https://doi.org/10.5755/j02.eie.35119 ELEKTRONIKA IR ELEKTROTECHNIKA, ISSN 1392-1215, VOL. 29, NO. 5, 2023

Identification of Wiener Systems with Recursive Gauss-Seidel Algorithm

Metin Hatun
Department of Electrical-Electronics Engineering, Bursa Uludag University, Gorukle Campus, 16059 Bursa, Turkey
metinh@uludag.edu.tr

***Abstract*—The Recursive Gauss-Seidel (RGS) algorithm is presented that is implemented in a one-step Gauss-Seidel iteration for the identification of Wiener output error systems. The RGS algorithm has lower processing intensity than the popular Recursive Least Squares (RLS) algorithm due to its implementation using one-step Gauss-Seidel iteration in a sampling interval. The noise-free output samples in the data vector used for implementation of the RGS algorithm are estimated using an auxiliary model. Also, a stochastic convergence analysis is presented, and it is shown that the presented auxiliary model-based RGS algorithm gives unbiased parameter estimates even if the measurement noise is coloured. Finally, the effectiveness of the RGS algorithm is verified and compared with the equivalent RLS algorithm by computer simulations.**

***Index Terms*—Auxiliary model; Gauss-Seidel; Recursive estimation; System identification; Wiener system.**

I. INTRODUCTION

Wiener systems have static nonlinear blocks at the output of dynamic linear subsystems and are widely used in engineering for modelling, identification and control applications [1]–[16]. These nonlinear blocks can be modelled using both parametric models and nonparametric models. Linear subsystems are modelled using impulse responses or transfer functions, and nonlinear blocks are modelled by piecewise linear functions or polynomial basis functions [1]–[3]. In recent years, the identification of parametric models has received more attention from researchers. The parametric Wiener models are linear-in-parameters and allow a direct application of the known algorithms used for identification of the linear models. Several methods have been presented for parameter estimation of Wiener models in the system identification literature. Offline methods are implemented using one-time processing after input-output samples are collected [17]–[19]. Iterative algorithms are implemented using multiple iterations after input-output samples are collected [20]–[33]. Unlike iterative methods, parameter estimates are updated during every sampling interval in recursive methods [34]–[57]. The recursive algorithms are implemented using one-step iteration in a sampling period and thus are more suitable for real-time applications. Gradient-based algorithms are preferred because of their lower computational complexity, despite their slow convergence rates. Although the Recursive Least Squares (RLS) algorithm is computationally intensive, it converges faster than gradient-based methods [57]. In addition, a new recursive algorithm is proposed that uses one-step Gauss-Seidel iteration is as an alternative method to adjust a self-tuning controller [58]. The Recursive Gauss-Seidel (RGS) algorithm has a faster convergence rate than gradient-based algorithms and has a lower computational burden than the RLS algorithm. This paper aims to transfer the advantages of the RGS algorithm to parameter estimation of nonlinear Wiener systems.

In this paper, Section II presents an identification model for Wiener output error models. The RGS algorithm for Wiener models is presented in Section III. The convergence of the RGS estimations is analysed in Section IV. Computer simulations are presented in Section V, and conclusions are given in Section VI.

II. IDENTIFICATION MODEL

A Wiener system includes linear and nonlinear subsystems, as seen in Fig. 1, and is formulated as follows:

$$x(k)=\frac{B(z^{-1})}{A(z^{-1})}u(k), \tag{1}$$

$$y(k)=f[x(k)]+v(k), \tag{2}$$

where $u(k)$, $x(k)$, and $v(k)$ are input, noise-free output, and measurement noise sequences, respectively.

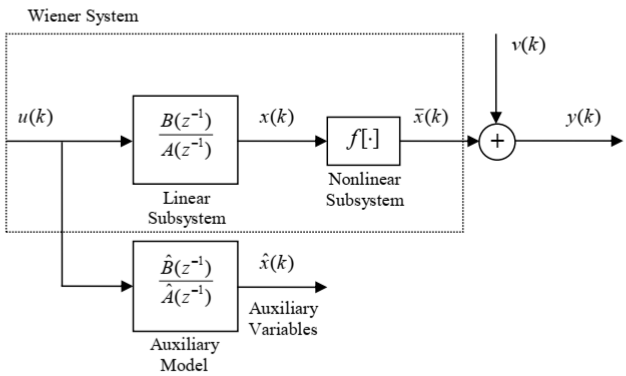

Fig. 1. Wiener system structure.

The linear subsystem polynomials are defined as:

Manuscript received 11 June, 2023; accepted 24 August, 2023.

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a}, \tag{3}$$

$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b}. \tag{4}$$

The nonlinear subsystem can be expressed using known polynomial basis functions

$$f[x(k)] = c_0 + c_1 x(k) + c_2 x^2(k) + \cdots + c_{n_c} x^{n_c}(k). \tag{5}$$

Using the models defined above and taking $c_1 = 1$, the Wiener model can be written as

$$y(k) = -\sum_{i=1}^{n_a} a_i x(k-i) + \sum_{i=1}^{n_b} b_i u(k-i) + \\ + c_0 + \sum_{i=2}^{n_c} c_i x^i(k) + v(k). \tag{6}$$

Defining the parameter and data vectors as:

$$\boldsymbol{\theta} = [a_1 \quad \cdots \quad a_{n_a} \quad b_1 \quad \cdots \quad b_{n_b} \quad c_0 \quad c_2 \quad \cdots \quad c_{n_c}]^T \in R^M, \tag{7}$$

$$\boldsymbol{\varphi}(k) = [-x(k-1) \quad \cdots \quad -x(k-n_a) \quad u(k-1) \quad \cdots \\ \cdots \quad u(k-n_b) \quad 1 \quad x^2(k) \quad \cdots \quad x^{n_c}(k)]^T \in R^M, \tag{8}$$

an identification model is written as

$$y(k) = \boldsymbol{\varphi}^T(k)\boldsymbol{\theta} + v(k), \tag{9}$$

where $M = n_a + n_b + n_c$.

