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Identification of Nonlinear Systems Using the Hammerstein-Wiener Model with Improved Orthogonal Functions

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***Abstract*—Hammerstein-Wiener systems present a structure consisting of three serial cascade blocks. Two are static nonlinearities, which can be described with nonlinear functions. The third block represents a linear dynamic component placed between the first two blocks. Some of the common linear model structures include a rational-type transfer function, orthogonal rational functions (ORF), finite impulse response (FIR), autoregressive with extra input (ARX), autoregressive moving average with exogenous inputs model (ARMAX), and output-error (O-E) model structure. This paper presents a new structure, and a new improvement is proposed, which is consisted of the basic structure of Hammerstein-Wiener models with an improved orthogonal function of Müntz-Legendre type. We present an extension of generalised Malmquist polynomials that represent Müntz polynomials. Also, a detailed mathematical background for performing improved almost orthogonal polynomials, in combination with Hammerstein-Wiener models, is proposed. The proposed approach is used to identify the strongly nonlinear hydraulic system via the transfer function. To compare the results obtained, well-known orthogonal functions of the Legendre, Chebyshev, and Laguerre types are exploited.**

***Index Terms*—Hammerstein-Wiener models; Identification system; Improved orthogonal functions; Nonlinear systems.**

I. INTRODUCTION

When the system's output is nonlinearly dependent on its inputs, it is possible that the input-output link is broken down into two or more interconnected elements. In this case, we can represent the dynamics using a linear transfer function and capture the nonlinearity using nonlinear input and output functions. The Hammerstein-Wiener (H-W) model realises this configuration as a serial connection of a static nonlinear block with a dynamic linear block [1]–[4]. The first and third blocks are the input and output nonlinearities. On the other hand, common linear model structures include rational transfer functions, polynomial functions, finite impulse response (FIR), autoregressive with extra input (ARX), autoregressive moving average with exogenous inputs (ARMAX), Bock-Jenkins model structures, and output-error (O-E) models [5], [6]. These nonlinear blocks can be either continuous or discontinuous, where some continuous nonlinearities can be represented using a function expansion with the polynomial representation [7], [8]. Some authors have also investigated the application of new techniques such as neural networks, using two independent networks with one hidden layer whose parameters were identified by the extreme learning machine (ELM) algorithm. In this case, the proposed solution is with an invertible output nonlinearity, which actually implies the basic structure of the H-W model.

Many authors have proposed identification algorithms for H-W model estimation [5]–[10]. Some use identification algorithms divided into two stages: recursively calculating the least squares estimation of the parameter vector and singular value decomposition of fixed-size matrices [11]–[17]. Also, in the literature, we can find a blind approach, where the structure of the input nonlinearity does not necessarily have to be known [11]. For H-W ARMAX models, an extended stochastic gradient algorithm is developed, and two types are proposed: recursive generalised extended least squares algorithm and generalised extended stochastic gradient algorithm [10].

The H-W model applies in several areas, such as modelling electro-mechanical systems and radio frequency components, sound and speech processing, chemical processes control, and nonlinear industrial systems identification. Also, the H-W model can be used as a black-box model because it allows flexible parameterisation for nonlinear models. For example, we can improve the accuracy of a linear model by adding input or output nonlinearity to this model [12]. We can use the H-W as a grey-box model to have an idea of the characteristics of the process through it. For example, the nonlinearity of the

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input may represent a typical physical transformation in actuators, and the nonlinearity of the output may describe typical sensor characteristics [12], [17].

In [18]–[21], authors proposed a new class of almost orthogonal polynomials with application in the modelling of dynamical systems. Further, the authors in [22]–[25] present possible applications in orthogonal endocrine adaptive neurofuzzy systems. Also, the authors in [26]–[29] suggest new classes of quasiorthogonal polynomials and their applications in modelling, neural networks, sliding mode control, and control of dynamical systems. In this paper, the authors consider the possible application of improved orthogonal polynomials of Legendre type [18], [19] to represent an H-W system's input and output nonlinearities.

