$$\hat{H}(n) = \frac{p_p(n)}{d_p(n)}. \tag{9}$$

Equalization is done by scalar division of received samples $p(n)$ to channel estimate

$$d(n) = \frac{p(n)}{\hat{H}(n)} = \frac{\hat{H}(n)^*}{|\hat{H}(n)|^2} p(n). \tag{10}$$

Obviously, if we choose other than Fourier transform, channel with CP will not decompose into series of independent channels. Both Hadamard transform and Phi transform lack their own-domain equalization methods, so we had to choose between time domain and “frequency” (base function) domain equalization. Our aim is show in this paper some advantages (or drawbacks) of this approach.

Timing and frequency synchronization in OFDM is a big hurdle [6]. Synchronization mechanisms our models were simplified to minimize impact of the synchronization issues on performance of the model. Transmitter and receiver clocks were same and both units were launched simultaneously. Therefore frequency, symbol timing and frame synchronization was ideal.

One of the major disadvantages of DFT based OFDM modulation scheme is large peak to average power ratio (PAPR), since time domain signal is sum of sinusoids with different frequencies. PAPR (in dB) of signal *s(t)* is defined as follows:

$$PAPR = 10\log_{10}\frac{\max[s(t)^2]}{E[s(t)^2]}. \qquad (11)$$

System performance degradation due nonlinearities of communication equipment have larger effect on signal with larger PAPR. In our experiments we tested impact of nonlinear channel with additive noise and clipping at 50% of peak level.

Results and discussion

Our OFDM communication system model was built using Mathworks Simulink environment. Block diagram of the communications system is depicted in Fig. 1.

OFDM time-domain signal consists of frames. Each frame consists of 4 pilot (training) symbols at the beginning at the frame and 20 payload symbols. Each symbol (including pilots) consists of 80 samples (subcarriers in frequency domain), and 16 from them are CP.

To measure impact of various communication channel impairments, we tested each transform with four different channels:

- Additive white Gaussian noise (AWGN) channel;
- Channel with clipping at 0.5 of peak magnitude with AWGN;
- Frequency selective channel with AWGN. Impulse and frequency characteristics of this channel are given in Fig. 2.

In all experiments we adjusted variance of the additive noise to achieve required signal to noise ratio (SNR) and then measured bit error ratio (BER) of the system.

Simulation results are depicted in Fig. 3, Fig. 4 and Fig. 5. In the legend, TDE stands for “time domain equalizer”. Notice, that in all cases, except identity transform, it is not pure TDE since equalization is applied in the transform domain, but not in the time domain (see model scheme in Fig. 1). FDE, in turn, means “frequency domain equalization”. Again, it is not pure FDE, except DFT mode, since we are not using frequency division. In other words, we just tested two different approaches – deconvolution and multiplication. PAPR of transmitted signals is summarized in Table 1.