III. RGS Algorithm for Wiener Models

The inner variable $x(k)$ in (8) cannot be measured but can be estimated using the auxiliary model

$$\hat{x}(k) = \hat{\boldsymbol{\varphi}}_a^T(k)\hat{\boldsymbol{\theta}}_a(k-1). \tag{10}$$

The data and parameter vectors in (10) are defined as:

$$\hat{\boldsymbol{\varphi}}_a(k) = [-\hat{x}(k-1) \quad \cdots \quad -\hat{x}(k-n_a) \quad u(k-1) \quad \cdots \quad u(k-n_b)]^T \in R^{n_a+n_b}, \tag{11}$$

$$\hat{\boldsymbol{\theta}}_a(k) = [\hat{a}_1(k) \quad \cdots \quad \hat{a}_{n_a}(k) \quad \hat{b}_1(k) \quad \cdots \quad \hat{b}_{n_b}(k)]^T \in R^{n_a+n_b}. \tag{12}$$

Thus, the data and the parameter estimation vectors of the Wiener model in (9) can be formed as follows:

$$\hat{\boldsymbol{\varphi}}(k) = [\hat{\boldsymbol{\varphi}}_a(k) \quad 1 \quad \hat{x}^2(k) \quad \cdots \quad \hat{x}^{n_c}(k)]^T \in R^M, \tag{13}$$

$$\hat{\boldsymbol{\theta}}(k) = [\hat{\boldsymbol{\theta}}_a(k) \quad \hat{c}_0(k) \quad \hat{c}_2(k) \quad \cdots \quad \hat{c}_{n_c}(k)]^T \in R^M, \tag{14}$$

and an output error is defined for the Wiener model as

$$e(k) = y(k) - \hat{\boldsymbol{\varphi}}^T(k)\hat{\boldsymbol{\theta}}(k-1). \tag{15}$$

The RGS algorithm can be obtained by minimising the following error function

$$J[\boldsymbol{\theta}] = \sum_{i=1}^{k} \lambda^{k-i} e^2(i) = \sum_{i=1}^{k} \lambda^{k-i} [y(i) - \hat{\boldsymbol{\varphi}}^T(i)\boldsymbol{\theta}]^2, \tag{16}$$

where λ is the forgetting factor. The optimal parameter vector

$$\hat{\boldsymbol{\theta}}(k) = \mathbf{R}^{-1}(k)\mathbf{p}(k) \tag{17}$$

is the solution of the normal equation

$$\mathbf{R}(k)\hat{\boldsymbol{\theta}}(k) = \mathbf{p}(k). \tag{18}$$

After the estimates $\mathbf{R}(k)$ and $\mathbf{p}(k)$ are updated as:

$$\mathbf{R}(k) = \lambda\mathbf{R}(k-1) + \hat{\boldsymbol{\varphi}}(k)\hat{\boldsymbol{\varphi}}^T(k), \tag{19}$$

$$\mathbf{p}(k) = \lambda\mathbf{p}(k-1) + \hat{\boldsymbol{\varphi}}(k)y(k), \tag{20}$$

the normal equation (18) can be solved by a single Gauss-Seidel cycle as follows

$$\hat{\theta}_i(k) = \left[p_i(k) - \sum_{j=1}^{i-1} R_{ij}(k)\hat{\theta}_j(k) \sum_{j=i+1}^{M} R_{ij}(k)\hat{\theta}_j(k-1) \right] \Big/ R_{ii}(k), \tag{21}$$

for $i = 1, 2, \ldots, M$, where $R_{ij}(k)$, $p_i(k)$, and $\hat{\theta}_i(k)$ are the elements of $\mathbf{R}(k)$, $\mathbf{p}(k)$, and $\hat{\boldsymbol{\theta}}(k)$. Thus, the computational burden of the RGS algorithm is lower than that of the RLS. The RGS algorithm is implemented using (19), (20), and (21) in a sampling interval, recursively [58]. The implementation steps and computational complexity of the RGS and RLS algorithms for identifying nonlinear Wiener output error systems are summarised in Table I and Table II, respectively.

TABLE I. IMPLEMENTATION AND COMPUTATIONAL COMPLEXITY OF RGS ALGORITHM FOR WIENER MODELS.

Implementation Steps of RGS Algorithm:	**Number of Operations:**	
Initial values: $\mathbf{R}(0) = \alpha\mathbf{I}_{M\times M}$, $\alpha > 0$, $\mathbf{p}(0) = \mathbf{0}_{M\times 1}$	Multiplications and Divisions	Summations and Subtractions
Estimate: $\hat{x}(k) = \hat{\boldsymbol{\varphi}}_a^T(k)\hat{\boldsymbol{\theta}}_a(k-1)$	$n_a + n_b$	$n_a + n_b - 1$
Compute: $\hat{x}^2(k), \ldots, \hat{x}^{n_c}(k)$	$n_c - 1$	-
$\mathbf{R}(k) = \lambda\mathbf{R}(k-1) + \hat{\boldsymbol{\varphi}}(k)\hat{\boldsymbol{\varphi}}^T(k)$	$2M^2$	M^2
$\mathbf{p}(k) = \lambda\mathbf{p}(k-1) + \hat{\boldsymbol{\varphi}}(k)y(k)$	$2M$	M
$\hat{\theta}_i(k) = \left[p_i(k) - \sum_{j=1}^{i-1} R_{ij}(k)\hat{\theta}_i(k) - \sum_{j=i+1}^{M} R_{ij}(k)\hat{\theta}_i(k-1) \right] \Big/ R_{ii}(k)$, $i = 1, 2, \ldots, M$, $M = n_a + n_b + n_c$.	M^2	$M(M-1)$
Total Number of Operations:	$3M^2 + 2M + n_a + n_b + n_c - 1$	$2M^2 + n_a + n_b - 1$
In simulation example: $M = n_a + n_b + n_c = 2 + 2 + 3 = 7$	167	101

TABLE II. IMPLEMENTATION AND COMPUTATIONAL COMPLEXITY OF RLS ALGORITHM FOR WIENER MODELS.

