Investigation of the Spectrum Decomposition Technique for Group Velocity Measurement of Lamb Waves Propagating in CFRP Composite Plate

L. Draudviliene[1], L. Mazeika[1]

[1]*Ultrasound Institute, Kaunas University of Technology,*
Studentu St. 50, LT–51368 Kaunas, Lithuania
lina.draudviliene@ktu.lt

Abstract—**The technique for measurement of the group velocity of guided waves was proposed. The technique is based on the spectrum decomposition method and was investigated in the case of A_0 and S_0 modes of Lamb waves propagating in CFRP composite plate. The obtained measurement results were compared with the theoretical dispersion curves obtained by SAFE method.**

Index Terms—**Composite plate, spectrum decomposition technique, guided waves, group velocity.**

I. INTRODUCTION

At recent time the composite materials are used in various transport areas such as aviation and aerospace, boats and yachts or buses. Also they are used in other industrial structures, such as wind turbines, towers and etc. These objects usually should be tested due to safety or economic reasons. The application of the guided waves for inspection of the composite materials is very attractive as they are sensitive to variation of elastic properties or different type of defects.

One of the main parameters of the ultrasonic wave is velocity, which is used to estimate deviation or variation of the properties of the composite materials. Detection of these variations gives information about non – uniformity of elastic properties of the materials or presence of the defects such as delaminations or breaks of fibbers.

The phase and the group velocities of guided waves usually are different and depend essentially on frequency. This phenomenon is called dispersion and it is characterized by phase and group velocities dispersion curves. Therefore measurements of these velocities are complicated. Another feature of guided waves complicating measurement is presence of infinite number of modes. Even at low frequency three fundamental modes exist: asymmetric, symmetric and shear horizontal. All listed above factors complicate analysis of the guided waves signals and the novel measurement techniques enabling to reconstruct the dispersion curves in various frequency ranges are needed.

In previous articles [1], [2] the method for measurement of the group velocity of guided waves based on the application of the spectrum decomposition technique was presented. The spectrum decomposition technique was already used for investigation of ultrasonic signals propagating in attenuating media [3] and for estimation of phase velocity [4]. The proposed method of group velocity measurement was verified using modelled and experiment acquired guided waves signals in aluminium plate. Investigations demonstrated that the spectrum decomposition technique enables with some uncertainties to reconstruct the segments of the group velocity dispersion curves.

The objective of this work was to investigate performance of the proposed group velocity measurement technique in the case of composite materials.

II. THE FINITE ELEMENT MODEL OF THE CFRP PLATE

The thin CFRP (carbon fibber reinforced plastic) plate was selected for analysis. It is unidirectional anisotropic composite. In general, such plates are used as material for manufacturing of components with complicated geometry by gluing different number of plates. In order to obtain signals for analysis of propagation of the A_0 and the S_0 modes in the CFRP plate was simulated using 3D finite element model (Fig. 1) [5]. The geometric parameters of the plate are: the length – 100 mm, the width – 40 mm and the thickness – 0.2mm.

The carbon fibbers were oriented along axis *z*. The density of the CFRP plate was assumed to be 1570 kg/m^3. The elastic coefficients have been defined by the following stiffness matrix:

$$C_{CFRP} = \begin{bmatrix} 13.59 & 6.63 & 5.46 & 0 & 0 & 0 \\ 6.63 & 13.59 & 5.46 & 0 & 0 & 0 \\ 5.46 & 5.46 & 144.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.48 \end{bmatrix}, \text{[GPa]. (1)}$$

