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Stability Analysis of Control Systems with the Asymmetric Controllers using Frequency Domain Criterion

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Introduction

There are lot of control algorithms developed for feedback control of plants and processes. The classical control algorithms as Proportional (P), Integral (I) and widely used Proportional-Integral (PI) [1] and Proportional-Integral-Derivative (PID) do not guaranty in some cases such a good control quality as nonlinear control algorithms provide [2]. The stability of control systems using the nonlinear algorithms, which are developed by authors [3, 4], is analyzed in this work. These control algorithms are asymmetric Proportional (aP)

$$U(t) = K_p(t)\ e(t), \quad \begin{vmatrix} K_p(t) = K_{pp}, & e(t) \geq 0, \\ K_p(t) = K_{pn}, & e(t) < 0 \end{vmatrix} \tag{1}$$

and Proportional – asymmetric Integral (PaI)

$$U(t) = K_p e(t) + \int_{t_0}^{t} K_i(\tau)\ e(\tau)\ d\tau, \quad \begin{vmatrix} K_i(t) = K_{ip}, & e(t) \geq 0, \\ K_i(t) = K_{in}, & e(t) < 0, \end{vmatrix} \tag{2}$$

where *U(t)* is the controller output, K_p is the proportional constant, K_i is the integral constant, *t* is time, *e(t)* is the error, i.e. the difference between desired and actual values of plant parameter, K_{pp}, K_{pn} and, K_{ip}, K_{in} are proportional constant and integral constant that act at positive and at negative *e(t)*, respectively, t_0 is point in time, at which the algorithm starts to operate.

The controllers based on algorithms (1) and (2) have variable structure [5–7], which changes when the error changes the sign. Therefore, the control system based on these controllers is nonlinear even in the case when the controlled plant is linear. The employment of these control algorithms allow us to achieve higher load disturbance rejection of the linear plant as compared to the case when classical P and PI controllers are used [8, 9].

The purpose of this work is to investigate the stability of control systems based on the aP and PaI controllers for the case when linear plants are controlled. However, the analysis of nonlinear control systems is complicated because does not exist the complete theory of nonlinear control systems analysis. There are several trends and lot of methods in nonlinear control systems stability research. The methods of nonlinear system stability analysis based on the frequency domain criterion [10], which are introduced by Popov [11], are used in this work for the analysis of control systems based on the aP and PaI controllers. They are dedicated for the systems with the one nonlinear element (part) (Fig.1), for the case when it is time invariant and the characteristic of this element is inside of some sector. The Y_d is desired (set point) and $Y_a(t)$ is actual value of the plant parameter in Fig.1.

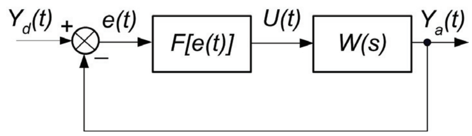

Fig. 1. The control system with the separated linear *W(s)* and nonlinear *F[e(t)]* parts

Later the Popov method was adopted for the systems with the non-stationary nonlinearity and the Popovs‘s criterion was corrected for the cases when the special requirements are raised for the nonlinear element. As an example, Cho and Narendra developed the method for the case when the characteristic of the nonlinear element have to be monotonic [12].

The advantages of the frequency domain methods are following:

– Possibility to find the stability boundaries not only for the one concrete system but for the several systems with the nonlinearities that belong to the same class;

– Possibility to apply the criterion to nonlinear systems that include one nonlinearity and linear part is *n* order or *n* order with the transport delay;

– Possibility to apply for the analysis of such nonlinear systems some instruments that are used in the stability analysis of the linear systems.

The disadvantage of these methods is fact that they define only sufficient but not obligatory stability condition of the nonlinear system. It means that if the frequency domain criterion shows that control system is not stable it does not mean that really it is unstable. Additionally, these criterions are not universal and can be applied for the systems, which include only one nonlinear element.

The purpose of this work is to find and investigate the suitable method for stability analysis of the nonlinear control system containing the linear plant and nonlinear aP or PaI controller.