Implementation Steps of RLS Algorithm:	**Number of Operations:**	
Initial values: $\mathbf{P}(0)=\mathbf{R}^{-1}(0)=\beta\mathbf{I}_{M\times M}$, $\beta>0$	Multiplications and Divisions	Summations and Subtractions
Estimate: $\hat{x}(k)=\hat{\boldsymbol{\varphi}}_a^T(k)\hat{\boldsymbol{\theta}}_a(k-1)$	n_a+n_b	n_a+n_b-1
Compute: $\hat{x}^2(k),\ldots,\hat{x}^{n_c}(k)$	n_c-1	-
$e(k)=y(k)-\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(k)\hat{\boldsymbol{\theta}}(k-1)$	M	M
$\mathbf{q}(k)=\mathbf{P}(k-1)\hat{\boldsymbol{\varphi}}(k)$	M^2	$M(M-1)$
$\mathbf{K}(k)=\mathbf{q}(k)/[\lambda+\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(k)\mathbf{q}(k)]$	$2M$	M
$\hat{\boldsymbol{\theta}}(k)=\hat{\boldsymbol{\theta}}(k-1)+\mathbf{K}(k)e(k)$	M	M
$\mathbf{P}(k)=[\mathbf{P}(k-1)-\mathbf{K}(k)\mathbf{q}^{\mathrm{T}}(k)]/\lambda$	$2M^2$	M^2
Total Number of Operations:	$3M^2+4M+n_a+n_b+n_c-1$	$2M^2+2M+n_a+n_b-1$
In simulation example: $M=n_a+n_b+n_c=2+2+3=7$	181	115

IV. CONVERGENCE ANALYSIS OF RGS ALGORITHM

Decomposing the correlation matrix as $\mathbf{R}(k)=\mathbf{R}_L(k)+\mathbf{R}_D(k)+\mathbf{R}_U(k)$, the RGS iteration (21) is rewritten as follows [58]

$$\hat{\boldsymbol{\theta}}(k)=\hat{\boldsymbol{\theta}}(k-1)+[\mathbf{R}_L(k)+\mathbf{R}_D(k)]^{-1}[\mathbf{p}(k)-\mathbf{R}(k)\hat{\boldsymbol{\theta}}(k-1)]. \quad (22)$$

Defining the parameter errors as

$$\tilde{\boldsymbol{\theta}}(k)=\hat{\boldsymbol{\theta}}(k)-\boldsymbol{\theta}, \quad (23)$$

the following iteration is obtained from (22)

$$\tilde{\boldsymbol{\theta}}(k)=\{\mathbf{I}-[\mathbf{R}_L(k)+\mathbf{R}_D(k)]^{-1}\mathbf{R}(k)\}\tilde{\boldsymbol{\theta}}(k-1)+ \\ +[\mathbf{R}_L(k)+\mathbf{R}_D(k)]^{-1}[\mathbf{p}(k)-\mathbf{R}(k)\boldsymbol{\theta}]. \quad (24)$$

Taking into account (11) and (18), the vector quantity in the second term of (24) is reduced to

$$\mathbf{p}(k)-\mathbf{R}(k)\boldsymbol{\theta}=\sum_{i=1}^{k}\lambda^{k-i}\hat{\boldsymbol{\varphi}}(i)[y(i)-\hat{\boldsymbol{\varphi}}^T(i)\boldsymbol{\theta}]= \\ =\sum_{i=1}^{k}\lambda^{k-i}\hat{\boldsymbol{\varphi}}(i)v(i). \quad (25)$$

Assuming that λ is close to 1 and k is high enough; the following assumption can be used

$$\frac{1}{k}\sum_{i=1}^{k}\lambda^{k-i}\hat{\boldsymbol{\varphi}}(i)v(i)\cong E\{\hat{\boldsymbol{\varphi}}(k)v(k)\}. \quad (26)$$

Because the noise signal $v(k)$ is not correlated with the other signals, the correlation vector in (26) becomes a zero vector as follows

$$E\{\hat{\boldsymbol{\varphi}}(k)v(k)\}=\begin{bmatrix} -E\{\hat{x}(k-1)v(k)\} \\ \vdots \\ -E\{\hat{x}(k-n_a)v(k)\} \\ E\{u(k-1)v(k)\} \\ \vdots \\ E\{u(k-n_b)v(k)\} \\ E\{v(k)\} \\ E\{\hat{x}^2(k)v(k)\} \\ \vdots \\ E\{\hat{x}^{n_c}(k)v(k)\} \end{bmatrix}=\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (27)$$

and thus the expected value of (24) is reduced to

$$E\{\tilde{\boldsymbol{\theta}}(k)\}=E\{\mathbf{I}-[\mathbf{R}_L(k)+\mathbf{R}_D(k)]^{-1}\mathbf{R}(k)\tilde{\boldsymbol{\theta}}(k-1)\}. \quad (28)$$

Considering that the estimate $\mathbf{R}(k)$ converges to its deterministic value, i.e., $E\{\mathbf{R}(k)\}=\mathbf{R}$, the iteration in (28) becomes

$$E\{\tilde{\boldsymbol{\theta}}(k)\}=\{\mathbf{I}-[\mathbf{R}_L+\mathbf{R}_D]^{-1}\mathbf{R}\}E\{\tilde{\boldsymbol{\theta}}(k-1)\}. \quad (29)$$

A positive definite matrix $\mathbf{R}$ allows all eigenvalues of the matrix $\{\mathbf{I}-[\mathbf{R}_L+\mathbf{R}_D]^{-1}\mathbf{R}\}$ to be less than 1 and, therefore, the RGS algorithm converges [58]. Consequently, $E\{\tilde{\boldsymbol{\theta}}(k)\}$ converges to zero vector, thereby letting $E\{\hat{\boldsymbol{\theta}}(k)\}=\boldsymbol{\theta}$, due to (23). This result shows that the RGS algorithm gives unbiased estimates without considering the structure of the noise model.