This paper is organised as follows. Section I gives a detailed introduction to the H-W models and improved almost orthogonal functions of the Müntz-Legendre type. In Section II, H-W models are given. Section III describes a process of obtaining improved almost orthogonal functions and polynomials of Müntz-Legendre type. A short description of the multi-tank hydraulic system is given in Section IV. Experimental results that verify the proposed approach and comparison with similar methods are presented in Section V. In Section VI, the latest works in which the H-W models are combined with intelligent controls are discussed. Finally, the last section consists of concluding remarks.

II. HAMMERSTEIN-WIENER MODELS

The Hammerstein-Wiener model structure is made up of two nonlinear blocks and a linear block. This structure combines the Hammerstein (Fig. 1(a)) and Wiener model structures (Fig. 1(b)). The Hammerstein model is described using input nonlinearity followed by a linear system model. In contrast, the Wiener model consists of a linear block followed by output nonlinearity. Depending on the specific case, these models can be combined and the Hammerstein-Wiener (H-W) or Wiener-Hammerstein (W-H) structure can be formed. The H-W model structure is presented in Fig. 1(c) and Fig. 2, while the W-H structure is presented in Fig. 1(d).

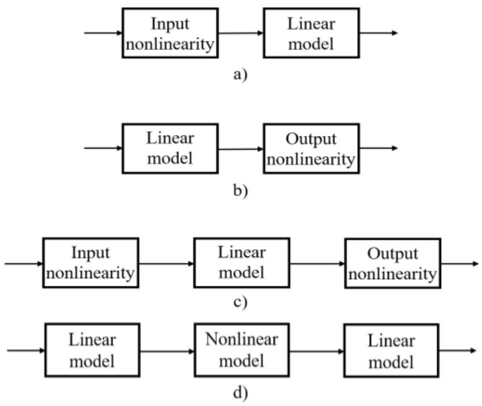

Fig. 1. (a) Hammerstein model structure; (b) Wiener model; (c) Hammerstein-Wiener model; (d) Wiener-Hammerstein model.

In Fig. 2, $u(t)$ is the system input and $y(t)$ is the system output. The nonlinear functions f_1 and f_2 represent the input and output nonlinearity, respectively.

The linear block can be described using various model structures [12] where the output signal of the structure presented in Fig. 2 can be determined using the following equations:

$$w_1(t) = f_1(u(t)), \quad (1)$$

$$w_2(t) = G(z) f_1(u(t)), \quad (2)$$

$$y(t) = f_2(w_2(t)) = f_2(G(z) f_1 u(t)). \quad (3)$$

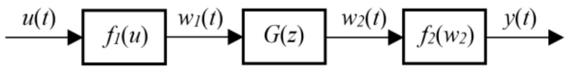

Fig. 2. H-W model structure.

One of the generally used models for the representation of the linear block is the ARMAX model (see Fig. 3) [3].

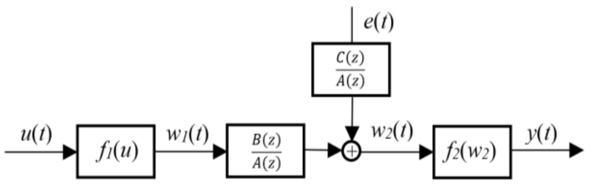

Fig. 3. H-W ARMAX system.

The linear part of the system can be described with the following equation [3]

$$w_2(t) = \frac{B(z)}{A(z)} w_1(t) + \frac{C(z)}{A(z)} e(t), \quad (4)$$

where $e(t)$ represents the disturbance term and $A(z)$, $B(z)$, and $C(z)$ are polynomials of order n_a, n_b, and n_c, respectively:

$$A(z) = 1 + \sum_{i=1}^{n_a} a_i z^{-i}, \quad (5)$$

$$B(z) = \sum_{i=1}^{n_b} b_i z^{-i}, \quad (6)$$

$$C(z) = 1 + \sum_{i=1}^{n_c} c_i z^{-i}. \quad (7)$$

Substituting (1) and (4) into (3), we obtain the output signal

$$y(t) = f_2\left(\frac{B(z)}{A(z)} f_1(u(t)) + \frac{C(z)}{A(z)} e(t) \right). \quad (8)$$

The influence of measurement noise ε on the output signal can also be considered [8], [9]

$$y(t) = f_2(w_2(t)) + \varepsilon(t). \quad (9)$$

In this paper, the O-E model has been used for the parameterisation of the linear part of the system (see Fig. 4).