Selection of stability analysis method

The linear and nonlinear part of the control system should be separated to prepare the system for the stability analysis using Popov method. The block diagrams of the control systems with the separated linear and nonlinear parts based on the aP and PaI controllers are given in the Fig. 2. The plants in these control systems are linear.

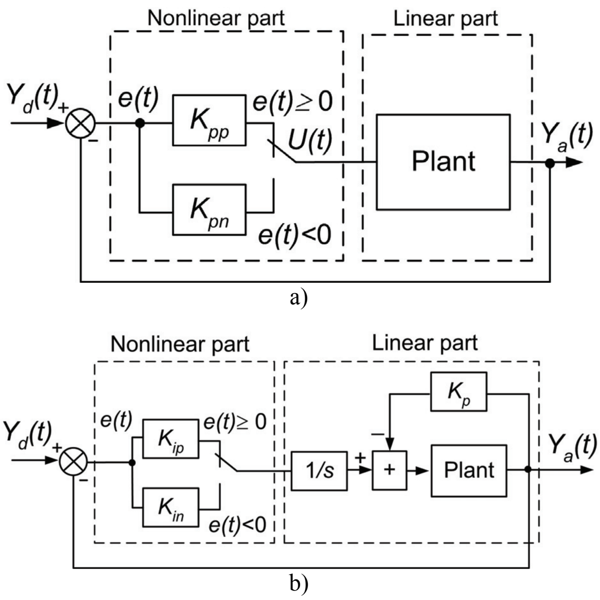

Fig. 2. The block diagrams of the control system with the separated linear and nonlinear parts based on the aP (a) and PaI (b) controllers

The nonlinear parts of the analyzed control systems presented in Fig.2 have the same structure. They can be described by the function

$$F\left[e(t)\right]=K(t)\cdot e(t)\ \left|\begin{array}{ll}K(t)=K_p, & e(t)\geq 0,\\ K(t)=K_n, & e(t)<0,\end{array}\right. \quad (3)$$

where K_p corresponds to K_{pp} and K_{ip}, K_n – to K_{pn} and K_{in}

The graph of the function (3) is given in Fig.3.

The analyzed control systems with the one nonlinear element, which is described by the function *F[e(t)],* meet the Popov criterion requirements. The nonlinearity have to be nonstationary and must be inside of region $0<F[e(t)]<\alpha$ according this criterion.

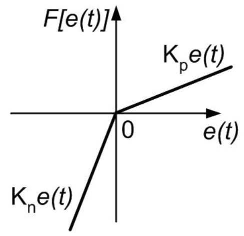

Fig. 3. Graph of the nonlinear function *F[e(t)]*

The analyzed control systems are stable if they meet the condition

$$\mathrm{Re}[(1+j\omega q)\cdot W(j\omega)]+\frac{1}{\alpha}>0, \quad (4)$$

where *q* is real number. The Circle criterion [13] can be used for the stability investigation in the analyzed situation. The definition of this criterion states that the control system with the one nonlinearity is stable in the boundary limited by the α and β if the function of the nonlinear element is inside of the region $\beta<F[e(t)]<\alpha$ and the condition

$$\mathrm{Re}[(1+\alpha\cdot W^{*}(j\omega))\cdot(1+\beta\cdot W(j\omega))]\geq 0 \quad (5)$$

is satisfied, where $W(j\omega)=U(\omega)+jV(\omega)$, $W^*(j\omega)=U(\omega)-jV(\omega)$.

The method discussed above is good for analysis of control systems with the linear plants based on the aP and PaI controllers. If the function *F[e(t)]* is monotonic the Cho and Narendra (Off-Axis) criterion can be applied for such a control systems as well. This criterion is similar to Circle criterion and in the particular cases coincides with them. The system is stable according the Off-Axis criterion if the real-positive function *Z(jω)* exists and condition

$$\mathrm{Re}\left[\frac{\alpha\cdot W(j\omega)+1}{\beta\cdot W(j\omega)+1}Z(j\omega)\right]>0 \quad (6)$$

is satisfied.

