

# Capacitor Allocation Using Multiobjective Water Cycle Algorithm and Fuzzy Logic

Menna Allah E. Mohamed El-Saeed<sup>1,\*</sup>, Amal F. Abdel-Gwaad<sup>2</sup>, Mohamed A. Farahat<sup>1</sup>

<sup>1</sup>*Department of Electrical Power and Machines Engineering, Zagazig University,  
Zagazig 44519, Egypt*

<sup>2</sup>*Faculty of Computers and Informatics, Zagazig University,  
Zagazig 44519, Egypt  
m.gouda021@eng.zu.edu.eg*

**Abstract**—Radial distribution systems are susceptible to a lack of voltage profile and increase system losses, particularly at the distant ends of the distribution feeder. This manuscript proposes an approach to solve the optimal capacitor placement problem in radial distribution networks to minimize system losses, improve the voltage profile of all buses, promote total voltage stability, and improve net savings. The optimal capacitor placement problem is solved in two stages. Firstly, normalized loss sensitivity factor and voltage magnitude are used as inputs to build fuzzy expert rules to arrange the most candidate buses for capacitor placement. Secondly, a multiobjective water cycle algorithm is applied to determine the optimal sizes and locations of capacitors within the predefined search space using fuzzy expert rules. The multiobjective function is formulated with operational constraints considering fixed and switched capacitors. To validate the effectiveness of this methodology, it is demonstrated on IEEE 33 and IEEE 94-bus radial distribution networks. Clearly, the findings show the improvement in the voltage profile and static voltage stability, the significant reduction in system losses, as well as the enhancement in overall savings. Furthermore, a comprehensive evaluation is also carried out by comparing the numerical results with other methods such as interior point algorithm, a combination fuzzy real coded genetic algorithm method, water cycle algorithm for IEEE 33-bus system and artificial bee colony algorithm for IEEE 94-bus system which prove the viability and effectiveness of the proposed methodology.

**Index Terms**—Radial distribution network; Capacitor placement; Fuzzy logic; Multiobjective optimization; Water cycle algorithm.

## I. INTRODUCTION

The three main components of the electrical power system are production units, transmission system, and distribution system. There are several different types of loads connected to the power network through the distribution system, such as domestic, commercial, and industrial. Thus, service quality is related to continuous supply of electricity and keeping the supply voltage within the desired limits with the identified frequency. Due to the continuous increase in loads, long distances, and the high R/X ratio of transmission lines [1], it is observed that the loss of power in the distribution system is about 13 % of the

total power generation [2]. This loss is due to the reactive power flow through the feeder, which adversely affects the voltage profile and costs the system more [3]. The installation of capacitors in the distribution network is considered one of the well-recognized solutions which helps to keep the voltage at the desired limits, enhance system stability, reduce system loss [4], and release the line capacity. Taking into account energy loss, capacitors have proven their ability to minimize overall current by cancelling the reactive component of the current provided by the substation [5]. It should be mentioned that the incorrect allocation of capacitors leads to bad results in terms of voltage profile, power losses, and system costs. Thus, the optimal locations and sizes of the shunt capacitors should be calculated to achieve maximum results without violating the constraints of the system [3].

Many methods and optimization algorithms have been introduced in the literature to determine the optimal sitting and sizing of shunt capacitors. In [2], two stages are applied to obtain the optimal allocation of capacitor banks with different load factors: first, the optimal locations of the capacitors are determined based on the voltage stability index (VSI), second, the cuckoo search algorithm (CSA) is proposed to find the optimal capacitor sizes. However, the optimum solution may not be determined, as the optimization method is limited only to the optimal calculation of sizes. Two loss sensitivity indices (LSIs) and the ant colony optimization algorithm are investigated in [1] to solve the optimal capacitor placement problem considering fixed, switched capacitors and their combination. However, it has more parameters to tune. The two bio-inspired methods, the CSA and the bat algorithm (BA), are implemented in [4] to find the optimal allocation of variable locations fixed capacitors (VLFQ) and variable locations variable sizing of capacitors (VLVQ) to reduce real power losses and increase network savings. However, it requires a large number of iterations. A fuzzy-based approach is suggested in [6] for capacitor allocation in the distribution circuit with the aim of improving voltage sensitivity and minimizing real power loss. However, the proposed membership functions are based on weighting factors. In [7], a fuzzy-reasoning method is applied to solve the same issue. However, the loss membership function

used to obtain the optimal locations depends on a parameter, which is calculated from experiences, and the optimal sizes are calculated using a local variational method, which takes a long time, and it may also lead to inefficient solutions. Genetic algorithms and optimal power flow are investigated in [8] to simultaneously place voltage regulators and shunt capacitors in radial distribution networks (RDNs). However, complicated objective functions and more parameters are used. In [9], a coordinated optimal location and size of distributed generations (DGs), and controllable devices including soft open points (SOPs) and capacitors in the participation of the active distribution network in annual cost reduction using bi-level programming is proposed. However, the objective functions used in the upper and lower levels are complicated. In [10], the Analytical Method is applied for the optimal capacitor allocation problem. However, it takes a long computational time to find the best solution.

Recently, many researchers have been the most inclined to use multiobjective algorithms. This kind of problem has many conflicting purposes that need to be solved simultaneously. The addition of the weighting factor with each objective function is considered the initial approach to solving these problems. The main disadvantage of the aforementioned method is wasting time in choosing appropriate weight factors. In contrast, multiobjective optimization techniques use the Pareto front approach as an alternative solution. A set of nondominated solutions is kept in an archive updated every iteration [11]–[13]. The best compromise of these nondominated possible solutions with proper decisions is chosen to be an optimal solution [14], [15].

Many optimization techniques have been applied in literature to solve multiobjective problems such as the multiobjective stochastic simulation optimization algorithm [16], which shows low applicability in settings with a lot of noise, the multiobjective whale optimization algorithm (MOWOA) [17], which has a long computation time, the multiobjective differential evolution [18], which lacks the preservation of diversity, the multiobjective dragonfly algorithm (MODA) [19], which has a large number of iterations, the multiobjective ant lion optimizer (MOALO) [20], which uses complicated objective functions, the multiobjective particle swarm optimization (MOPSO) [21], which does not deal with the constrained objective functions, the nondominated sorting of genetic algorithm (NSGA-II) [22], which has a range of parameters that can lead to slow convergence, the multiobjective binary cat swarm optimization (MOBCSO) [23], which has a large number of function evaluations, and multiobjective cuckoo search algorithm [24], [25], which needs adding Lévy flights to improve search capability.