The structure of the O-E model can be described using the following equation

$$y(t) = \frac{B(z)}{A(z)} u(t - n_k) + e(t), \tag{10}$$

where n_k is an input delay.

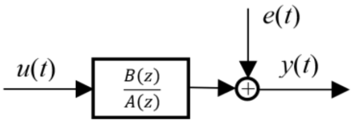

Fig. 4. O-E model structure.

The advantages of the Wiener system with orthogonal Hermite polynomials used for nonlinearity representation have been given in [29]. Hence the authors of this paper use a class of improved almost orthogonal polynomials of Müntz-Legendre type for the nonlinearity functions of the H-W model, i.e., f_1 and f_2.

III. Improved Almost Orthogonal Functions

The authors of this paper developed several new types of orthogonality in the last ten years. Some of them are almost orthogonality [18], improved orthogonality [26], quasiorthogonality [19], generalised quasiorthogonality [23], and a new type of trigonometric orthogonality [27]. On the other hand, the generalisation of Malmquist-Takenaka polynomials is developed for the modelling purposes of an industrial protector cooling system [20]. This type of polynomial is modified in the *z*-domain [28]. All these types of orthogonality, in combination with intelligent systems (genetic algorithms, neural networks, fuzzy systems), have been used very successfully in the field of automation and electronics [18]–[28]. This paper provides an extension of generalised Malmquist polynomials that represent appropriate Müntz polynomials. Additionally, a combination of extended Müntz-Legendre polynomials is proposed, whose coefficients contain a measure of imperfection in the form of the parameter δ.

To start with, the definition of improved almost orthogonality can be represented via the rational transfer function

$$\left(W_n^{(\delta)}, \overline{W}_m^{(\delta)}\right) = \oint_C W_n^{(\delta)}(s) \overline{W}_m^{(\delta)}(s) w(s) ds, \tag{11}$$

with weight function $w(s) = 1$, $m > n$. The transfer functions have the following form

$$\begin{bmatrix} W_n^{(\delta)}(s) = \dfrac{\prod_{i=1}^{n}(s - i\delta)}{\prod_{i=0}^{n}(s + i)}, \\ \overline{W}_m^{(\delta)}(s) = \dfrac{\prod_{i=1}^{m}(s + i\delta)}{\prod_{i=0}^{m}(s - i)}. \end{bmatrix} \tag{12}$$

For obtaining the transfer function $\overline{W}_m^{(\delta)}(s)$, we used a well-known transformation $\overline{s} = f(s)$ and the property of symmetry [30]. Now, a sequence of Müntz orthogonal rational functions is used in the following form [31]

$$W_n(s) = \frac{\prod_{k=0}^{n-1}(s - \alpha_k^*)}{\prod_{k=0}^{n}(s - \alpha_k)}, \quad n = 1, 2, 3, \ldots, \tag{13}$$

where the zeros α_k^* are obtained by mapping the poles α_k, and the poles α_k by mapping the zeros α_k^*, using the symmetric transformation $\alpha_k^* = f(\alpha_k)$, $\alpha_k = f(\alpha_k^*)$. Therefore, using the symmetric transformation in (13), we obtain

$$\overline{W_n(s)} = \frac{\prod_{k=0}^{n-1}(s - \alpha_k)}{\prod_{k=0}^{n}(s - \alpha_k^*)}. \tag{14}$$