The sampling step in the spatial domain was $\Delta x=0.1$mm and $\Delta t=0.1\mu$s in time domain. The A_0 mode was excited by applying the tangential force (Fig. 1, (a)) and the S_0 mode – by applying the normal force (Fig. 1, (b)) to one of the plate edges. The excitation zone was in the centre the one edges of the plate and was 10 mm in length along y axis. The propagating signals of the guided waves modes were modelled during 70 µs time interval. The waveform of the excitation signal was 3 period, 400 kHz burst with the Gaussian envelope.
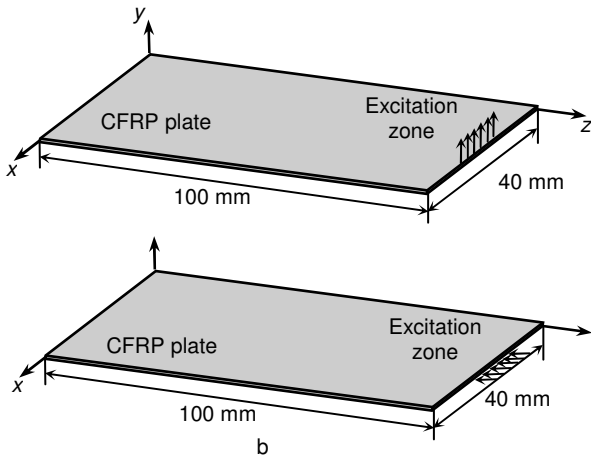

Fig. 1. The finite element model for investigation of the A_0 (a) and the S_0 (b) modes of guided waves propagating in thin CFRP plate.

The distribution of the normal component of the particle velocity on the top surface of the plate was used as the set of the signals for analysis. The B – scan (Fig. 2) of propagating A_0 mode demonstrates that the phase and the group velocities are different and the phase velocity is smaller than the group velocity. The zoomed B – scan of the S_0 mode is presented in Fig. 3. It can be seen that the S_0 mode is about five times faster than the A_0 mode (Fig. 2) and during the same modelling time was several times reflected by the plate edges therefore for better analysis the reflected waves were filtered using 2D Fourier transform.
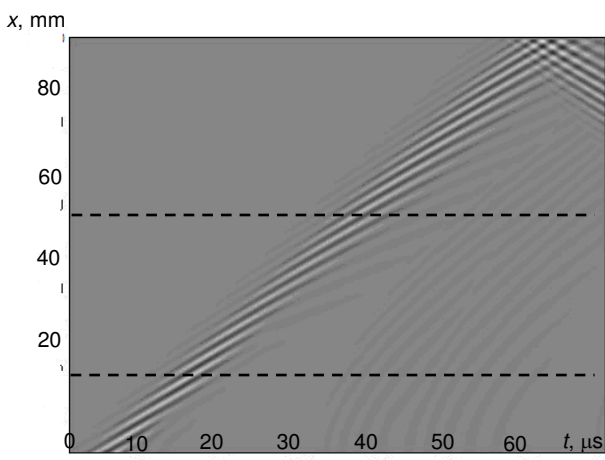

Fig. 2. The B-scan image of the asymmetric A_0 Lamb waves mode propagating in CFRP plate and the investigation distance 10…50 mm from excitation point.
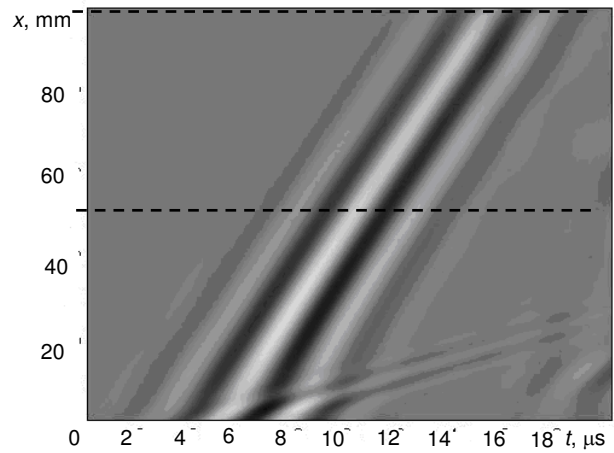

Fig. 3. Zoomed image of the B-scan of the symmetric S_0 Lamb waves mode propagating in CFRP plate and the investigation distance 50…100 mm from excitation point.