The multiobjective optimization algorithms are applied as a solution to various problems in electric power systems, including determining the optimal placement of capacitors in RDNs. In [26], a nondominated sorting genetic algorithm (NSGA-II) is implemented to solve network reconfiguration, optimal placement of the distributed generator and capacitor; however, the parameters should be adjusted well to not fall into local optima. In [27], NSGA II

is also proposed to solve the proposed issue taking into account the total harmonic distortion constraint; however, there are some solutions on the Pareto front, which exceed the security boundaries in terms of voltage resonance. In [28], a comparison is made between NSGA II and Strength Pareto Evolutionary Algorithm (SPEA 2) to solve this issue taking into account the limitations of harmonic distortion. However, NSGA-II results in lower quality solutions, and SPEA 2 yields several solutions that violate voltage constraints. In [29], the multiobjective immune algorithm is proposed to solve the capacitor allocation problem in a distorted electrical distribution network. However, more parameters are used. In [30], a multiobjective flower pollination algorithm is applied for the optimal placement of shunt capacitors and solar distributed generations (DGs) along with the optimal network reconfiguring. However, interest rates impact annual economic savings. In [31], the whale optimization algorithm (WOA) and the moth swarm algorithm (MSA) are used and compared to solve the optimal allocation problem of shunt capacitors and DG units; however, WOA results in lower quality solutions and MSA has complicated objective functions. In [32], an improved multiobjective harmony search algorithm (IMOHSA) is used to find the optimal allocation of DG and capacitor; however, it has a high number of iterations. In [33], a combined approach is proposed to solve the proposed issue; however, complicated objective functions are used.

The water cycle algorithm (WCA) is used to solve single-optimization problems. A new WCA, the so-called “multiobjective water cycle algorithm” (MOWCA), has recently been proposed to solve multi-optimization problems. The WCA was introduced in 2012 by Sadollah, Eskandar, and Kim [11]. The WCA simulates the water cycle process, including the flowing water from rivers and streams to the sea [34]. After the evaporation process occurs, the clouds form, which later turn into rain. Then again, new streams and rivers are created. Compared to other well-known algorithms, this method offered superior and effective performance for solving discrete and continuous problems. WCA has encouraged researchers to apply it due to its accuracy and robustness in addition to its exploratory ability to provide the optimal solution [11], [13], [35]–[38]. In [34], the WCA is applied to solve the capacitor and DG allocation problem for single and multiobjective scenarios, and the WCA achieves high-grade solutions and offers great performance in addition to good convergence characteristics.

The main contributions of this article are minimization of total power loss, enhancement of voltage stability, and maximization of net savings with satisfying the voltage profile at all buses within the allowable limits through solving the optimal capacitor placement problem in radial distribution systems. Fixed and switched capacitors are considered. At first, the initial potential buses for capacitor placement are obtained using fuzzy expert rules. The bus voltage magnitude and the loss sensitivity factor are the inputs of these rules. For this, two membership functions are utilized. Afterwards, MOWCA is proposed to give a set of Pareto front solutions, and the decision marker is then used

to pick the best compromise among the final stored members. It should be mentioned that the proposed method of calculating a list of possible solutions for capacitor placement takes into account the bus voltages and losses of the system, unlike other approaches that only consider the voltage or losses. Numerical simulations are performed in MATLAB. The proposed method is tested on IEEE 33 and IEEE 94-bus RDNs. To validate the capabilities of the proposed methodology, the cropped results are compared with interior point algorithm (IP), a combination fuzzy real coded GA (FRCGA) method, and WCA for the IEEE 33-bus system and the artificial bee colony (ABC) algorithm for the IEEE 94-bus system obtained from the literature with comprehensive discussions.

The rest of this paper is organized as follows. Section II introduces the problem formulation. Section III explains the proposed techniques, including the normalized loss sensitivity factor  $LSF_{norm}$  and fuzzy expert rules. Section IV defines the multiobjective optimization problems. Section V explains in detail the concept of a multiobjective water cycle algorithm. Section VI displays the numerical results and discussion. Finally, Section VII presents the conclusions.

## II. PROBLEM FORMULATION

A constrained optimization problem has been investigated to mathematically determine the optimal allocation of fixed and switched capacitors, and that problem consists of two objectives with operational constraints.

### A. Total Cost

The formulation of  $f_1$  aims to reduce the total energy cost and increase the savings and can be determined using (1)

$$f_1 = K_p \times P_{loss} + K_c \sum_{j=1}^{N_c} Q_{c_j}, [\$/\text{year}], \quad (1)$$

where  $K_p$  is the annual cost per  $kw$  ( $\$/kW/\text{year}$ ),  $P_{loss}$  is the total active power loss ( $kw$ ),  $N_c$  is the number of installed capacitors,  $K_c$  is the sum of purchase and installation costs of capacitor per  $kvar$  ( $\$/kVAr$ ),  $Q_{c_j}$  is the reactive power of capacitor installed on bus  $j$  ( $kVAr$ ). The total annual cost of capacitors (TCC) can be determined by [1]

$$TCC = K_c \sum_{j=1}^{N_c} Q_{c_j} / \text{Life expectancy } (\$/\text{year}). \quad (2)$$

### B. Total Voltage Stability Index (TVSI)

TVSI is determined based on (3), which should be increased, whereas the voltage stability is calculated at the receiving end bus according to (4) [39]:

$$TVSI = \sum_{j=2}^{N_B} VSI(j), \quad (3)$$

$$VSI(j) = |v_i|^4 - 4 \times (P_j \times R_{ij} + Q_j \times X_{ij}) \times |v_i|^2 - 4 \times (P_j \times X_{ij} - Q_j \times R_{ij})^2, \quad (4)$$

where  $N_B$  is a total number of nodes,  $v_i$  is the voltage value of the sending end node  $i$ ,  $P_j$  and  $Q_j$  represent effective

active and reactive powers at the receiving end node  $j$ , respectively,  $R_{ij}$  and  $X_{ij}$  represent the resistance and reactance of the connecting line between nodes  $i$  and  $j$ , respectively.