If the poles and zeros from (11) are substituted in (13) and (14), a new class of improved almost Müntz-Legendre orthogonal polynomials $W_{nML}^{(\delta)}(s)$ defined via the transfer function and the parameter δ will be obtained

$$W_{nML}^{(\delta)}(s) = \frac{\prod_{\substack{k=0 \\ j=k+1}}^{n-1}(s - j\delta\alpha_k^*)}{\prod_{\substack{k=0 \\ j=k+1}}^{n}(s - j\delta\alpha_k)}, \quad n = 1, 2, 3, \ldots, \tag{15}$$

where the zeros α_k^* and poles α_k are $\alpha_k^* = \dfrac{b}{\overline{\alpha_k}}$ and $\alpha_k = \dfrac{b}{\overline{\alpha_k^*}}$, respectively. Parameter b represents the real constant of the symmetric bilinear transformation $s \to (as + b)/(cs - a)$ [30]. Classical orthogonal polynomials are obtained depending on the parameter values a, b, and c. When parameter $c = 0$, Laguerre and Legendre orthogonal polynomials on the interval (0, 1) are obtained. The Malmquist-Takenaka polynomials are derived for the values $a = 0$ and $b \neq 1$.

If we substitute (15) into (13) and (14), improved almost orthogonal polynomials of Müntz-Legendre type $P_{nML}^{(\delta)}(x)$ can be obtained

$$\begin{bmatrix} P_{nML}^{(\delta)}(x) = \dfrac{1}{2\pi i} \oint_{C_P} W_{nML}^{(\delta)}(s) x^s ds, \\ P_{nML}^{(\delta)}(x) = \sum_{k=0}^{n} A_{n,k}^{(\delta)} x^{\alpha_k}, \end{bmatrix} \tag{16}$$

where C_p involves all the poles of $W_{nML}^{(\delta)}(s)$, and

$$A_{n,k}^{(\delta)} = \frac{\prod\limits_{\substack{f=0\\ j=k+1}}^{n-1}\left(\alpha_k - j\delta\dfrac{b}{\alpha_f}\right)}{\prod\limits_{\substack{f=0,\, f\neq k\\ j=k+1}}^{n}\left(\alpha_k - j\delta\alpha_f\right)}.$$

An example of a sequence (the first four members) of improved almost orthogonal polynomials of Müntz-Legendre type (for $b=-1,\ \ \alpha_0=\delta,\ \ \alpha_1=2\delta,\ \ \alpha_2=3\delta,$ $\alpha_3=4\delta,\ \ \alpha_4=5\delta$) is given as

$$\begin{bmatrix} P_{0ML}^{(\delta)}(x)=\delta x, \\ P_{1ML}^{(\delta)}(x)=3\delta^2x^2-2\delta x, \\ P_{2ML}^{(\delta)}(x)=7\delta^3x^3-\dfrac{15}{2}\delta^2x^2+\dfrac{3}{2}\delta x, \\ P_{3ML}^{(\delta)}(x)=\dfrac{65}{4}\delta^4x^4-\dfrac{70}{3}\delta^3x^3+\dfrac{35}{4}\delta^2x^2-\dfrac{2}{3}\delta x. \end{bmatrix} \quad (17)$$

If we apply substitution $x=e^{-t}$ in the previous expressions, the appropriate functions for H-W models are obtained in the form $\varphi_{iML}^{(\delta)}(x)$ and $\varphi_{iML}^{(\delta)}(x)^{-1}=\dfrac{1}{\varphi_{iML}^{(\delta)}(x)}, i=0,1,2,\ldots n.$

Hence, to identify nonlinear systems in the form of transfer functions, we will develop a new type of polynomial (17). The newly developed polynomials will be used in the structure of the H-W model as a replacement for the standard functions contained in these models. Unknown systems are identified by the transfer function using the mean square error (MSE) method [18], [23]. In identification systems, we minimise the error value between classical H-W models with our proposed H-W structure with improved almost orthogonal polynomials of Müntz-Legendre type.

IV. Experimental Setup-Multi-Tank System

To apply newly improved almost Müntz-Legendre orthogonal functions in combination with H-W models, we used a multi-tank hydraulic system shown in Fig. 5.

Fig. 5. Multi-tank system by INTECO.

The multi-tank system [32] consists of three separate tanks equipped with drain valves. The first tank has a constant cross section, while the second and third are spherical and conical and have a variable cross section. These features introduce the main nonlinearities in the utilised system. Each tank is equipped with a level sensor based on hydraulic pressure measurement. Finally, a speed pump is variable and is used to fill the upper tank with a constant cross section.