V. COMPUTER SIMULATION

The following Wiener nonlinear output error model is used in computer simulations:

$$x(k)=\frac{0.4z^{-1}+0.5z^{-2}}{1-0.9z^{-1}+0.8z^{-2}}u(k), \quad (30)$$

$$y(k)=f[x(k)]+v(k), \quad (31)$$

$$f[x(k)]=-0.1+x(k)-0.4x^2(k)+0.3x^3(k). \quad (32)$$

The vectors $\boldsymbol{\theta}$, $\hat{\boldsymbol{\theta}}(k)$, and $\hat{\boldsymbol{\varphi}}(k)$ are formed as follows:

$$\boldsymbol{\theta}=[a_1 \quad a_2 \quad b_1 \quad b_2 \quad c_0 \quad c_2 \quad c_3]^T= \\ =[-0.9 \quad 0.8 \quad 0.4 \quad 0.5 \quad -0.1 \quad -0.4 \quad 0.3]^T, \quad (33)$$

$$\hat{\boldsymbol{\theta}}(k)=[\hat{a}_1(k) \quad \hat{a}_2(k) \quad \hat{b}_1(k) \quad \hat{b}_2(k) \quad \hat{c}_0(k) \quad \hat{c}_2(k) \quad \hat{c}_3(k)]^T, \quad (34)$$

$$\hat{\boldsymbol{\varphi}}(k)=[-\hat{x}(k-1) \quad -\hat{x}(k-2) \quad u(k-1) \quad u(k-2) \quad 1 \quad \hat{x}^2(k) \quad \hat{x}^3(k)]^T. \quad (35)$$

A two-level signal with amplitude ±0.5 and period 100 samples was used as input signal.

In the first simulation, a white measurement noise $w(k)$ with variance $\sigma_w^2=0.15^2$ is used. The corresponding noise-to-signal ratio (NSR) is computed as about

$\%\,\mathrm{NSR} = 100 \times \sqrt{\sigma_w^2 / \sigma_{\bar{x}}^2} \cong \%\,24.4$. The following initial values are used for RGS: $\mathbf{R}(0) = \mathbf{I}_{(7\times7)}$, $\mathbf{p}(0) = \mathbf{0}_{(7\times1)}$, and for RLS: $\mathbf{R}^{-1}(0) = \mathbf{I}_{(7\times7)}$. The initial parameter estimates are taken as $\boldsymbol{\theta}(0) = \mathbf{0}_{(7\times1)}$, and the forgetting factor $\lambda = 0.9995$ is used in the both algorithms. The parameter estimates obtained are plotted in Fig. 2.

In the second simulation, a coloured measurement noise sequence $v(k)$ with its variance computed as $\sigma_v^2 \cong 0.256^2$ is contaminated to the output measurements. The coloured noise sequence $v(k)$ is obtained using the following Auto Regressive Moving Average (ARMA) model, which has the same white noise input $w(k)$ in the first simulation,

$$v(k) = 0.65v(k-1) - 0.4v(k-2) + w(k) + \\ + 0.5w(k-1) + 0.2w(k-2), \tag{36}$$

where the corresponding NSR is computed as about $\%\,\mathrm{NSR} = 100 \times \sqrt{\sigma_v^2 / \sigma_{\bar{x}}^2} \cong \%\,40.74$. The same initial values of the algorithms used in the first simulation are also used in the second simulation. The parameter estimations obtained are shown in Fig. 3.

The estimation errors for the algorithms used are illustrated in Fig. 4; these errors are computed using $\delta(k) = \| \hat{\theta}(k) - \theta \| / \| \theta \|$ and $\| x \| = \sqrt{x^T x}$.

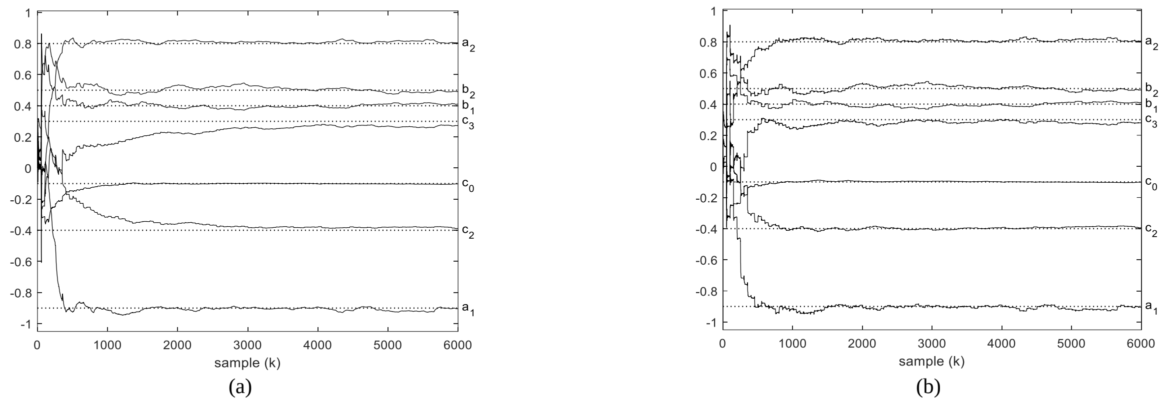

Fig. 2. (a) RGS parameter estimations with white measurement noise; (b) RLS parameter estimations with white measurement noise.

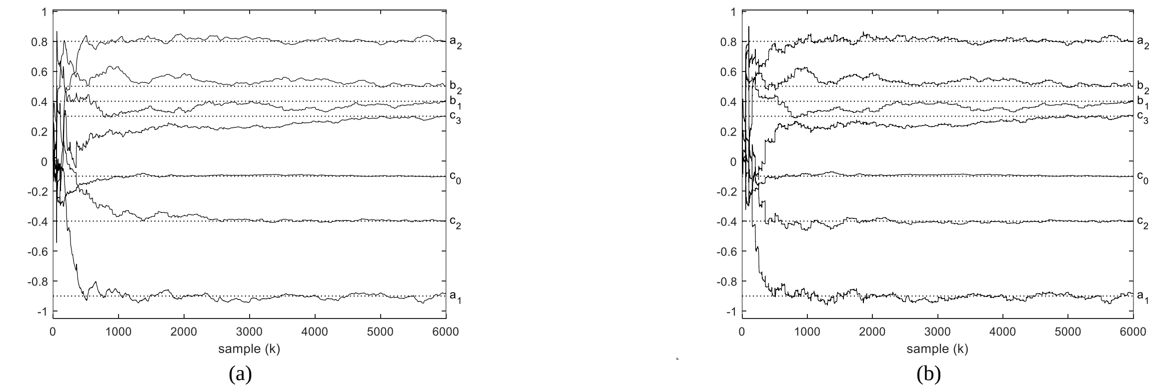

Fig. 3. (a) RGS parameter estimations with coloured measurement noise; (b) RLS parameter estimations with coloured measurement noise.