Figure 1 presents a simple radial distribution line. The voltage stability index (VSI) is used to identify the buses that are probably going to collapse. For stable operation, the VSI values must be greater than zero. The node, which has the smallest value of VSI, is known as the weakest node, and the voltage collapse phenomenon will begin at that node. To minimize the chance of voltage collapse, VSI values for all nodes must be increased [2], [39], [40]. Optimizing this indicator means that the security level of the power system has become better.

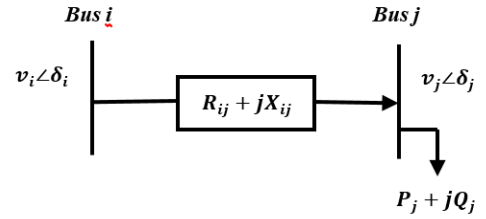


Fig. 1. A simple radial distribution line.

The objective function used to enhance TVSI is formulated as follows

$$f_2 = 1/TVSI. \quad (5)$$

### C. Multiobjective Function

The optimal capacitor placement problem aims to reduce power losses, enhance net savings, and total voltage stability index while satisfying system constraints. The mathematical formulation of the capacitor allocation problem can be expressed as multiple objectives using (6)

$$\text{minimize } (f_1; f_2). \quad (6)$$

### D. Operational Constraints

#### 1. Power balance constraints.

Power balance constraints for active and reactive powers are considered equality constraints. The active power supplied from the slack bus should be equal to the total active power demand and the total active power losses. The reactive power supplied from the slack bus and the total reactive compensation should be equal to the total reactive power demand and the total reactive power losses. They can be expressed as follows:

$$P_{slack} = \sum_{i=1}^{n_l} P_{D_i} + \sum_{j=1}^n P_{L_j}, \quad (7)$$

$$Q_{slack} + \sum_{i=1}^{N_c} Q_{c_i} = \sum_{i=1}^{n_l} Q_{D_i} + \sum_{j=1}^n Q_{L_j}, \quad (8)$$

where  $P_{slack}$  and  $Q_{slack}$  are active and reactive powers fed from the slack bus, respectively,  $P_{D_i}$  and  $Q_{D_i}$  are active and reactive load demands of bus  $i$ , respectively,  $P_{L_j}$  and  $Q_{L_j}$  are active and reactive power losses of line  $j$ , respectively,  $n_l$ ,  $n$ , and  $N_c$  are total numbers of load buses, lines, and

capacitors, respectively [4], [40].

## 2. Voltage limits.

The magnitudes of voltages for all buses must be kept within the allowable minimum and maximum limits

$$V_i^{\min} \leq |V_i| \leq V_i^{\max}, \quad (9)$$

where  $V_i^{\min}$  and  $V_i^{\max}$  are the upper and lower limits of bus  $i$ , respectively [41]–[43].

## 3. Reactive compensation limits.

Reactive power injection at each nominated bus should be restricted by lower and upper limits

$$Q_{cj}^{\min} \leq Q_{cj} \leq Q_{cj}^{\max}, \quad (10)$$

where  $Q_{cj}^{\min}$  and  $Q_{cj}^{\max}$  are the minimum and maximum limits of the reactive compensation on bus  $j$ , respectively.

## 4. Line capacity limits.

The apparent power flow for each line must be kept below or equal to the permitted capacity of each line

$$S_{li} \leq S_{li}^{\text{rated}}, \quad (11)$$

where  $S_{li}^{\text{rated}}$  is the permitted power flow through line  $i$ .

## 5. Total reactive power compensation.

Total reactive power compensation must not exceed total reactive load demand

$$Q_C^{\text{Total}} \leq Q_D^{\text{Total}}, \quad (12)$$

where  $Q_D^{\text{Total}}$  donates the total reactive load demand of the system [4], [40].

## III. DETERMINATION OF THE CANDIDATE BUSES FOR CAPACITOR PLACEMENT WITH FUZZY EXPERT RULES

The most potential buses for capacitor placement in radial distribution systems are determined to limit the effort of the search process for the MOWCA. Therefore, the CPU processing time and the probability of the load flow to diverge become lower. The normalized loss sensitivity factor  $LSF_{norm}(j)$  and  $|v_i|$  are developed herein for this target.  $LSF$  can identify which bus has the least loss when reactive compensation is placed. Buses with lower  $|v_i|$  and higher  $LSF$  have a higher priority to be candidate buses [39]. As shown in Fig. 1, the active power loss in branch  $ij$  can be determined as [1]

$$P_{\text{loss-}ij} = \frac{(P_j^2 + Q_j^2)}{|v_j|^2} \times R_{ij}. \quad (13)$$

$LSF$  is derived from [1]

$$LSF(j) = \frac{\partial P_{\text{loss-}ij}}{\partial Q_j} = \frac{2 \times Q_j \times R_{ij}}{|v_j|^2}. \quad (14)$$

$LSF$  is normalized to be modeled by fuzzy membership

functions, and it is determined by

$$LSF_{norm}(j) = \frac{LSF(j) - LSF_{min}}{LSF_{max} - LSF_{min}}, \quad j \in [2, N], \quad (15)$$

where  $LSF_{min}$  and  $LSF_{max}$  represent the smallest and largest  $LSF$  values, respectively,  $N$  is a total number of buses.  $LSF_{norm}$  values range from 0 to 1 [39].

The candidate locations for capacitor placement are selected on the basis of a set of fuzzy rules.  $|v_i|$  and  $LSF_{norm}$  are inputs for fuzzy expert rules, and suitability is the output consequent for these rules. Table I presents a summary of fuzzy rules in a decision matrix. These variables are expressed in linguistic terms: Small (S), Small-medium (SM), Medium (M), Large-medium (LM), Large (L), Small-Normal (SN), Normal (N), and Large-Normal (LN) [44]. The membership functions (MFs) for these variables with trapezoidal and triangular shapes are depicted in Figs. 2–4.

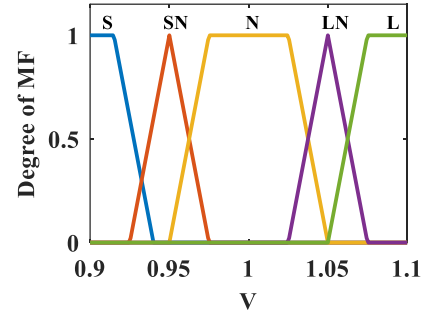


Fig. 2. Membership functions for voltage.