The multi-tank system can be described using the nonlinear equations [27]

$$\begin{bmatrix} \dfrac{dH_1}{dt}=\dfrac{1}{\beta_1(H_1)}q-\dfrac{1}{\beta_1(H_1)}C_1H_1^{\mu_1}, \\ \dfrac{dH_2}{dt}=\dfrac{1}{\beta_2(H_2)}C_1H_1^{\mu_1}-\dfrac{1}{\beta_2(H_2)}C_2H_2^{\mu_2}, \\ \dfrac{dH_3}{dt}=\dfrac{1}{\beta_3(H_3)}C_2H_2^{\mu_2}-\dfrac{1}{\beta_3(H_3)}C_3H_3^{\mu_3}, \end{bmatrix} \quad (18)$$

where q represents the entry into the first upper tank, H_i is the level of fluid in the i-th tank (i = 1, 2, 3), C_i is the resistance of the output orifice of i-th tank, and μ_i represents the flow coefficient of the i-th tank. Finally, $\beta_1(H_1)$ represents the cross-sectional area of the i-th tank at the level H_i. The specified parameter values can be found in [27]: a = 0.25 m, b = 0.345 m, c = 0.1 m, w = 0.035 m, R = 0.364 m, $H_{1max} = H_{2max} = H_{3max}$ = 0.35 m.

Now, we rewrite the right sides of (18) in the form $F(x, q) = [F_1, F_2, F_3]$, where

$$\begin{bmatrix} F_1(q,H_1)=\dfrac{1}{\beta_1(H_1)}q-\dfrac{1}{\beta_1(H_1)}C_1H_1^{\alpha_1}, \\ F_2(H_1,H_2)=\dfrac{1}{\beta_2(H_2)}C_1H_1^{\alpha_1}-\dfrac{1}{\beta_2(H_2)}C_2H_2^{\alpha_2}, \\ F_3(H_2,H_3)=\dfrac{1}{\beta_3(H_3)}C_2H_2^{\alpha_2}-\dfrac{1}{\beta_3(H_3)}C_3H_3^{\alpha_3}. \end{bmatrix} \quad (19)$$

For the model (18), for fixed $q = q_0$, we can define an equilibrium state (steady-state points) given by $q_0=C_1H_{10}^{\alpha_1}=C_2H_{20}^{\alpha_2}=C_3H_{30}^{\alpha_3}$. The linearized model is obtained by the Taylor expansion of (19) around the assumed equilibrium state

$$\frac{dh}{dt}=J_Hh+J_qu, \quad (20)$$

where $h = H\text{-}H_0$ is the modified state vector (deviation from equilibrium state H_0), $u = q\text{-}q_0$ is deviation of the control, relative to q_0, J_H and J_q are Jacobians of function (19) and (20):

$$\begin{aligned} J_H &= \left[\frac{\partial F(H,q)}{\partial H}\right]_{H=H_0,q=q_0}, \\ J_q &= \left[\frac{\partial F(H,q)}{\partial q}\right]_{H=H_0,q=q_0}, \end{aligned} \quad (21)$$

i.e.,

$$\begin{cases} J_H = \begin{bmatrix} A & 0 & 0 \\ B & C & 0 \\ 0 & D & E \end{bmatrix}, \\ A = \dfrac{-C_1\alpha_1}{(H_{10})^{1-\alpha_1}\beta_1(H_{10})}, \\ B = \dfrac{C_1\alpha_1}{(H_{10})^{1-\alpha_1}\beta_2(H_{20})}, \\ C = \dfrac{-C_2\alpha_2}{(H_{20})^{1-\alpha_2}\beta_2(H_{20})}, \\ D = \dfrac{C_2\alpha_2}{(H_{20})^{1-\alpha_2}\beta_3(H_{30})}, \\ E = \dfrac{C_3\alpha_3}{(H_{30})^{1-\alpha_3}\beta_3(H_{30})}, \\ J_q = \begin{bmatrix} \dfrac{1}{\beta_1(H_{10})} \\ 0 \\ 0 \end{bmatrix}. \end{cases} \tag{22}$$

This linear model (22) can be used for analysis and for the design of local controllers of the pump-controlled system. The model of this system is described by (18) and (22) used in our experiments.