Summarizing the presented analysis it is necessary to stress that first of all the Circle criterion should be applied for the stability analysis of the control systems with the linear plants based on the aP and PaI controllers. If the Circle criterion does not allow us to solve the problem and function *F[e(t)]* is monotonic, the Off-Axis criterion can be used.

Stability investigation

The graphical-analytical analysis of control systems stability was used for the investigation. The control systems with the linear plants, which are described by the following transfer functions, were analyzed:

$$G_1(s)=\frac{e^{-sT_d}}{4s^2+s+1}, \quad (7)$$

$$G_2(s)=\frac{3(s+1)}{s^2(s^2+s+25)}. \quad (8)$$

The analyzed control systems are based on the aP and PaI controllers.

The analysis was provided using software Mathcad. The Nyquist plot of the control system linear part should be plotted for this purpose. The Nyquist stability criterion could be used for the stability boundaries estimation if the control system would be linear. However, the analyzed control systems (Fig. 2) are nonlinear. Because of this, the Circle or Off-Axis criterions have to be applied.

The function of the nonlinear element, which is inside of the region $\beta<F[e(t)]<\alpha$, is presented by the circle for the case when the Circle criterion is used. The circle crosses the real axis in points (0,-1/α) and (0,-1/β) and centre of the circle lies on the real axis as well. It is enough for the stability of the analyzed control systems that the Nyquist plot that presents the linear part of the system would not pass through the circle.

The Off-Axis criterion is similar to Circle criterion. The only difference is that the circle centre should not lie on the real axis.

The analysis results of the control system with the plant $G_1(s)$ based on the aP controller for the various values of α and β are presented in Fig 4. It is seen that when the difference between the coefficients α and β (they correspond to K_{pp} and K_{pn}) decreases, i.e. the nonlinearity of

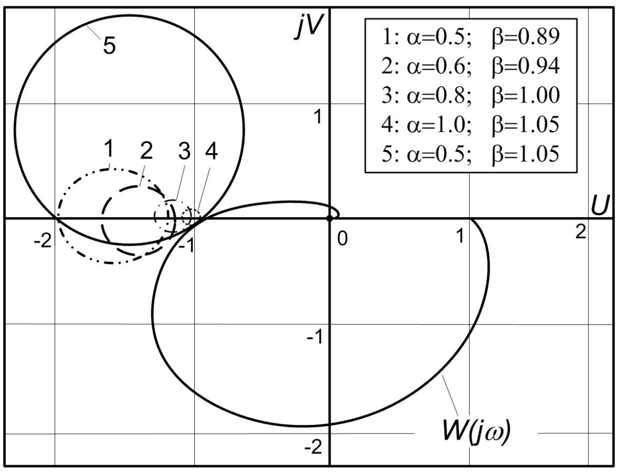

Fig. 4. The stability analysis plots using Circle and Off-Axis criterions for the control system with the plant $G_1(s)$ and aP controller

aP controller is lowered, the stability of the analyzed system approaches to the stability of the linear system based on the P controller.

The Nyquist plot of the plant $G_1(s)$ control system linear part changes if instead of aP controller the PaI controller is used. The $W(j\omega)$-plane encircles the point $(-1;+j0)$ in such a situation. This fact shows that the linear part is not stable. Because of this, the Circle and Off-Axis criterions can not be applied.

The plots of the stability analysis of the control systems based on the aP and PaI controllers with the plant $G_2(s)$ are given in Figs. 5 and 6.

The analyzed control systems were investigated using Matlab/Simulink software. The obtained results prove the fact that the Circle and Off-Axis criterions define only sufficient but not obligatory stability condition of the nonlinear system.

Additionally, it was found during the analysis that there are control systems, which do not operate stable with the P and PI controllers, however, they become stable when aP and PaI controllers are employed.