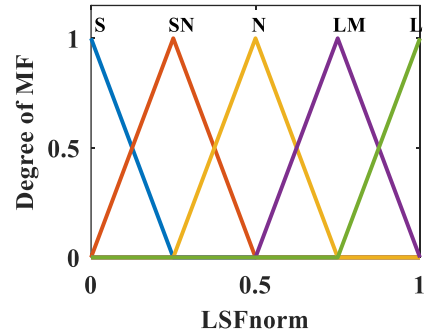


Fig. 3. Membership functions for  $LSF_{norm}$ .

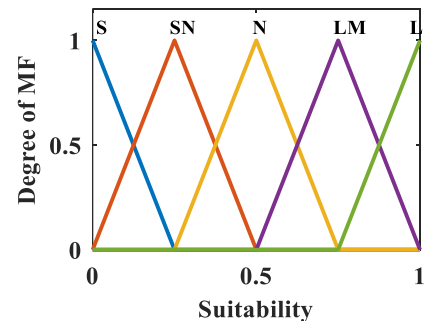


Fig. 4. Membership functions for suitability.

The surface view is shown in Fig. 5. The centroid method is used as a defuzzification technique to obtain the preliminary list of the most candidate locations for capacitor placement. Buses that have the highest suitability values are selected as candidate locations for capacitor placement.

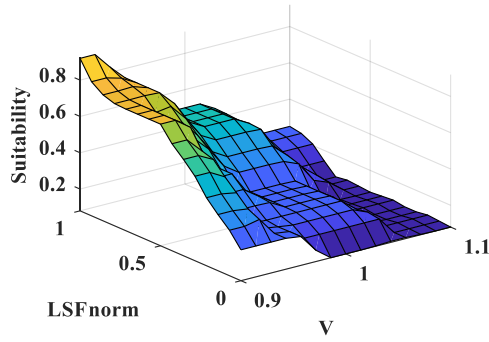


Fig. 5. The surface graph of fuzzy expert rules.

TABLE I. DECISION MATRIX FOR CALCULATING CANDIDATE BUSES.

AND		$ v_i $				
		S	SN	N	LN	L
$LSF_{norm}$	S	SM	SM	S	S	S
	SM	M	SM	SM	S	S
	M	LM	M	SM	S	S
	LM	LM	LM	M	SM	S
	L	L	LM	M	SM	SM

#### IV. MULTIOBJECTIVE OPTIMIZATION PROBLEMS (MOPs)

In MOPs, there are at least two objective functions that must be solved concurrently. MOPs can be mathematically written as:

$$\text{Minimize : } F(x) = \{f_1(x), f_2(x), \dots, f_m(x)\}, \quad (16)$$

$$\text{Subject to : } g_i(x) \geq 0, \forall i \in [1, n], \quad (17)$$

$$h_i(x) = 0, \forall i \in [1, p], \quad (18)$$

$$L_i \leq x_i \leq U_i, \forall i \in [1, k], \quad (19)$$

where  $x$  is a vector of decision variables,  $n$ ,  $p$ , and  $k$  are numbers of inequality constraints, equality constraints, and decision variables, respectively,  $g_i$  and  $h_i$  represent the  $i^{\text{th}}$  inequality and equality constraints, respectively,  $L_i$  and  $U_i$  are the lower and upper limits for the  $i^{\text{th}}$  variable, respectively [17].

The multiobjective optimization aims to obtain as many nondominated solutions as possible. They can be defined as:

– Pareto dominance:

Consider two vectors  $U$  and  $V$  in the objective space. A solution  $U$  is said to dominate solution  $V$  (i.e.,  $U < V$ ) if and only if solution  $U$  is partially less than solution  $V$ , which means [38]:

$$\begin{cases} f_i(U) \leq f_i(V), & \forall i \in [1, m], \\ f_i(U) < f_i(V), & \exists i \in [1, m], \end{cases} \quad (20)$$

where  $m$  represents the total number of objective functions.

– Pareto optimal solution:

$U$  is considered a Pareto optimal solution if and only if no solution has been calculated to dominate  $U$ . A set of Pareto-optimal solutions is known as a Pareto front [13].

#### V. WATER CYCLE ALGORITHM (WCA)

The WCA [11] is inspired by the influx of water from

rivers and streams to the sea and participates in the monitoring of the water cycle procedure [45].

Like other heuristic optimization techniques, a random population of streams of size  $N_{pop} \times N$  is initially created after the raining process as written in (21) [13], [38]

$$\text{Population} = [x_{ij}]_{N_{pop} \times N}, \forall i \in N_{pop}, \forall j \in N, \quad (21)$$

where  $x_{ij}$  is the decision variable,  $N_{pop}$  is the population size, and  $N$  is a number of decision variables.

$$\vec{x}_{initial} = \overline{LB} + rand \times (\overline{UB} - \overline{LB}), \quad (22)$$

where  $rand$  is a random value distributed between 0 and 1,  $\overline{LB}$  and  $\overline{UB}$  are the lower and upper limits of the design variables, respectively [13].

$N_{pop}$  is the sum of  $N_{sr}$  (i.e., the number of one sea and a total number of rivers  $N_r$ ) and the remaining number of streams  $N_{stream}$  as clarified in (23) and (24):

$$N_{sr} = I + N_r, \quad (23)$$

$$N_{pop} = N_{sr} + N_{stream}. \quad (24)$$

Each river and sea have assigned a number of streams determined by the following formula [35]

$$NS_n = round \left\{ \left\{ \frac{cost_n}{\sum_{i=1}^{N_{sr}} cost_i} \right\} \times N_{stream} \right\}, \forall n \in N_{sr}. \quad (25)$$

The distance  $X$  between the river and the stream may be changed randomly according to (26)

$$X \in (0, C \times d), C > 1, \quad (26)$$

where  $1 < C < 2$  and it is preferably equal to 2,  $d$  is the existing distance between the river and the stream [11].