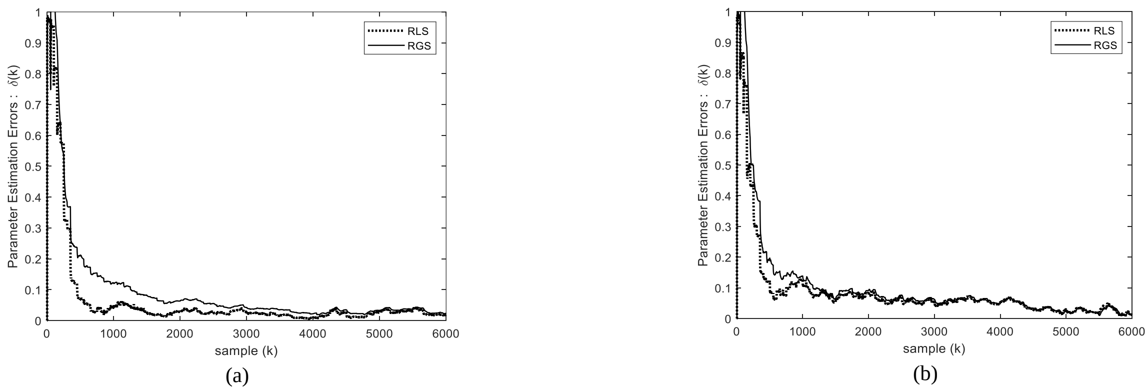

Fig. 4. (a) Parameter estimation errors for white measurement noise; (b) Parameter estimation errors for coloured measurement noise.

Some numerical values of the obtained results are also summarised in Tables III and IV. Considering the obtained results given in Figs. 2–4, Table III, and Table IV, it is seen that the auxiliary model-based RGS algorithm gives unbiased parameter estimations, even if the measurement noise is coloured, and produces close and comparable results to the RLS results with lower computational complexity as seen in Tables I and II.

TABLE III. ESTIMATED PARAMETERS AND THEIR ERRORS WITH WHITE MEASUREMENT NOISE.

Algorithm	k	$\hat{a}_1(k)$	$\hat{a}_2(k)$	$\hat{b}_1(k)$	$\hat{b}_2(k)$	$\hat{c}_0(k)$	$\hat{c}_2(k)$	$\hat{c}_3(k)$	$\delta(k)\%$
RGS algorithm with white measurement noise	50	0.0219	0.0714	0.1118	0.1159	0.2529	0.0197	0.0051	97.1050
	200	-0.2957	0.4461	0.5294	0.7162	-0.2584	0.1033	0.1178	63.8929
	500	-0.9254	0.8322	0.4335	0.5198	-0.1469	-0.1991	0.0776	21.1951
	1000	-0.9127	0.8146	0.4167	0.5064	-0.1119	-0.3149	0.1462	12.2318
	2000	-0.8954	0.8146	0.3885	0.5319	-0.1016	-0.3590	0.2264	6.3269
	3000	-0.8997	0.8107	0.3796	0.5348	-0.0994	-0.3777	0.2607	4.2245
	4000	-0.8925	0.7982	0.4057	0.4965	-0.1013	-0.3793	0.2777	2.2038
	5000	-0.9171	0.8083	0.4119	0.4772	-0.1015	-0.3811	0.2698	3.2905
	6000	-0.9045	0.8046	0.4112	0.4907	-0.1027	-0.3881	0.2721	2.3597
RLS algorithm with white measurement noise	50	0.0251	0.0629	0.1224	0.1224	0.2448	0.0022	0.0003	96.8675
	200	-0.3246	0.4696	0.5841	0.6845	-0.2407	0.0688	-0.0510	64.0971
	500	-0.9047	0.7519	0.4162	0.4545	-0.1159	-0.3578	0.2351	7.1743
	1000	-0.9150	0.8121	0.4067	0.4997	-0.0993	-0.4049	0.2437	4.1260
	2000	-0.8921	0.8107	0.3821	0.5305	-0.0965	-0.4020	0.2836	2.8396
	3000	-0.9000	0.8087	0.3761	0.5315	-0.0965	-0.4031	0.2961	2.8100
	4000	-0.8914	0.7976	0.4030	0.4972	-0.0999	-0.3939	0.2995	0.7965
	5000	-0.9164	0.8081	0.4114	0.4770	-0.1007	-0.3897	0.2825	2.5755
	6000	-0.9052	0.8056	0.4108	0.4911	-0.1021	-0.3939	0.2787	1.8824
True Parameter Values:		-0.9000	0.8000	0.4000	0.5000	-0.1000	-0.4000	0.3000	-

TABLE IV. ESTIMATED PARAMETERS AND THEIR ERRORS WITH COLOURED MEASUREMENT NOISE.