III. MEASUREMENT OF THE GROUP VELOCITY

According to proposed algorithm of the group velocity measurement the two signals at two different distances from the excitation zone are selected $u_x(t)$ and $u_{x+\Delta x}(t)$. The waveforms of both A_0 mode signals are presented in Fig. 4 and the waveform of the S_0 mode in Fig. 5.
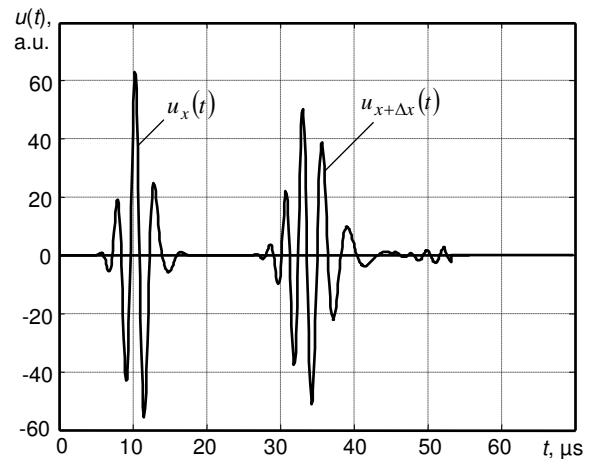

Fig. 4. The two waveforms of the A_0 mode signals measured at distances 10mm (solid line) and 50 mm (dashed line) from excitation zone.
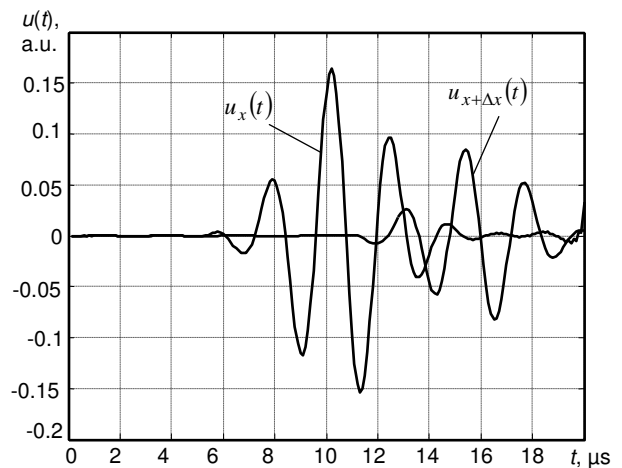

Fig. 5. The two waveforms of the S_0 mode signals measured at distances 50mm (solid line) and 100 mm (dashed line) from excitation zone.

The distances from excitation point in the case of A_0 mode were 10 mm and 50 mm. In the case of S_0 mode the signals were measured at 50 mm to 100 mm in order to avoid influence of the A_0 mode (the weak pattern of which can be observed in the B – scan image presented in Fig. 3).

For estimation of the frequency bandwidth in which signal processing should be performed the frequency spectrums of the propagating signals were estimated using Fourier transform [6]:

$$U_x(f) = \text{FT}[u_x(t)], \qquad (2)$$

$$U_{x+\Delta x}(f) = \text{FT}[u_{x+\Delta x}(t)], \qquad (3)$$

where FT – denotes Fourier transform.

Obtained frequency spectrum of the A_0 mode waves generated in the CFRP plate is presented in Fig. 6 and is in the range of 120 – 680 kHz at – 40 dB levels. Corresponding to frequency bandwidth of analysed signals the central frequencies of the filters used in spectrum decomposition approach were changed from 200 kHz up to 600 kHz with a step of 20 kHz. The frequency spectrum of S_0 mode signals is presented in Fig. 7 and possesses slightly higher frequency bandwidth – from 170 kHz up to 670 kHz at – 40dB levels.
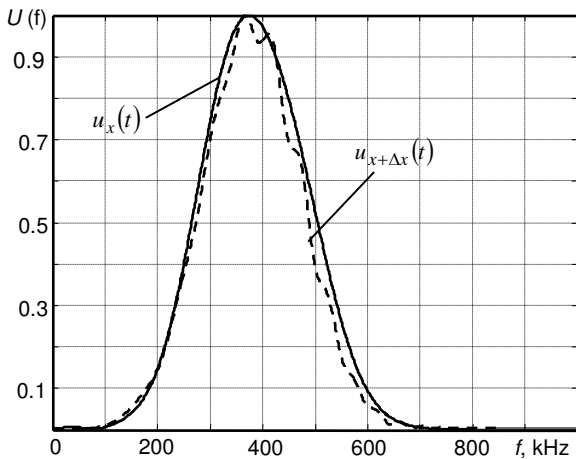