For the  $N_{pop}$  dimensional search space, the new locations for streams and rivers can be updated by [13]:

$$\vec{x}_{Stream}^{i+1} = \vec{x}_{Stream}^i + rand \times C \times (\vec{x}_{River}^i - \vec{x}_{Stream}^i), \quad (27)$$

$$\vec{x}_{Stream}^{i+1} = \vec{x}_{Stream}^i + rand \times C \times (\vec{x}_{Sea}^i - \vec{x}_{Stream}^i), \quad (28)$$

$$\vec{x}_{River}^{i+1} = \vec{x}_{River}^i + rand \times C \times (\vec{x}_{Sea}^i - \vec{x}_{River}^i). \quad (29)$$

The exchange between the locations of both the river and the sea will take place if the solution of the sea is worse than that of the river. This is also done for the stream and its connecting river [35].

After updating the locations of the streams and rivers, the evaporation process will be addressed. The evaporation process prevents early convergence. Basically, the evaporation process makes seawater evaporate as rivers/streams flow to the sea, which in turn has resulted in new precipitations. So, we must verify if the distance between river/stream and sea is close enough to make the evaporation process happen. The following pseudocode is used for this purpose

$$\begin{aligned} & \text{if } \text{norm}(x_{\text{Sea}}^i - x_{\text{River}}^i) < d_{\text{max}}, \\ & \quad \text{execute raining process,} \\ & \quad \text{end,} \end{aligned} \quad (30)$$

where  $d_{\text{max}}$  is a low value near zero.

After the evaporation process occurs, the raining process creates new streams in different positions. The positions of the streams that are lately creating are determined by [11], [37], [46]

$$\vec{x}_{\text{new stream}} = \overline{LB} + \text{rand} \times (\overline{UB} - \overline{LB}). \quad (31)$$

It is worth stating that extra searches are avoided by a high value of  $d_{\text{max}}$  while the low value of  $d_{\text{max}}$  promotes the search close to the sea intensively. The value of  $d_{\text{max}}$  is adaptively reduced by [11]

$$d_{\text{max}}^{i+1} = d_{\text{max}}^i - \frac{d_{\text{max}}^i}{\text{Max}_{\text{iter}}}. \quad (32)$$

*Proposed MOWCA.* A crowding distance mechanism is applied to pick the most possible solutions in the population as a sea and rivers. The crowding distance can express the density of nondominant solutions in the archive. To estimate crowding distance values, the population in the archive must be arranged in ascending order depending on each cost function. The periphery solutions for each cost function are customized. The set distance value for all other intermediate nondominant solutions can be calculated as the absolute value of the normalized difference of two adjacent solutions. Before determining the crowding distance values, each cost function is normalized. The total crowding distance value is calculated as the sum of crowding distance values for each objective [13].

Once we obtain Pareto front archive, which includes nondominant solutions, we need to choose one of these solutions as the optimal solution by calculating  $\mu_i$  as

$$\mu_i = \frac{F_i - F_i^{\min}}{F_i^{\max} - F_i^{\min}}, \quad \forall i \in m, \quad (33)$$

where  $\mu_i$  is the  $i^{\text{th}}$  membership function,  $F_i$  is the  $i^{\text{th}}$  objective function,  $m$  is a number of solutions,  $\mu_i$  varies between 0 and 1. The normalized membership function is determined for each optimal Pareto solution  $k$  by

$$\mu^k = \frac{\sum_{i=1}^n \mu_i^k}{\sum_{k=1}^m \sum_{i=1}^n \mu_i^k}, \quad (34)$$

where  $n$  is the number of objective functions.

The optimal solution to the capacitor placement problem can be determined according to (35) [39]

$$\text{Best Solution} = \min\{\mu^1, \mu^2, \dots, \mu^k\}. \quad (35)$$

Figure 6 presents the MOWCA flow chart.

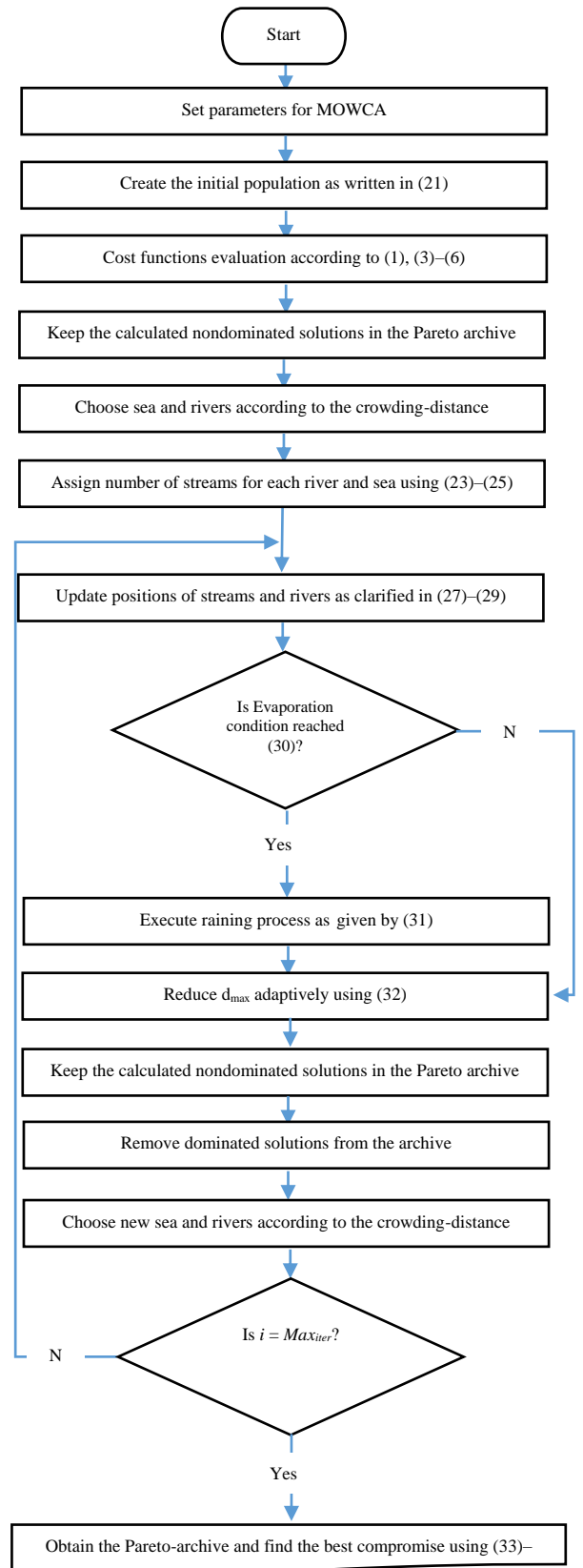


Fig. 6. MOWCA flow chart.

