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The New Equations of p-n Junction Carrier Injection Level

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***Abstract*—The new equations of minority carrier hole and electron injection levels k_p and k_n valid at high-level injection have been derived. They relate the k_p and k_n and the voltage drop across the p-n junction depletion region U_d. At low U_d, i.e. at low-level injection the obtained equations coincide with well known exponential equations of injection level. However, at high-level injection when U_d becomes high and is close to the potential barrier of junction, the derived equations give increased steepness of k_p and k_n dependence on U_d as compared with the exponential law. The dependences of k_p and k_n of concrete silicon p-n junctions with different impurity concentrations have been analyzed using derived equations of injection level.**

***Index Terms*—*p-n* junction, minority carriers, injection level, high-level injection, integrated circuit.**

I. INTRODUCTION

The p-n junction is still the basic building block of many semiconductor devices. The parameter, which indicates the operating conditions of forward biased p-n junction, is injection level of minority carriers. It is important to know the injection level of analyzed junctions since it determines the characteristics of device and validity of models used for analysis. For example, if injection level is low the voltage drop across the quasi-neutral regions of the p-n junction can be neglected and exponential (Shockley) equation-based models can be used for simulation. If injection level is high, the operating conditions of p-n junction are changed greatly and more complex models should be used for adequate simulation of such devices.

The injection level is ratio of excess minority carrier concentration to equilibrium majority carrier concentration [1]–[3]. Equilibrium majority carrier concentration of real p-n junctions is very close to concentration of impurity, consequently, the injection level of holes and electrons k_p and k_n can be presented as follows:

$$k_p = \frac{p_n(x_n) - p_{n0}}{N_D}, \tag{1}$$

$$k_n = \frac{n_p(-x_p) - n_{p0}}{N_A}, \tag{2}$$

where $p_n(x_n)$ and $n_p(-x_p)$ are hole and electron (minority carrier) boundary concentrations at the edges of the depletion region $x=x_n$ and $x=-x_p$ in n and p region, respectively, p_{n0} and n_{p0} are minority carrier boundary concentrations at the equilibrium when voltage drop across the depletion region U_d=0 and N_D, N_A are impurity (donor and acceptor) concentrations.

The minority carrier boundary concentrations can be expressed by the commonly used exponential boundary conditions:

$$p_n(x_n) = p_{n0} \exp\left(\frac{U_d}{V_T}\right), \tag{3}$$

$$n_p(-x_p) = n_{p0} \exp\left(\frac{U_d}{V_T}\right), \tag{4}$$

where V_T is thermal potential.

Using (1), (2) and (3), (4), knowing that $p_{n0} = n_i^2/N_D$ and $n_{p0} = n_i^2/N_A$, (n_i is intrinsic carrier concentration) the hole and electron injection levels:

$$k_p = \frac{n_i^2}{N_D^2}\left[\exp\left(\frac{U_d}{V_T}\right) - 1\right], \tag{5}$$

$$k_n = \frac{n_i^2}{N_A^2}\left[\exp\left(\frac{U_d}{V_T}\right) - 1\right]. \tag{6}$$

The (5) and (6) are important since they allow us to estimate the injection level of minority carriers at given U_d, and U_d can be related with current of the junction.

Unfortunately, the equations (3), (4) and consequently, equations (5), (6) are valid only at low-level injection when $p_n(x_n) - p_{n0} << N_D$ and $n_p(-x_p) - n_{p0} << N_A$. On the other hand, the junctions of the semiconductor devices especially those used in the integrated circuits and power semiconductor devices operate at high-level injection of minority carriers. Because of this it is of interest to relate the injection level of minority carriers with U_d for case of high-level injection.

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II. THE EQUATIONS OF INJECTION LEVEL VALID AT ARBITRARY INJECTION

The boundary conditions that relate the minority carrier boundary concentrations with U_d at arbitrary injection level including high-level injection, derived on basis of commonly accepted Boltzmann relations and quasi-neutrality conditions are following [4]:

$$p_n(x_n) = \frac{p_{n0}\exp\left(\frac{V_B}{V_T}\right) + n_{p0}\exp\left(\frac{U_d}{V_T}\right)}{2\,\mathrm{sh}\left(\frac{V_B - U_d}{V_T}\right)}\ , \quad (7)$$

$$n_p(-x_p) = \frac{n_{p0}\exp\left(\frac{V_B}{V_T}\right) + p_{n0}\exp\left(\frac{U_d}{V_T}\right)}{2\,\mathrm{sh}\left(\frac{V_B - U_d}{V_T}\right)}\ , \quad (8)$$

where V_B is the potential barrier of junction.