V. Experimental Results

To evaluate the efficiency of the proposed H-W structure with improved almost orthogonal functions of the Müntz-Legendre type, we conducted experiments with a given multi-tank system.

To obtain the transfer function of the hydraulic system, the method shown in Fig. 6 was used. The unknown hydraulic system is identified through six different types of polynomials.

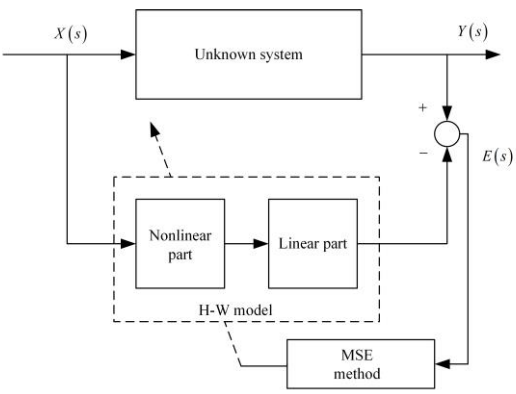

Fig. 6. Block scheme of system identification.

First, the input-output data set has been formed using the model of the multi-tank system given by (18). The data set used for this purpose is shown in Fig. 7. We consider a proposed method with the H-W structure with orthogonal functions described by (17), and the values for $\delta = 1.01$. This H-W structure consists of two nonlinear blocks presented by orthogonal functions from (17). The proposed improved H-W structure was further applied to identify the hydraulic multi-tank system. For this purpose, the first half of the generated data set was used to determine the parameters of the H-W model. Subsequently, the second half of the data set was used to test the performance of the obtained structure. Finally, function substitutions were applied using the newly derived improved almost orthogonal functions.

To demonstrate the effectiveness of the proposed method, various combinations of functions f_1 and f_2 are prepared. In the first example, the following functions for input nonlinearity f_1 and output nonlinearity f_2 are used

$$\begin{bmatrix} f_1 = \varphi_{1ML}^{(\delta)}(t) = 3\delta^2 e^{-2t} - 2\delta e^{-t}, \\ f_2 = f_1^{-1} = \dfrac{1}{f_1} = \varphi_{1ML}^{(\delta)\,-1}(t) = \dfrac{1}{3\delta^2 e^{-2t} - 2\delta e^{-t}}. \end{bmatrix} \tag{23}$$

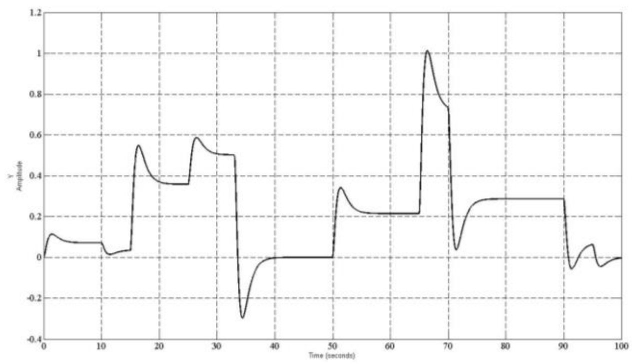

Fig. 7. Generated data set used for the system identification.

The results in Fig. 8 show that the proposed H-W structure with input nonlinearity and output nonlinearity can capture the multi-tank system's behaviour with an accuracy of 99.32 %.

In this case, we obtain the following transfer function

$$W(s) = \frac{0.0268s^2 + 1.1891s + 0.4302}{s^3 + 5.99s^2 + 11.01s + 5.98}. \tag{24}$$

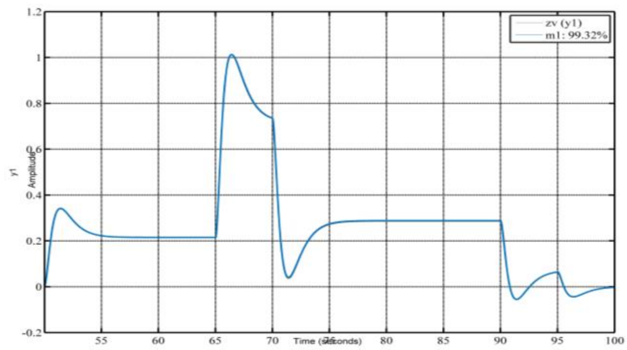

Fig. 8. Comparison of the simulated response of the multi-tank system (*zv* signal) and the H-W model of the system (m_1). The input and output nonlinearities of an H-W model have been represented using $\varphi_{1ML}^{(\delta)}(t)$ and $\varphi_{1ML}^{(\delta)\,-1}(t)$ functions.