## VI. NUMERICAL RESULTS AND DISCUSSION

To illustrate the features of the proposed method and check its viability, the 33-bus and actual Portuguese 94-bus radial test systems are used. Two matrixes are used to solve the load flow problem in radial distribution systems: the bus injection to branch current (BIBC) matrix and the branch

current to bus voltage (BCBV) matrix [47]. The backward/forward sweep algorithm [48] is suggested to iteratively solve the equations and determine the node voltages. The proposed MOWCA and load flow method are coded and developed in the MATLAB R2017b version. The simulations are implemented in a Dell laptop with processor Intel(R) Core (TM) i3-3217U CPU @ 1.80 GHz, a 4.0 GB of RAM & 64-bit operating system.

The optimal capacitor placement problem is treated as a multiobjective problem that seeks to improve total voltage stability and net savings while satisfying all operating system constraints. Comparisons are made with other published researches on the voltage profile of system buses, total power loss, total voltage stability, and net savings.

Two scenarios are studied to determine the optimal solution:

- Scenario 1: Optimal allocation of fixed capacitors;
- Scenario 2: Optimal allocation of switched capacitors.

The values of  $K_p$ ,  $K_c$ , and life expectancy are supposed to be 168 \$/(kW/year), 5 \$/kVAr, and 10 years, respectively. Maintenance and running costs are omitted. Bus number 1 is considered a slack bus with voltage 1 p.u for all test systems.

The following list of MOWCA tuned parameters with the required inequality constraints is presented in Table II.

TABLE II. MOWCA PARAMETERS WITH THE SETTINGS OF REQUIRED CONSTRAINTS.

Set parameters	33-bus RDN	94-bus RDN
$N_{pop}$	150	
$N_{sr}$	25	
$d_{max}$	$10^{-16}$	
$Max\_iter$	300	500
Limit of voltage magnitude	$0.95 \leq  V_i  \leq 1.05$ p.u	$0.9 \leq  V_i  \leq 1.1$ p.u
Permissible fixed capacitor limit	50 to 1500 kVAr	
Permissible switched capacitor limit	50 to 1500 kVAr by step 50 kVAr	

#### A. IEEE 33-Bus System

This system consists of 33 buses and 32 lines. Bus and load data are obtained from [49]. The single-line diagram of this system is portrayed in Fig. 7. The simulation calculations adopt the per-unit system. The base values of this system are 12.66 kV and 10 MVA. The total load demand is 3715 kW and 2300 kVAr. The first top 9 buses, depending on the fuzzy rankings, are nominated as potential locations for capacitor placement. They are 28, 6, 29, 8, 30, 9, 13, 10, and 3.

This system is characterized by heavy inductive loads that result in lower voltage values on system buses. The total power loss before compensation is 210.998 kW and 143.033 kVAr. The annual cost of the kW loss in the base case is 35447.66 \$. Bus voltages and system losses are improved by installing capacitor units, which provide reactive compensation resulting in a reduction in the current flow of the line.

Table III summarizes the cropped results in the base case and after applying the MOWCA against the results of other competing algorithms (e.g., the interior point algorithm (IP)

[50] and a combination fuzzy real coded GA (FRCGA) method [51]) in the 33-bus system to reduce power losses, improve the voltage profile, total voltage stability, and net savings.

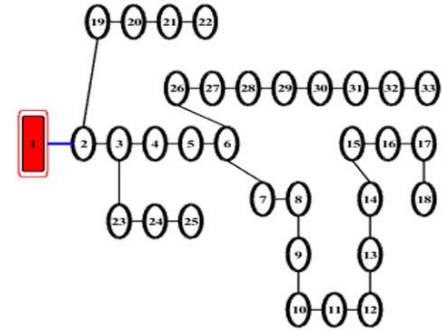


Fig. 7. Single-line diagram of the 33-bus radial test system.

The proposed MOWCA is superior to other existing techniques in that it achieves the least active power loss and the highest annual net savings due to allocating capacitors at the appropriate sizes and optimal locations. It can also be observed that scenario 2 can reduce power losses to 145.9700 kW and 100.2083 kVAr with net savings of 9999.7519 \$/year corresponding to percentage savings of 28.2137 % that are slightly better than those obtained by scenario 1. It can be noted that TVSI is enhanced after MOWCA is implemented. Scenario 1 can improve TVSI to 28.0289, which is slightly more than 27.9532 of scenario 2. Figures 8 and 9 present the graphs of Pareto solutions and the best compromise in the 33-bus RDN for scenarios 1 and 2, respectively. The comparison of voltage profiles before and after compensation for both scenarios is shown in Fig. 10. Figure 10 highlights the improvement in voltage profiles in all buses after proposing capacitors and both scenarios give almost the same results for most buses. The minimum voltage is recorded in the base case of 0.9039 p.u in bus number 18. After proposing a MOWCA-based approach, the lowest voltage is better improved and meets the voltage limits. The lowest voltage at bus number 18 is 0.9500 p.u and 0.9502 p.u for scenarios 1 and 2, respectively, which are lower than the FRCGA of 0.9665 p.u.

The total reactive compensation using MOWCA is 1934 kVAr for scenario 1 and 1850 kVAr for scenario 2, which are significantly lower than IP of 2150 kVAr and FRCGA of 2250 kVAr. Although the total injected kVAr, total active and reactive power losses, and total cost using MOWCA are lower than those using IP and FRCGA, the minimum voltage using MOWCA is lower than those using FRCGA.

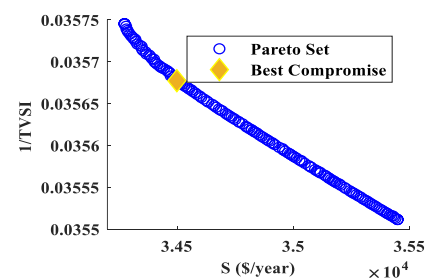


Fig. 8. Best compromise among the Pareto optimal set for scenario 1 in IEEE 33-bus.