On basis of equations (1), (2) and (7), (8), taking into account that $p_{n0} = n_i^2 / N_D$ and $n_{p0} = n_i^2 / N_A$, the hole and electron injection levels:

$$k_p = \frac{n_i^2}{N_D^2}\left[\frac{\exp\left(\frac{V_B}{V_T}\right) + \frac{N_D}{N_A}\exp\left(\frac{U_d}{V_T}\right)}{2\,\mathrm{sh}\left(\frac{V_B - U_d}{V_T}\right)} - 1\right]\ , \quad (9)$$

$$k_n = \frac{n_i^2}{N_A^2}\left[\frac{\exp\left(\frac{V_B}{V_T}\right) + \frac{N_A}{N_D}\exp\left(\frac{U_d}{V_T}\right)}{2\,\mathrm{sh}\left(\frac{V_B - U_d}{V_T}\right)} - 1\right]. \quad (10)$$

In contrast to exponential equations (5) and (6), the high-level injection of minority carriers is taken into account in derived new equations (9) and (10). The validity of (9) and (10) is limited by the validity of Boltzmann relations and quasi-neutrality conditions. The analysis of obtained equations shows that they coincide with equations (5), (6) at low-level injection when $[N_D \exp(U_d / V_T)] / N_A$, and $[N_A \exp(U_d / V_T)] / N_D] << \exp(V_B / V_T)$, i.e. when U_d is significant lower than V_B.

If junction operates at high-level injection when U_d is so close to V_B that assumptions $\exp(U_d / V_T) \approx \exp(V_B / V_T)$ and $\mathrm{sh}[(V_B - U_d)/ V_T] \approx (V_B - U_d)/ V_T$ can be made, the equations (9), (10) simplifies us follows:

$$k_p \approx \frac{n_i^2}{2N_D^2}\left(1+\frac{N_D}{N_A}\right)\exp\left(\frac{V_B}{V_T}\right)\frac{V_T}{V_B - U_d}, \quad (11)$$

$$k_n \approx \frac{n_i^2}{2N_A^2}\left(1+\frac{N_A}{N_D}\right)\exp\left(\frac{V_B}{V_T}\right)\frac{V_T}{V_B - U_d}. \quad (12)$$

It is seen (11), (12) that k_p and k_n dependence on U_d at high-level injection, when U_d is close to V_B, become not exponential and is determined by function $V_T / (V_B - U_d)$. By this is meant that voltage drop across the depletion region can not exceed V_B, i.e. the inequality $U_d < V_B$ is valid. On the other hand, according to the exponential equations (5) and (6), the wrong conclusion can be drawn that there is no limitation on U_d.

On basis of (5) and (6) can be estimated that ratio of hole and electron injection levels at low-level injection is independent of U_d and is determined by square of impurity concentrations ratio

$$k_p / k_n = (N_A / N_D)^2. \quad (13)$$

The equation of ratio k_p / k_n obtained on basis of (9) and (10)

$$\frac{k_p}{k_n} = \frac{1+\frac{N_A}{N_D}\exp\frac{V_B - U_d}{V_T}}{1+\frac{N_D}{N_A}\exp\frac{V_B - U_d}{V_T}} \quad (14)$$

shows that in general case the ratio k_p / k_n is U_d dependent (14) and varies in range from value presented by equation (13) at low-level injection when $\exp[-(V_B - U_d)/ V_T] << N_D / N_A$, N_A / N_D to value $k_p / k_n \approx N_A / N_D$ at high–level injection when U_d is so close to V_B that $\exp[(V_B - U_d)/ V_T]$ is close to unity. Equation (14) is derived taking into account fact that the fractional term in the angle brackets of equations (9) and (10) in case of forward-biased junction is much more higher than unit.

III. ANALYSIS OF CONCRETE P-N JUNCTIONS

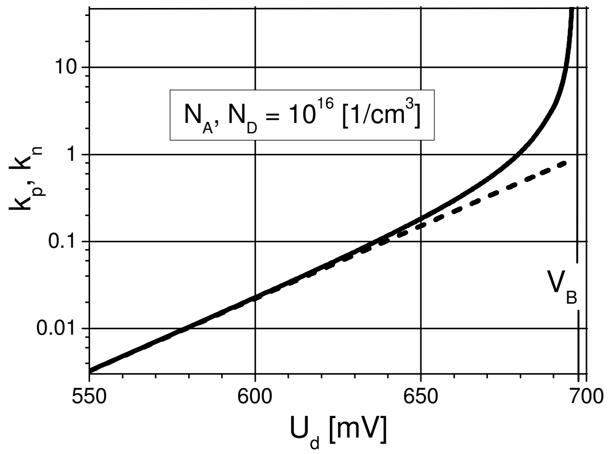
To examine the derived equations of injection level, three silicon p-n junctions with different impurity concentrations (Table I) were analyzed. The analysis was performed at room temperature assuming that junctions are abrupt and homogeneously doped.