A second example of the use case of a nonlinear multi-tank system bases the H-W model on the following function

$$f_1 = \varphi_{2ML}^{(\delta)}(t) = 7\delta^3 e^{-3t} - \frac{15}{2}\delta^2 e^{-2t} + \frac{3}{2}\delta e^{-t}. \quad (25)$$

Figure 9 shows the results obtained when both the input and output nonlinearities are represented using $\varphi_{2ML}^{(\delta)}(t)$ function with an accuracy of 99.62 %.

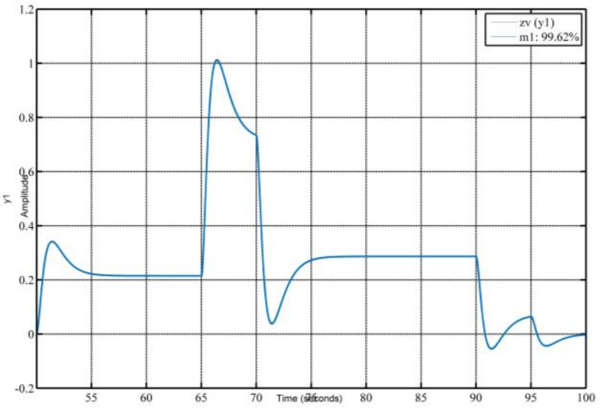

Fig. 9. Comparison of the simulated response of the multi-tank system (*zv* signal) and the H-W model of the system (m_1). Both the input and output nonlinearities of an H-W model have been represented using $\varphi_{2ML}^{(\delta)}(t)$ function.

It can be noticed that the structure with both input and output nonlinearities realised using $\varphi_{2ML}^{(\delta)}(t)$ gives slightly better results and follows the multi-tank system response with slightly higher precision.

To demonstrate the effectiveness of the proposed method, the experiments were repeated with well-known orthogonal polynomials. We used generalised quasiorthogonal polynomials (order $k = 1$) of the Legendre type [18], Chebyshev polynomials of the second kind [19], classical Legendre polynomials [13], and Laguerre polynomials [23].

In the case when we used generalised quasiorthogonal polynomials (order $k = 1$) of Legendre type and input and output nonlinearities, the following functions exist

$$f_1 = f_2 = \varphi_2^{(1,\delta)}(t) = -\frac{(\delta+2)}{2}e^{-2t} + (\delta+1)e^{-t} - \frac{\delta}{2}. \quad (26)$$

On the other hand, Chebyshev polynomials of the second kind [24] and input and output nonlinearities are given by

$$\begin{bmatrix} f_1 = T_1(t) = 2e^{-t}, \\ f_2 = T_2(t)^{-1} = \dfrac{1}{4e^{-2t}-1}. \end{bmatrix} \quad (27)$$

Classical Legendre polynomials [18] and input and output nonlinearities are given by

$$\begin{bmatrix} f_1 = \varphi_2(t) = 6e^{-2t} - 6e^{-t} + 1, \\ f_2 = \varphi_3(t) = 20e^{-3t} - 30e^{-2t} + 12e^{-t} - 1. \end{bmatrix} \quad (28)$$

Finally, the experiment was performed with Laguerre polynomials [12], [33], [34] where nonlinearities are described as follows

$$\begin{bmatrix} f_1 = L_2(t) = \dfrac{1}{2}\left(e^{-2t} - 4e^{-t} + 2\right), \\ f_2 = L_3(t) = \dfrac{1}{6}\left(-e^{-3t} + 9e^{-2t} + 18e^{-t} + 6\right). \end{bmatrix} \quad (29)$$

A summarisation of all the results obtained is given in Table I.