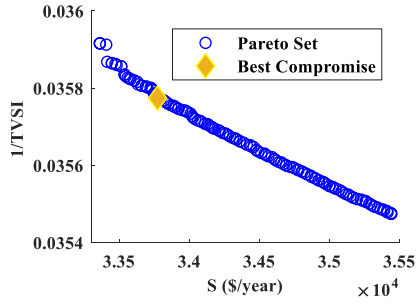


Fig. 9. Best compromise among the Pareto optimal set for scenario 2 in IEEE 33-bus.

The proposed method outweighs other methods in annual cost and net savings, which means better performance of the system. It should be highlighted that the total capacitor cost (TCC) also decreases consequently as the performance of the system has been enhanced. The processing time for the proposed MOWCA-based approach is 877 seconds. The water cycle algorithm is also proposed in [34] to solve the problem of capacitor and DG allocation considering single and multiple objectives. Table IV presents the results of the

proposed MOWCA and WCA [34] when optimizing only the power losses using shunt capacitors.

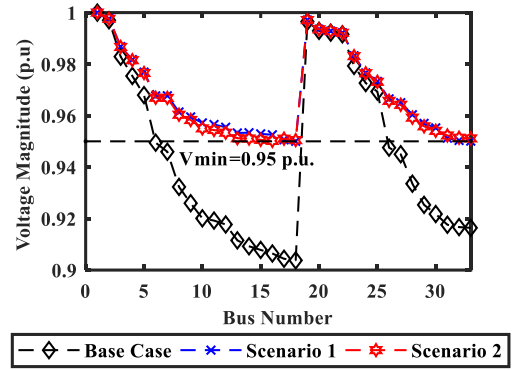


Fig. 10. Voltage profile for the 33-bus system with and without compensation.

Although the minimum voltage and total injected reactive power using scenario 2 and WCA are almost equal, the reduction in power loss (%) using WCA is more than that obtained using the proposed approach.

TABLE III. RESULTS AT BASE CASE AND AFTER APPLYING MOWCA IN 33-BUS RDN.

Item	Base Case	IP [50]	FRCGA [51]	Proposed MOWCA	
				Scenario 1	Scenario 2
$\sum Q_C$ (kVAr)	-	2150	2250	1934	1850
$V_{min}$ (p.u)	0.9039 (18)	-	0.9665	0.9500	0.9502
$V_{max}$ (p.u)	0.9970 (2)	-	-	0.9976	0.9976
$\sum P_{Loss}$ (kW)	210.998	171.78	148.6951	147.7899	145.9700
$\sum Q_{Loss}$ (kVAr)	143.033	-	-	101.5347	100.2083
$VSI_{min}$	0.6672 (18)	-	0.8652	0.8146	0.8151
$VSI_{max}$	0.9881 (2)	-	-	0.9906	0.9905
$TVSI$	25.5401	-	-	28.0289	27.9532
Optimal size in kVAr (location)	-	450 (9)	100 (28)	50 (28)	650 (30)
		800 (29)	325 (6)	1004 (30)	350 (9)
		900 (30)	425 (29)	187 (9)	50 (13)
			350 (8)	203 (10)	350 (31)
			675 (30)	223 (14)	200 (14)
		375 (9)	267 (16)	250 (17)	
Annual cost of kW loss (\$/year)	35447.66	28859.04	24980.78	24828.71	24522.97
Total capacitors cost (TCC) (\$/year)	-	1075	1125	967	925
Total cost (C) (\$/year)	-	29934.04	26105.777	25795.71	25447.968
Net savings (\$/year)	-	5513.7	9341.9	9652.0094	9999.7519
% Savings	-	15.55%	26.35 %	27.2325%	28.2137 %

TABLE IV. COMPARISON BETWEEN MOWCA AND WCA.

Item	WCA [34]	Proposed method	
		Scenario 1	Scenario 2
$\sum Q_C$ (kVAr)	1848.4	1934	1850
$V_{min}$ (p.u)	0.951	0.9500	0.9502
Reductions in $P_{Loss}$ (%)	35.4063	29.9568	30.8194
Optimal size in kVAr (location)	397.3 (14) 451.1 (24) 1000 (30)	50 (28)	650 (30)
		1004 (30)	350 (9)
		187 (9)	50 (13)
		203 (10)	350 (31)
		223 (14)	200 (14)
	267 (16)	250 (17)	

### B. IEEE 94-Bus System

This system has 94 buses and 93 lines. The total load demand is 4797 kW and 2323.9 kVAr with a voltage level of 15 kV. The bus and load data for this system are found in [52]. Figure 11 shows the single-line diagram of this system. According to fuzzy rankings, the first 25 top locations are designated as candidate buses for capacitor placement. They are 11, 90, 10, 18, 21, 54, 52, 15, 83, ...

The total power losses and the annual cost of the kW loss in the base case are 362.86 kW, 504.04 kVAr, and 60960.48 \$, respectively. The minimum voltage in the base case is 0.8485 at bus number 92. The minimum voltage is



not maintained within the allowable bounds because this system has demanding characteristics (i.e., high length of line and heavy load).

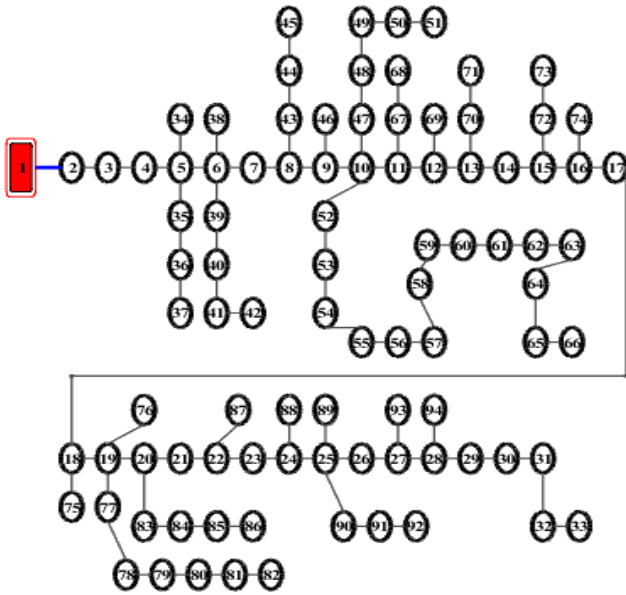


Fig. 11. Single-line diagram of the 94-bus radial test system.