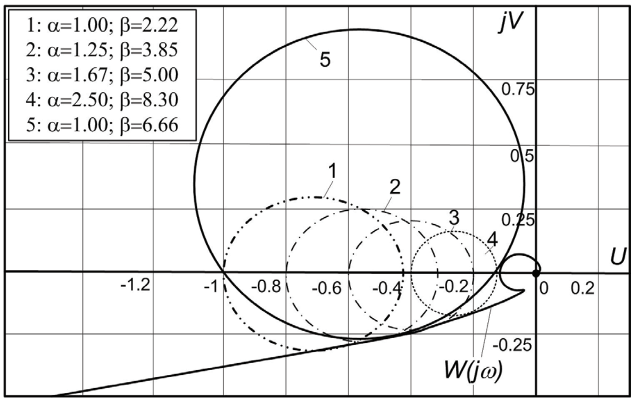

Fig. 5. The stability analysis plots using Circle and Off-axis criterions for the control system with the plant $G_2(s)$ and aP controller

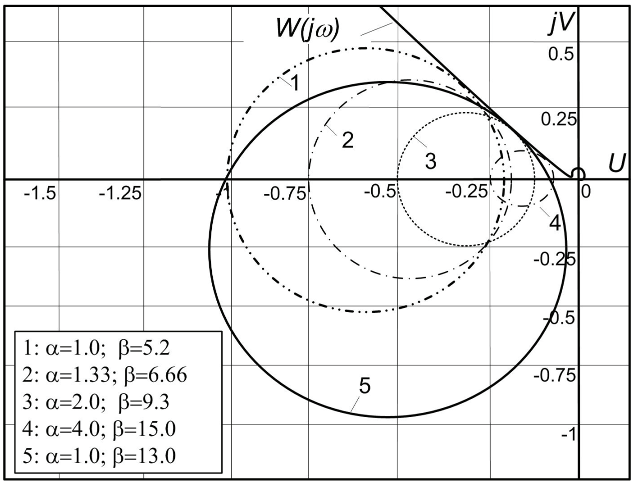

Fig. 6. The stability analysis plots using Circle and Off-axis criterions for the control system with the plant $G_2(s)$ and PaI controller

Conclusions

The Circle criterion should be applied for the stability analysis of the control systems with the linear plants based on the aP and PaI controllers. If the Circle criterion does not allow us to solve the problem and the nonlinear element function is monotonic, the Off-Axis criterion can be used.

The investigation of the analyzed control systems using Matlab/Simulink software prove the fact that the Circle and Off-Axis criterions define only sufficient but not obligatory stability condition of the nonlinear system.

There are control systems, which do not operate stable with the P and PI controllers, however, they become stable when aP and PaI controllers are employed.

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The stability of control systems with the linear plants using asymmetric Proportional and Proportional-asymmetric Integral controllers is analyzed in this work. The methods of nonlinear systems stability analysis based on the frequency domain criterion proposed by Popov and modified by the Cho and Narendra were employed. They are dedicated for the systems with the one nonlinear element. The investigation has been performed using Off-Axis and Circle criterions by employing graphical-analytical analysis methods, which were realized using software Mathcad. The obtained analysis results were verified using dynamic systems simulation software Matlab/Simulink. Ill. 6, bibl. 13 (in English; abstracts in English and Lithuanian).

V. Zlosnikas, A. Baškys, V. Gobis. Valdymo sistemų su asimetriniais reguliatoriais stabilumo analizė dažninių charakteristikų metodais // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 5(111). – P. 63–66.

Darbe tiriamas tiesinių objektų, valdomų asimetriniu proporciniu ir proporciniu–asimetriniu integraliniu reguliatoriais, stabilumas. Tyrimui taikytas dažninių charakteristikų tyrimo metodas, skirtas netiesinėms sistemoms. Jį pasiūlė Popovas, o patobulino Cho ir Narendra. Šie metodai skirti sistemoms, turinčioms vieną netiesinį elementą. Tyrimas atliktas naudojant apskritiminį ir neašinį (Off-axis) kriterijus. Jam atlikti taikytas grafinis-analitinis metodas, sukurtas naudojantis programų paketu „Mathcad“. Gauti rezultatai patikrinti dinaminių sistemų modeliavimo programa „Matlab/Simulink“. Il. 6, bibl. 13 (anglų kalba; santraukos anglų ir lietuvių k.).