Fig. 6. The frequency spectra of the A_0 mode modeling signals measured at distances 10 mm (solid line) and 50 mm (dashed line) from excitation zone.
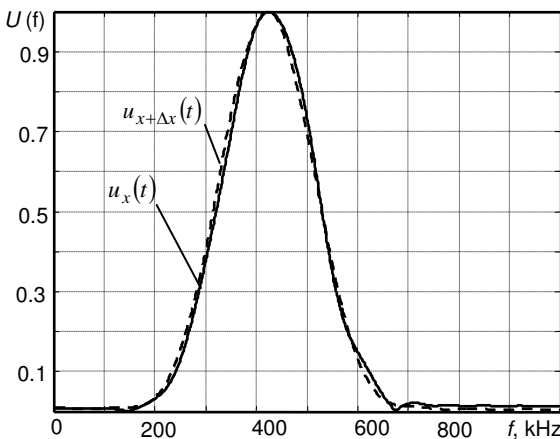

Fig. 7. The frequency spectra of the S_0 mode modeling signals measured at distances 50mm (solid line) and 100 mm (dashed line) from excitation zone.

In this case the central frequencies of the filter were changed from 200 kHz up to 500 kHz with the step of 20 kHz.

Reconstruction of the group velocity dispersion curve according proposed technique [1] is performed in the following steps:

– The filter spectrum is calculated according

$$H(f) = e^{-K \cdot (f - f_n)^2}, \qquad (4)$$

where K is the coefficient which defines the bandwidth of the filter and f_n is the central frequency of the filter;

– The signals measured at two distances are filtered:

$$U_{x,F}(f) = U_x(f) \cdot H(f), \qquad (5)$$

$$U_{x+\Delta x,F}(f) = U_{x+\Delta x}(f) \cdot H(f). \qquad (6)$$

– The waveforms of filtered signals are reconstructed:

$$u_{x,\text{F}}(t) = \text{FT}^{-1}\left[U_{x,F}(f)\right], \qquad (7)$$

$$u_{x+\Delta x,\text{F}}(t) = \text{FT}^{-1}\left[U_{x+\Delta x,F}(f)\right], \qquad (8)$$

where FT^{-1} – denotes inverse Fourier transform.

– The envelope of the signals is calculated using Hilbert transform:

$$u_{x,\text{FH}}(t) = \left|\text{Hilbert}\left[u_{x,\text{F}}(t)\right]\right|, \qquad (9)$$

$$u_{x+\Delta x,\text{FH}}(t) = \left|\text{Hilbert}\left[u_{x+\Delta x,\text{F}}(t)\right]\right|. \qquad (10)$$

– The delay time between two envelopes is estimated using cross-correlation according

$$t_{\Delta x} = \arg \max_t \left\{\text{corr}\left[u_{x,\text{FH}}(t), u_{x+\Delta x,\text{FH}}(t)\right]\right\}. \qquad (11)$$

– The group velocity of the A_0 or S_0 modes is estimated according

$$c_g = \Delta x / t_{\Delta x}, \qquad (12)$$

where Δx is the distance between points were the signals were measured.

However there are two important questions: what bandwidth of the filter should be selected and to which frequency calculated group velocity should be related. It was determined that most correct is to relate group velocity to the central frequency of the filtered signal [2]. More complicated question is selection of the filter bandwidth. As was shown in previous article [2] that the scattering of the results and the frequency ranges in which the group velocity dispersion curve is reconstructed essentially depends on the width of the filter used in spectrum decomposition technique. On the other hand it depends on the shape of the group dispersion curve or in what frequency ranges the measurement was carried out. In order to determine more suitable bandwidth of the filter the calculations were

performed using filters with different bandwidth. For the A_0 mode the filter bandwidth was varied in the range of 20 – 200 kHz and for the S_0 mode – 50 – 200 kHz with the step 20 kHz. The obtained results were compared to the CFRP plate dispersion curves of the guided wave calculated using SAFE method.