Algorithm	k	$\hat{a}_1(k)$	$\hat{a}_2(k)$	$\hat{b}_1(k)$	$\hat{b}_2(k)$	$\hat{c}_0(k)$	$\hat{c}_2(k)$	$\hat{c}_3(k)$	$\delta(k)\%$
RGS algorithm with coloured measurement noise	50	-0.0632	-0.0085	0.1095	0.1085	0.2582	0.0941	0.0264	98.0683
	200	-0.2911	0.4935	0.4671	0.7430	-0.2280	0.0519	0.2876	59.4411
	500	-0.9242	0.8332	0.4159	0.5381	-0.1505	-0.2210	0.0893	19.7140
	1000	-0.8763	0.8149	0.3079	0.6304	-0.1018	-0.3685	0.1841	13.8559
	2000	-0.8972	0.8315	0.3378	0.5839	-0.1004	-0.3637	0.2328	9.1463
	3000	-0.9381	0.8236	0.3893	0.5129	-0.0945	-0.3883	0.2263	6.1031
	4000	-0.8941	0.8071	0.3606	0.5459	-0.0948	-0.3967	0.2662	4.8198
	5000	-0.8797	0.7761	0.3812	0.4944	-0.0970	-0.3942	0.2989	2.5807
	6000	-0.8840	0.7998	0.4054	0.5019	-0.1040	-0.3970	0.2989	1.2160
RLS algorithm with coloured measurement noise	50	-0.0094	0.0574	0.1283	0.1283	0.2566	0.0340	0.0086	96.0782
	200	-0.4220	0.5553	0.5457	0.6935	-0.1955	-0.0796	0.0338	49.9847
	500	-0.9015	0.7832	0.4233	0.5086	-0.1062	-0.4044	0.1274	12.0420
	1000	-0.8836	0.8151	0.3076	0.6236	-0.0829	-0.4594	0.2227	12.6796
	2000	-0.8899	0.8251	0.3345	0.5870	-0.0949	-0.3937	0.2532	8.3715
	3000	-0.9355	0.8190	0.3875	0.5113	-0.0916	-0.4034	0.2389	5.1929
	4000	-0.8953	0.8079	0.3584	0.5472	-0.0934	-0.4052	0.2731	4.7796
	5000	-0.8750	0.7719	0.3813	0.4934	-0.0962	-0.3996	0.3076	2.9773
	6000	-0.8846	0.8007	0.4051	0.5023	-0.1034	-0.4008	0.3030	1.1727
True Parameter Values:		-0.9000	0.8000	0.4000	0.5000	-0.1000	-0.4000	0.3000	-

VI. CONCLUSIONS

The Recursive Gauss-Seidel (RGS) algorithm was presented for Wiener model identification. The least-squares-based RGS algorithm has reduced computational burden because of its implementation using a one-step Gauss-Seidel iteration in a sampling period. The RGS algorithm uses the output estimations of an auxiliary model instead of unmeasurable inner variables. Also, by performing a stochastic convergence analysis, it was shown that the RGS algorithm gives unbiased parameter estimates even if the measurement noise is coloured. The convergence and complexity of the presented algorithm were studied and evaluated with the popular Recursive Least Squares (RLS) algorithm by computer simulations. The following results were observed:

1. The auxiliary model-based RGS algorithm produced unbiased parameter estimates for coloured measurement noise without considering the noise model structure;
2. The RGS algorithm gives results comparable to those of the RLS algorithm, with a lower computational load.

CONFLICTS OF INTEREST

The author declares that he has no conflict of interest.

REFERENCES

[1] A. Janczak, *Identification of Nonlinear Systems Using Neural Networks and Polynomial Models: A Block-Oriented Approach*. Berlin, Heidelberg: Springer-Verlag, 2005. DOI: 10.1007/b98334.

[2] F. Giri and E.-W. Bai (Editors), *Block-Oriented Nonlinear System Identification*. London: Springer-Verlag, 2010. DOI: 10.1007/978-1-84996-513-2.

[3] G. Mzyk, *Combined Parametric-Nonparametric Identification of Block-Oriented Systems*. Switzerland: Springer International Publishing, 2014. DOI: 10.1007/978-3-319-03596-3.

[4] G. Pajunen, “Adaptive control of Wiener type nonlinear systems”, *Automatica*, vol. 28, no. 4, pp. 781–785, 1992. DOI: 10.1016/0005-1098(92)90037-G.

[5] S. Norquay, A. Palazoglu, and J. A. Romagnoli, “Model predictive control based on Wiener models”, *Chemical Engineering Science*, vol. 53, no 1, pp. 75–84, 1998. DOI: 10.1016/S0009-2509(97)00195-4.

[6] H. H. J. Bloemen, C. T. Chou, T. J. J. van den Boom, V. Verdult, M. Verhaegen, and T. C. Backx, “Wiener model identification and predictive control for dual composition control of a distillation column”, *Journal of Process Control*, vol. 11, no. 6, pp. 601–620, 2001. DOI: 10.1016/S0959-1524(00)00056-1.

[7] J. C. Gómez, A. Jutan, and E. Baeyens, "Wiener model identification and predictive control of a pH neutralisation process", *IEE Proceedings - Control Theory and Applications*, vol. 151, no. 3, pp. 329–338, 2004. DOI: 10.1049/ip-cta:20040438.

[8] A. D. Kalafatis, L. Wang, and W. R. Cluett, "Linearizing feedforward-feedback control of pH processes based on the Wiener model", *Journal of Process Control*, vol. 15, no. 1, pp. 103–112, 2005. DOI: 10.1016/j.jprocont.2004.03.006.

[9] J. Figueroa, J. E. Cousseau, S. Werner, and T. Laakso, "Adaptive control of a Wiener type system: Application of a pH Neutralization reactor", *International Journal of Control*, vol. 80, no. 2, pp. 231–240, 2007. DOI: 10.1080/00207170600977815.

[10] M. M. Arefi, A. Montazeri, J. Poshtan, and M. R. Jahed-Motlagh, "Wiener-neural identification and predictive control of a more realistic plug-flow tubular reactor", *Chemical Engineering Journal*, vol. 138, nos. 1–3, pp. 274–282, 2008. DOI: 10.1016/j.cej.2007.05.044.

[11] G. Shafiee, M. M. Arefi, M. R. Jahed-Motlagh, and A. A. Jalali, "Nonlinear predictive control of a polymerization reactor based on piecewise linear Wiener model", *Chemical Engineering Journal*, vol. 143, nos. 1–3, pp. 282–292, 2008. DOI: 10.1016/j.cej.2008.05.013.

[12] P. Cao and X. Luo, "Soft sensor model derived from Wiener model structure: Modeling and identification", *Chinese Journal of Chemical Engineering*, vol. 22, no. 5, pp. 538–548, 2014. DOI: 10.1016/S1004-9541(14)60077-X.

[13] P. Yuan, B. Zhang, and Z. Mao, "A self-tuning control method for Wiener nonlinear systems and its application to process control problems", *Chinese Journal of Chemical Engineering*, vol. 25, no. 2, pp. 193–201, 2017. DOI: 10.1016/j.cjche.2016.07.003.

[14] M. Kazemi and M. M. Arefi, "Nonlinear generalized minimum variance control and control performance assessment of nonlinear systems based on a Wiener model", *Transactions of the Institute of Measurement and Control*, vol. 40, no. 5, pp. 1538–1553, 2018. DOI: 10.1177/0142331216685395.