TABLE I. IMPURITY CONCENTRATIONS OF JUNCTIONS.

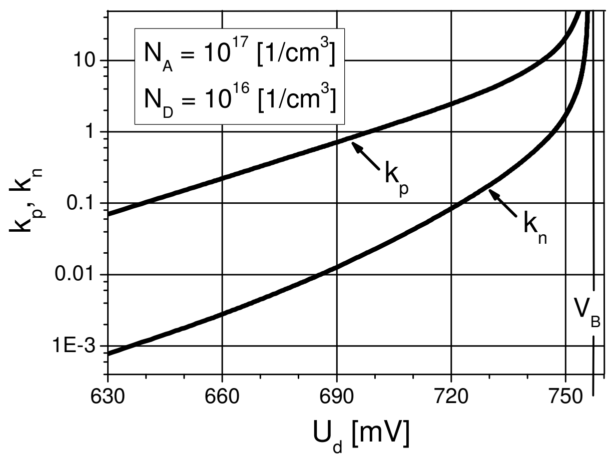
№	1	2	3
N_A, cm^{-3}	10^{16}	10^{17}	10^{18}
N_D, cm^{-3}	10^{16}	10^{16}	10^{15}

Using data presented in Table I and widely known relation $V_B = V_T \ln[N_A N_D / n_i^2]$, the dependences of k_p and k_n on U_d on the basis of equations (9) and (10) were computed (Fig. 1). Additionally, the k_p and k_n dependences using exponential equations (5), (6) were calculated for junction No.1 (Fig. 1). It is seen that at low U_d the dependences calculated using (9), (10) and (5), (6) coincide. When U_d becomes high and increases, equations (9), (10) give a higher rise of injection level as compared with equations (5), (6). At high values of U_d the dependences of k_p and k_n computed using (9), (10) become non-linear on *log* scale, i.e. they become non-exponential.

It is of interest to evaluate the current density of the junction at given injection level. The calculation of approximate values of current densities for analyzed junctions at given k_p and k_n was provided according to the following scheme. Firstly, the U_d for given k_p and k_n using graphs presented in Fig. 1 were estimated. Secondly, using classical equations $J_{sp} = en_i^2 D_p /(W_n N_D)$, $en_i^2 D_p /(L_p N_D)$ and $J_{sn}= en_i^2 D_n /(W_p N_A)$, $en_i^2 D_n /(L_n N_A)$ the hole and electron saturation current densities were calculated for case when junctions are with short (W_p, W_n =3x10^{-4} cm) and long ($W_p > L_n$, $W_n > L_p$) quasi-neutral regions, where W_p and W_n are lengths of p and n quasi-neutral regions, L_p and L_n are hole and electron diffusion lengths. The data for calculations were taken from [5].



a)



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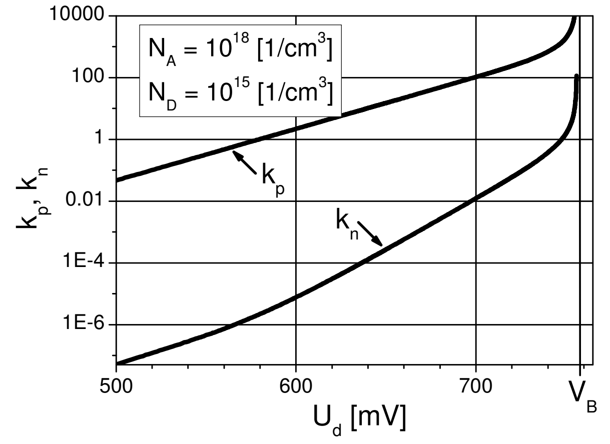

c)

Fig. 1. Dependences of minority carrier hole and electron injection levels on voltage drop across the depletion region for silicon junctions No1–No.3 calculated using equations (9) and (10) (solid lines). The dependence for junction No1 presented by the dashed line has been calculated using equations (5) and (6).

Knowing the J_{sp} and J_{sn}, the minority carrier hole and electron current densities of the junction at the edges of the depletion region J_p and J_n at given U_d were computed assuming that their dependences on U_d in respect of J_{sp} and J_{sn} are determined by law of boundary conditions (7) and (8), respectively.