The calculation results have demonstrated when is used of the narrowband filter (bandwidth 20 kHz) the dispersion curve is reconstructed almost in whole bandwidth of the signal – 200 – 600 kHz.

However obtained group velocity values are strongly scattered. In opposite, when the filter with wider bandwidth is used (200 kHz) the results are less scattered, but the reasonable reconstruction of dispersion is performed in more narrow frequency bandwidth only from 280 kHz up to 520 kHz. More suitable bandwidth of the filter for A_0 mode was 60 kHz. The results obtained using such filter are presented in Fig. 8. The group velocity dispersion curve of the A_0 mode is reconstructive with some systematic error in the frequency range from 218 kHz up to 584 kHz.

The more suitable bandwidth of the filter in the case of S_0 mode was 150 kHz. The measurement results of the group velocity of S_0 mode using this filter are presented in Fig. 9. It can be seen that the dispersion curve of group velocity of the S_0 mode is reconstructed in frequency ranges 280–477 kHz.
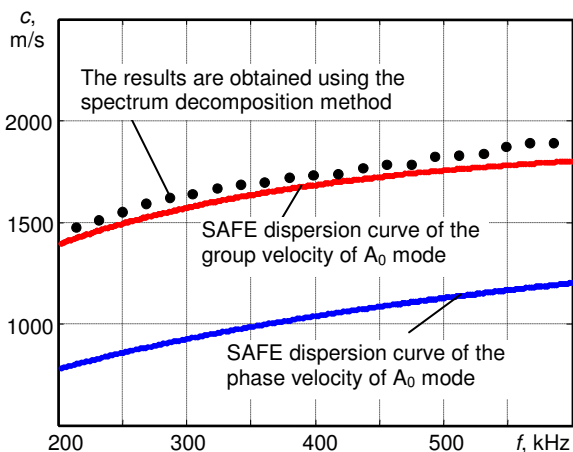

Fig. 8. The group velocity of the A_0 mode of Lamb wave obtained using modelling signals and theoretical (SAFE) dispersion curve.
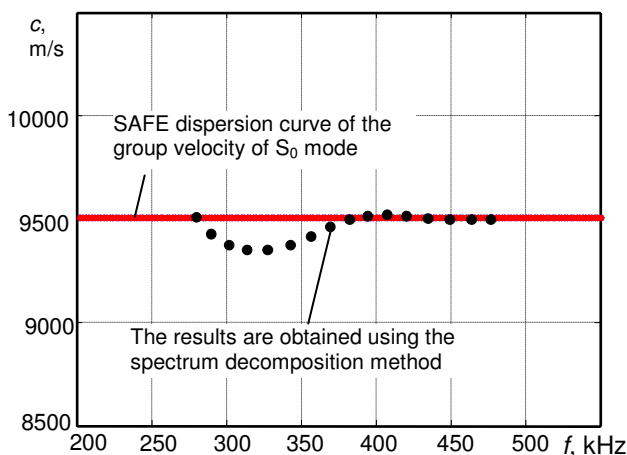

Fig. 9. The group velocity of the S_0 mode of Lamb wave obtained using modelling signals and theoretical (SAFE) dispersion curve.

The obtained results of the investigation demonstrated that the proposed group velocity method based on the spectrum decomposition technique enables reconstruction of the segments of the dispersion curves of the fundamental modes for anisotropic composite materials.

IV. CONCLUSIONS

The investigations demonstrated that the proposed group velocity calculation technique can be applied for more complicated materials and enables at least rough estimation of the dispersion curve. However some systematic errors have been observed which can be related not only to proposed technique but to the modelling approach using which the signals were obtained.

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