[15] I. Aliskan, "Adaptive model predictive control for Wiener nonlinear systems", *Iran. J. Sci. Technol. Trans. Electr. Eng.*, vol. 43, pp. 361–377, 2019. DOI: 10.1007/s40998-018-0159-0.

[16] Z. Wang, L. Zhao, and X. Luo, "Wiener structure based adaptive control for dynamic processes with approximate monotonic nonlinearities", *Journal of the Franklin Institute*, vol. 357, no. 18, pp. 13534–13551, 2020. DOI: 10.1016/j.jfranklin.2020.10.006.

[17] A. Janczak, "Instrumental variables approach to identification of a class of MIMO Wiener systems", *Nonlinear Dynamics*, vol. 48, pp. 275–284, 2007. DOI: 10.1007/s11071-006-9088-y.

[18] R. Pupeikis, "Parametric identification of linear systems followed by non-invertible piecewise nonlinearities", *International Frontier Science Letters*, vol. 6, pp. 16–27, 2015. DOI: 10.18052/www.scipress.com/IFSL.6.16.

[19] A. Janczak and J. Korbicz, "Two-stage instrumental variables identification of polynomial Wiener systems with invertible nonlinearities", *Int. J. Appl. Math. Comput. Sci.*, vol. 29, no. 3, pp. 571–580, 2019. DOI: 10.2478/amcs-2019-0042.

[20] J. Vörös, "Parameter identification of Wiener systems with discontinuous nonlinearities", *Systems & Control Letters*, vol. 44, no. 5, pp. 363–372, 2001. DOI: 10.1016/S0167-6911(01)00155-4.

[21] J. Vörös, "Modeling and identification of Wiener systems with two-segment nonlinearities", *IEEE Trans. Control Syst. Technol.*, vol. 11, no. 2, pp. 253–257, 2003. DOI: 10.1109/TCST.2003.809238.

[22] J. Vörös, "Parameter identification of Wiener systems with multisegment piecewise-linear nonlinearities", *Syst. Control Lett.*, vol. 56, no. 2, pp. 99–105, 2007. DOI: 10.1016/j.sysconle.2006.08.001.

[23] D. Wang and F. Ding, "Least squares based and gradient based iterative identification for Wiener nonlinear systems", *Signal Process.*, vol. 91, no. 5, pp. 1182–1189, 2011. DOI: 10.1016/j.sigpro.2010.11.004.

[24] M. Liu, Y. Xiao, and R. Ding, "Iterative identification algorithm for Wiener nonlinear systems using the Newton method", *Applied Mathematical Modelling*, vol. 37, no. 9, pp. 6584–6591, 2013. DOI: 10.1016/j.apm.2013.01.025.

[25] L. Zhou, X. Li, and F. Pan, "Gradient based iterative parameter identification for Wiener nonlinear systems", *Applied Mathematical Modelling*, vol. 37, nos. 16–17, pp. 8203–8209, 2013. DOI: 10.1016/j.apm.2013.03.005.

[26] L. Zhou, X. Li, and F. Pan, "Least-squares-based iterative identification algorithm for Wiener nonlinear systems", *Journal of Applied Mathematics*, vol. 2013, art. ID 565841, pp. 1–6, 2013. DOI: 10.1155/2013/565841.

[27] W. Xiong, J. Ma, and R. Ding, "An iterative numerical algorithm for modeling a class of Wiener nonlinear systems", *Applied Mathematics Letters*, vol. 26, no. 4, pp. 487–493, 2013. DOI: 10.1016/j.aml.2012.12.001.

[28] L. Zhou, X. Li, and F. Pan, "Gradient-based iterative identification for MISO Wiener nonlinear systems: Application to a glutamate fermentation process", *Applied Mathematics Letters*, vol. 26, no. 8, pp. 886–892, 2013. DOI: 10.1016/j.aml.2013.03.015.

[29] F. Ding, J. Ma, and Y. Xiao, "Newton iterative identification for a class of output nonlinear systems with moving average noises", *Nonlinear Dynamics*, vol. 74, pp. 21–30, 2013. DOI: 10.1007/s11071-013-0943-3.

[30] L. Zhou, X. Li, and F. Pan, "Gradient-based iterative identification for Wiener nonlinear systems with non-uniform sampling", *Nonlinear Dynamics*, vol. 76, pp. 627–634, 2014. DOI: 10.1007/s11071-013-1156-5.

[31] J. Chen, X. Lu, and R. Ding, "Gradient-based iterative algorithm for Wiener systems with saturation and dead-zone nonlinearities", *Journal of Vibration and Control*, vol. 20, no. 4, pp. 634–640, 2014. DOI: 10.1177/1077546312466563.

[32] L. Zhou, X. Li, H. Xu, and P. Zhu, "Gradient-based iterative identification for Wiener nonlinear dynamic systems with moving average noises", *Algorithms*, vol. 8, no. 3, pp. 712–722, 2015. DOI: 10.3390/a8030712.

[33] İ. Alışkan, "Gradient based identification of Wiener systems" (in Turkish), *Pamukkale University Journal of Engineering Sciences*, vol. 24, no. 8, pp. 1418–1424, 2018. DOI: 10.5505/pajes.2018.65392.

[34] T. Wigren, "Recursive prediction error identification using the nonlinear Wiener model", *Automatica*, vol. 29, no. 4, pp. 1011–1025, 1993. DOI: 10.1016/0005-1098(93)90103-Z.

[35] J. Vörös, "Recursive identification of time-varying Wiener systems with polynomial non-linearities", *International Journal of Automation and Control*, vol. 2, no. 1, pp. 90–98, 2008. DOI: 10.1504/IJAAC.2008.020421.

[36] J. Vörös, "Recursive identification of Wiener systems with two-segment polynomial nonlinearities", *Journal of Electrical Engineering*, vol. 59, no. 1, pp. 40–44, 2008.

[37] J. Vörös, "Recursive identification of systems with noninvertible output nonlinearities", *Informatica*, vol. 21, no. 1, pp. 139–148, 2010. DOI: 10.15388/Informatica.2010.278.