The total current density of junction at given U_d and, consequently, at given k_p and k_n was estimated as sum $J = J_p + J_n$. The J values at injection levels k_p, k_n = 0.1; 1 for analyzed junctions with short and long quasi-neutral regions are presented in Table II and Table III. The value k_p, k_n = 0.1 can be considered as limiting value of low-level injection of minority carriers holes and electrons, i.e. as limiting value for validity of Shockley equation-based models. Value k_p, k_n = 1 corresponds with high-level injection [3]. It is seen (Table II and Table III) that at given values of k_p and k_n the current density of junctions with short quasi-neutral regions is higher than that of junctions with long quasi-neutral regions. The reason for this is the fact that J_{sp} and J_{sn} are higher for junctions with short quasi-neutral regions.

TABLE II. CURRENT DENSITY OF SILICON JUNCTIONS WITH SHORT QUASI-NEUTRAL REGIONS (W_P, W_N = 3X10^{-4} CM) AT GIVEN INJECTION LEVELS K_P AND K_N.

No	N_A, N_D [1/cm^3]	Junction current density,[A/mm^2]			
		k_p		k_n	
		0.1	1	0.1	1.0
1	N_A = 10^{16} N_D=10^{16}	0.23	2.3	0.23	2.3
2	N_A = 10^{17} N_D = 10^{16}	0.072	0.82	2.7	18.6
3	N_A = 10^{18} N_D = 10^{15}	6.7x10^{-3}	0,067	25	130.6

TABLE III. CURRENT DENSITY OF SILICON JUNCTIONS WITH LONG QUASI-NEUTRAL REGIONS ($W_P > L_N$, $W_N > L_P$) AT GIVEN INJECTION LEVELS K_P AND K_N.

No	N_A, N_D [1/cm^3]	Junction current density, [A/mm^2]			
		k_p		k_n	
		0.1	1	0.1	1.0
1	N_A = 10^{16} N_D = 10^{16}	0.0016	0.016	0.002	0.02
2	N_A = 10^{17} N_D = 10^{16}	6.8x10^{-4}	0.008	0.028	0.2
3	N_A = 10^{18} N_D = 10^{15}	5.7x10^{-5}	6x10^{-4}	0.37	2.7

The p-n junctions of silicon diodes and BJT's often operate at current densities that may be as much as tens of A/mm^2 [6]. For emitter junction of n-p-n integral transistors, as an example, the current density can reach 100 A/mm^2 and more [7]. It is apparent that values of current density mentioned above exceed considerably the low-level injection limiting values presented in Table II and Table III.

The experimental voltage-current characteristic of base-emitter junction of p-n-p lateral transistor used in integrated circuit of voltage comparator [8] is presented in Fig. 2. The U_{BE} in Fig. 2 is voltage applied to the base – emitter junction and I_E is emitter current. The area of the junction is 1.3x10^{4} mm^2, the maximum operating current I_{Emax}= 5mA. This junction corresponds with junction No.2 with short quasi-neutral regions analyzed above. Using results presented in Table II, the approximate values of current that correspond with injection levels k_p, k_n =0.1; 1 are marked in Fig. 2.

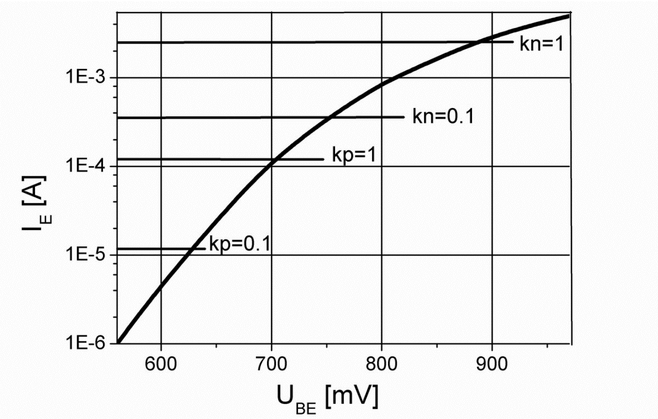

Fig. 2. Experimental voltage-current characteristic of the base-emitter junction of the p-n-p integral lateral transistor with junction area 1.3x10^{-4} mm^2.

It is seen that the low-level injection condition for holes (k_p< 0.1) and electrons (k_n< 0.1) is violated at about 0.01 and 0.35 mA, respectively. This fact shows that for the adequate simulation of circuit based on such transistors the model, which takes into account the high-level injection, should be used.

IV. Conclusions

1. In contrast to exponential equations (5) and (6), the derived equations (9) and (10) allow us to relate the injection level of minority carriers with U_d for case of high-level injection.

2. In general case the ratio of hole and electron injection levels is U_d dependent and varies in range from $(N_A / N_D)^2$ at low-level to approximately N_A / N_D at high-level injection.

3. The analysis of concrete p-n junctions shows that junctions of silicon semiconductor devices often operate at injection levels that are much greater than low-level injection limiting values.

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