TABLE I. IDENTIFICATION RESULTS OF THE H-W MODELS WITH ORTHOGONAL POLYNOMILAS.

Type of polynomials	f_1	f_2	Accuracy [%]
Improved almost Müntz-Legendre	$\varphi_{1ML}^{(\delta)}(t)$	$\varphi_{1ML}^{(\delta)-1}(t)$	99.32
Improved almost Müntz-Legendre	$\varphi_{2ML}^{(\delta)}(t)$	$\varphi_{2ML}^{(\delta)}(t)$	99.62
Generalised quasiorthogonal polynomials (order k = 1) of Legendre type	$\varphi_2^{(1,\delta)}(t)$	$\varphi_2^{(1,\delta)}(t)$	98.27
Chebyshev polynomials of second kind	$T_1(t)$	$T_2(t)^{-1}$	97.64
Classical Legendre polynomials	$\varphi_2(t)$	$\varphi_3(t)$	96.66
Laguerre polynomials	$L_2(t)$	$L_3(t)$	96.53

Based on the results from Table I, it can be concluded that H-W models with improved almost Müntz-Legendre polynomials implemented as nonlinearities are very robust. The experimental results of our research have shown that H-W models with proposed orthogonal functions implemented in nonlinear blocks effectively capture the behaviour of the solid nonlinear system.

The results obtained by the developed method using the improved almost orthogonal Müntz-Legendre polynomials in the H-W model have been compared with those obtained by the the iterative least squares and a recursive least squares [1], the method using the Genetic Algorithm (GA) combined with the Recursive Least Squares (RLS) method [4], and the method for estimating the parameters of the Hammerstein nonlinear model using a multi-signal approach [35]. The results are shown in Table II.

TABLE II. IDENTIFICATION RESULTS OF THE H-W MODELS.

Method	Accuracy [%]
Multi-signal approach	99.09
GA + RLS	98.74
Iterative least squares	98.07
Recursive least squares	96.12

The advantage of the method proposed in this paper compared to the others is that the implementation of the H-W model with orthogonal polynomials requires a much less complex mathematical background.

VI. Discussion

In the last 10 years, the application of the H-W model for the identification and estimation of parameters of nonlinear systems has mainly used classical functions for input and output nonlinearities. In [35], the authors propose a new solution to estimate the parameters of the Hammerstein nonlinear model using a multi-signal approach. A novel Adaptive Dual Nonlinear Model Predictive Control (ADNMPC) based on discrete-time block-oriented models is developed [36]. The proposed method uses a modern method of predictive control in combination with H-W models to model black-box models. On the other hand, in [37], a method for identifying the H-W model is proposed, Fourier decomposition, the frequency approach, and the spectrum analysis. The authors in [38], [39] propose an innovative identification approach of nonlinear stochastic systems using H-W model with Output-Error Autoregressive (OEA) noise with Improved Backpropagation Gradient (IBPG) algorithm, Multi-Innovation IBPG (MIIBPG) algorithm, a data filtering IBPG (FIBPG) algorithm, and a Multi-Innovation-based FIBPG (MIFIBPG) algorithm. On the other hand, in this paper, we have shown the possibility of applying of the almost improved orthogonal polynomials in combination with H-W models. Our approach developed showed high accuracy and small modelling error, which verifies the accuracy and efficiency of the developed structure.

VII. Conclusions

In the last twenty years, H-W models have proved to have a significant capability for robust and efficient control of highly-nonlinear systems. They have especially shown their potential for a wide range of strongly nonlinear system representations. This paper considers the application of newly derived improved almost orthogonal polynomials for the representation of nonlinearities of an H-W system. The aim of this paper is to derive a new type of improved almost Müntz-Legendre polynomials based on the bilinear transformation, which can be used for derivation of the all-type classical polynomials. A novel identification method for nonlinear processes is also discussed in detail. From a technical aspect, both nonlinear functions in H-W models are replaced with improved almost functions and classical polynomials. Their combinations are used to improve the accuracy of the proposed method to identify numerous engineering applications. In future work, we will consider the possibility of using deep learning techniques and orthogonal endocrine neural networks together with the improved H-W structure proposed in this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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