[38] B. Chen, Y. Zhu, J. Hu, and J. C. Principe, "Stochastic gradient identification of Wiener system with maximum mutual information criterion", *IET Signal Process*., vol. 5, no. 6, pp. 589–597, 2011. DOI: 10.1049/iet-spr.2010.0171.

[39] J. Yao, Y. Huang, and Z. Ji, "Parameter identification of Wiener models by multi-innovation algorithms", *International Journal of Control, Automation and Systems*, vol. 11, pp. 1170–1176, 2013. DOI: 10.1007/s12555-012-0261-4.

[40] J. Li, C. Hua, Y. Tang, and X. Guan, "Stochastic gradient with changing forgetting factor-based parameter identification for Wiener systems", *Applied Mathematics Letters*, vol. 33, pp. 40–45, 2014. DOI: 10.1016/j.aml.2014.03.003.

[41] Y. Hu, B. Liu, Q. Zhou, and C. Yang, "Recursive extended least squares parameter estimation for Wiener nonlinear systems with moving average noises", *Circuits, Systems, and Signal Processing*, vol. 33, pp. 655–664, 2014. DOI: 10.1007/s00034-013-9652-x.

[42] A. Atitallah, S. Bedoui, and K. Abderrahim, "Identification of Wiener time delay systems based on hierarchical gradient approach", *IFAC-PapersOnLine*, vol. 48, no. 1, pp. 403–408, 2015. DOI: 10.1016/j.ifacol.2015.05.040.

[43] F. Ding, X. Liu, and M. Liu, "The recursive least squares identification algorithm for a class of Wiener nonlinear systems", *Journal of the Franklin Institute*, vol. 353, no. 7, pp. 1518–1526, 2016. DOI: 10.1016/j.jfranklin.2016.02.013.

[44] R. Liu, T. Pan, S. Chen, and Z. Li, "Identification of non-uniformly sampled Wiener systems with dead-zone non-linearities", *Mathematical and Computer Modelling of Dynamical Systems*, vol. 23, no. 6, pp. 595–612, 2017. DOI: 10.1080/13873954.2016.1278392.

[45] S. Jing, T. Pan, and Z. Li, "Variable knot-based spline approximation recursive Bayesian algorithm for the identification of Wiener systems with process noise", *Nonlinear Dynamics*, vol. 90, pp. 2293–2303, 2017. DOI: 10.1007/s11071-017-3803-8.

[46] F. Ding, Y. Wang, J. Dai, Q. Li, and Q. Chen, "A recursive least squares parameter estimation algorithm for output nonlinear autoregressive systems using the input-output data filtering", *Journal of the Franklin Institute*, vol. 354, no. 15, pp. 6938–6955, 2017. DOI: 10.1016/j.jfranklin.2017.08.009.

[47] M. Kazemi and M. M. Arefi, "A fast iterative recursive least squares algorithm for Wiener model identification of highly nonlinear systems", *ISA Transactions*, vol. 67, pp. 382–388, 2017. DOI: 10.1016/j.isatra.2016.12.002.

[48] A. Atitallah, S. Bedoui, and K. Abderrahim, "Multistage for identification of Wiener time delay systems based on hierarchical gradient approach", *Mathematical and Computer Modelling of Dynamical Systems*, vol. 23, no. 2, pp. 222–239, 2017. DOI: 10.1080/13873954.2016.1242143.

[49] A. Atitallah, S. Bedoui, and K. Abderrahim, "System identification: Parameter and time-delay estimation for Wiener nonlinear systems with delayed input", *Transactions of the Institute of Measurement and Control*, vol. 40, no. 3, pp. 1035–1045, 2018. DOI: 10.1177/0142331216674772.

[50] J. Li, W. X. Zheng, J. Gu, and L. Hua, "A recursive identification algorithm for Wiener nonlinear systems with linear state-space subsystem", *Circuits, Systems, and Signal Processing*, vol. 37, pp. 2374–2393, 2018. DOI: 10.1007/s00034-017-0682-7.

[51] S. Dong, L. Yu, W.-A. Zhang, and B. Chen, "Recursive identification for Wiener non-linear systems with non-stationary disturbances", *IET Control Theory & Applications*, vol. 13, no. 16, pp. 2648–2657, 2019. DOI: 10.1049/iet-cta.2018.6413.

[52] R. Pupeikis, "Tracking coefficients of a nonstationary system, followed by static nonlinearity jointly with the time delay", *International Journal of Adaptive Control and Signal Processing*, vol. 34, no. 2, pp. 210–227, 2020. DOI: 10.1002/acs.3080.

[53] A. Atitallah, S. Bedoui, and K. Abderrahim, "Joint parameter and time-delay identification algorithm and its convergence analysis for Wiener time-delay systems", *Circuits, Systems, and Signal Processing*, vol. 39, pp. 199–222, 2020. DOI: 10.1007/s00034-019-01191-1.

[54] Z. Wang, H. An, and X. Luo, "Adaptive filtering-based recursive identification for time-varying Wiener output-error systems with unknown noise statistics", *Journal of the Franklin Institute*, vol. 357, no. 2, pp. 1280–1298, 2020. DOI: 10.1016/j.jfranklin.2019.11.005.

[55] S. Dong, L. Yu, W.-A. Zhang, and B. Chen, "Robust hierarchical identification of Wiener systems in the presence of dynamic disturbances", *Journal of the Franklin Institute*, vol. 357, no. 6, pp. 3809–3834, 2020. DOI: 10.1016/j.jfranklin.2020.02.009.

[56] L. Li *et al.*, "A novel recursive learning estimation algorithm of Wiener systems with quantized observations", *ISA Transactions*, vol. 112, pp. 23–34, 2021. DOI: 10.1016/j.isatra.2020.11.032.

[57] L. Ljung, *System Identification: Theory for the User*, 2nd ed., New Jersey, USA: Prentice-Hall, 1999.

[58] M. Hatun and O. H. Koçal, "Recursive Gauss-Seidel algorithm for direct self-tuning control", *International Journal of Adaptive Control and Signal Processing*, vol. 26, no. 5, pp. 435–450, 2012. DOI: 10.1002/acs.1296.