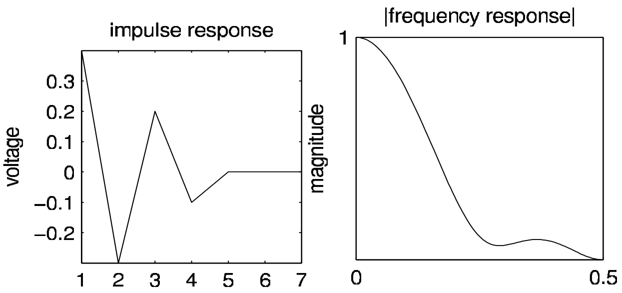

Fig. 2. Characteristics of requency selective channel

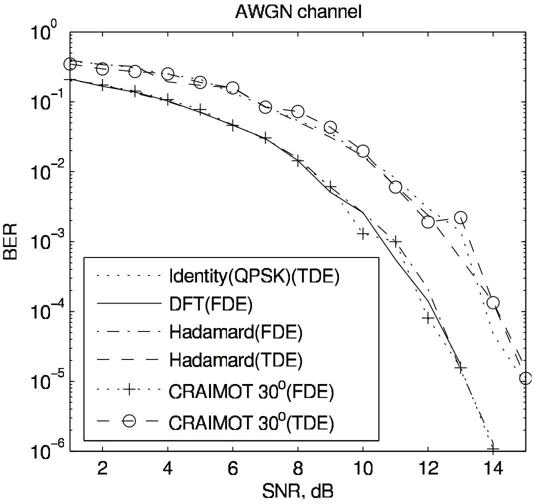

Fig. 3. Communication system performance in AWGN channel

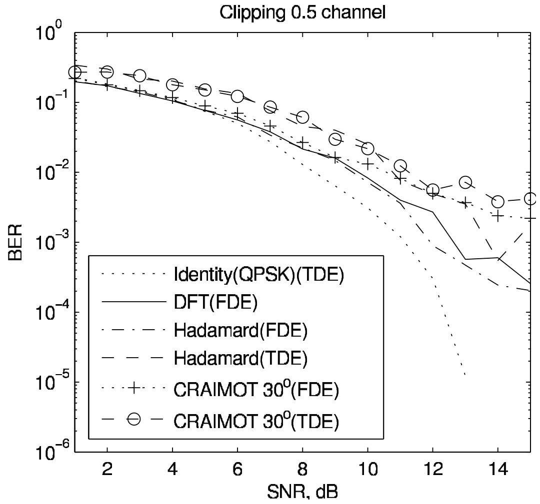

Fig. 4. Communication system performance in clipping channel

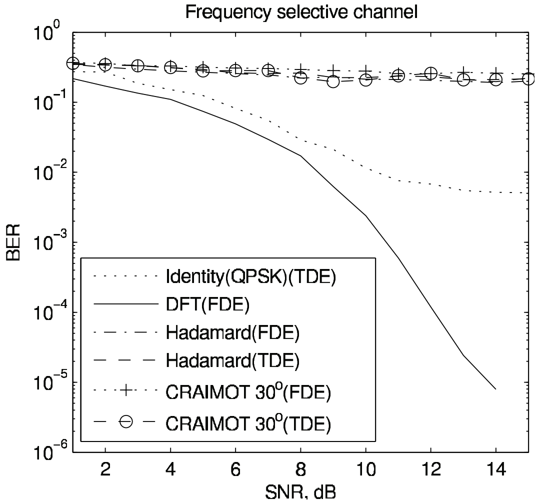

Fig. 5. Communication system performance in frequency selective channel

Table 1. PAPR of transmitted signals

Transform	PAPR, dB
Identity (QPSK)	0
Fourier transform	6
Hadamard transform	6.1
CRAIMOT 30^0 transform	3.2

Conclusions

In AWGN channel performance of all transforms is similar and is affected mostly by equalizer (which in this case does not help) faults. In accordance with [7] certain Phi configurations provide even some performance gain.

In frequency selective channels DFT significantly outperforms other orthogonal transforms due powerful equalization.

Given Phi transform configuration (CRAIMOT 30^0) is not suitable nor for deconvolution (TDE) nor for multiplication based (FDE) equalization.

To achieve acceptable BER results with Hadamard or Phi transform, appropriate transform domain equalization methods must be developed.

Frequency domain equalization is more BER efficient. Moreover, it is computationally more efficient as well. To overcome large PAPR problem, single carrier frequency domain equalizer modulation SC-FDE scheme can be used [8, 9].

References

1. **Bingham J**, Multicarrier modulation for data transmission: An idea whose time has come // IEEE Commun. Mag., 1990. – Vol. 28. – No. 5. – P. 5–14.
2. **Misans P., Terauds M., Aboltins A., Valters G.** MATLAB/SIMULINK implementation of Phi transforms – a new toolbox only or the rival of Wavelet Toolbox for the next decade? // Nordic MATLAB User Conference, 2008. – P. 1–8.
3. **Oka I., Fossorier M.** A General Orthogonal Modulation Model for Software Radios // IEEE Transactions on Communications, 2006. – Vol. 54. – No. 1. – P. 7–12.
4. **Jain V., Myers B.** OWSS: a new signaling system for 100–150 Mb/s wireless LANs // IEEE Wireless Communications, 2003. – P. 16–24.
5. **Peled A., Ruiz A.** Frequency domain data transmission using reduced computational complexity algorithms // Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP). – Denver, Colorado, 1980. – P. 964–967.
6. **Aboltins A., Klavins D.** Synchronization and Correction of Channel Parameters for an OFDM based Communication System // Automatic Control and Computer Sciences. – Allerton Press, Inc., 2010. – Vol. 44. – No. 3. – P. 160–170.
7. **Misans P., Valters G**. Initial FPGA design for generalized Orthogonal Nonsinusoidal Division Multiplexing // NORCHIP'2009, 2009. – P. 1–5.
8. **Devillers B., Louveaux J., Vandendorpe L.** About the diversity in cyclic prefixed single–carrier systems // Physical Communication. – Elsevier, 2008. – Vol. 1. – P. 266–276
9. **Radić J., Rožić N.** Adaptive Impulse Noise Suppression in OFDM Systems // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 1(97). – P. 3–8.

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A. Aboltins. Comparison of Orthogonal Transforms for OFDM Communication System // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 5(111). – P. 77–80.

Classical orthogonal frequency division (OFDM) employs fast Fourier transform (FFT) for elimination of inter-symbol interference (ISI) and expansion of time support of the transmitted signals. However FFT could not always be optimal in terms bit error ratio (BER) due channel variations, Doppler Effect, nonlinearities and high peak-to-average power ratio of the signal. Aim of this publication is to experimentally compare different orthogonal transforms and outline impact of these transforms on BER of the communication system with noisy, frequency selective and nonlinear communication channels. Simulation results of OFDM communication systems with equalization and no transform, with FFT, Hadamard and Generalized nonsinusoidal orthogonal transforms are presented. Ill. 5, bibl. 9, tabl. 1 (in English; abstracts in English and Lithuanian).

A. Aboltins. Ortogonaliųjų transformacijų palyginimas OFDM sistemose // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 5(111). – P. 77–80.

Esant trikdžiams pastebėta, kad OFDM sistemose vyksta Furjė transformacija. Įvertinus bitų klaidų koeficientą, Doplerio efektą, netiesiškumus, nustatyta, kad optimumo negalima pasiekti taikant Furjė transformaciją. Eksperimentiškai patikrintos ortogonaliosios transformacijos. Pateikti modeliavimo rezultatai. Il. 5, bibl. 9, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).