The results collected in the base case and after proposing MOWCA in addition to comparing it with the artificial bee colony (ABC) algorithm [40] are listed in Table V.

TABLE V. OPTIMAL RESULTS AFTER APPLYING MOWCA IN 94-BUS RDN.

Item	Base case	ABC algorithm [40]	Proposed method	
			Scenario 1	Scenario 2
$\sum Q_c$ (kVAr)	-	2100	2323	2300
$V_{min}$ (p.u)	0.8485 (92)	0.90721	0.9174	0.9168
$V_{max}$ (p.u)	0.9951 (2)	0.99699	0.9972	0.9972
$\sum P_{Loss}$ (kW)	362.86	271.3590	270.4281	269.5503
$\sum Q_{Loss}$ (kVAr)	504.04	374.5060	373.7962	373.1232
$VSI_{min}$	0.5183	0.6774	0.7082	0.7065
$VSI_{max}$	0.9804	0.9879	0.9887	0.9886
$TVSI$	62.2650	75.0565	75.9089	75.7675
Optimal size in kVAr (location)	-	600 (18) 450 (21) 1050 (54)	50 (10) 521 (15) 610 (20) 318 (23) 642 (57) 50 (22) 132 (56)	300 (11) 450 (18) 100 (21) 350 (83) 300 (24) 750 (57) 50 (53)
Annual cost of kW loss (\$/year)	60960.48	45588.31	45431.92	45284.45
Total capacitors cost (TCC) (\$/year)	-	1050	1161.5	1150
Total cost (C) (\$/year)	-	46638.31	46593.42	46434.45
Net savings (\$/year)	-	14322.17	14367.06	14526.03
% Savings	-	23.4942 %	23.5678 %	23.8286 %

The proposed method yields better results than the ABC algorithm in terms of reducing total active power losses

from 362.86 kW to 270.4281 kW and 269.5503 kW for scenarios 1 and 2, respectively, and enhancing the net savings to 14367.06 \$/year in the case of scenario 1 and 14526.03 \$/year in the case of scenario 2, corresponding to % savings 23.5678 % in the case of scenario 1 and 23.8286 % in the case of scenario 2. In addition, there is a marked improvement in TVSI of 75.9089 with scenario 1 and 75.7675 with scenario 2, which are better than 75.0565 with the ABC algorithm. Moreover, it is manifested that the minimum and maximum voltages and VSI after implementing MOWCA are enhanced compared to those obtained by the ABC algorithm. This is due to the total reactive compensation using MOWCA being higher than that of using the ABC algorithm. It should be highlighted that scenario 2 achieves slightly better results than scenario 1 for total power losses and net savings, but for TVSI, scenario 1 is slightly better. Figures 12 and 13 show the Pareto optimal set and the best compromise solution when scenarios 1 and 2 are used, respectively. The voltage profiles in the base case and after compensation for both scenarios using MOWCA are depicted in Fig. 14. It is noticeable that all bus voltages of this system are improved and both scenarios give almost the same results. The time it takes to implement the MOWCA-based method is about 4971 seconds.

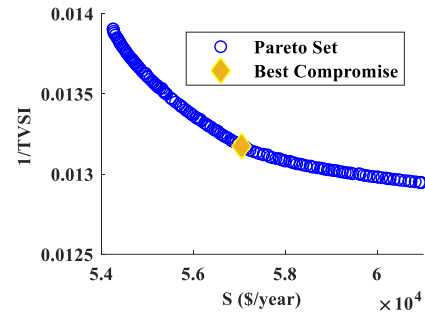


Fig. 12. Best compromise among Pareto optimal set for scenario 1 in IEEE 94-bus.

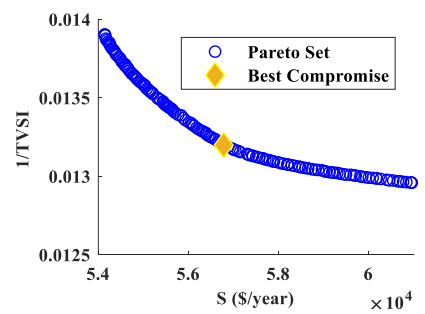


Fig. 13. Best compromise among the Pareto optimal set for scenario 2 in IEEE 94-bus.

The total injected power using MOWCA is 2323 kVAr and 2300 kVAr for scenarios 1 and 2, respectively, which is more than the ABC algorithm of 2100 kVAr. Although the power losses, minimum voltage, TVSI, annual cost of kW loss, and net savings are better using MOWCA than those using ABC algorithm, yet total capacitors cost (TCC) using MOWCA is more than that of using ABC algorithm due to more injected VARs using MOWCA. The proposed method gives better results than the ABC algorithm in terms of voltage profile, power losses, voltage stability, annual cost,

and net savings, which means better performance of the system.

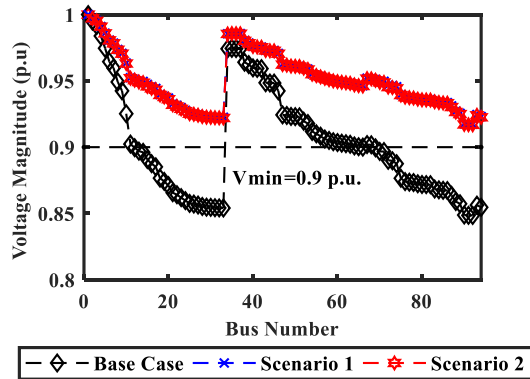


Fig. 14. Voltage profile for the 94-bus system with and without compensation.

## VII. CONCLUSIONS

In this article, a two-stage procedure is addressed to solve the optimal allocation problem of capacitors considering fixed and switched capacitors in IEEE 33 and IEEE 94-bus RDNs to increase the net savings and boost the static voltage stability index. A multiobjective scenario is considered. At first, bus voltages and loss sensitivity factor are employed as inputs for the fuzzy expert rules to find the most candidate buses for capacitor allocation. The MOWCA-based approach is then implemented to determine the optimal size and location of capacitors.

The proposed MOWCA is compared with IP, FRCGA, WCA for the 33-bus system, and the ABC algorithm for the 94-bus system reported from the literature. For the IEEE 33-bus system, MOWCA results in active and reactive power losses of 147.7899 kW, 101.5347 kVAr, 145.9700 kW, 100.2083 kVAr, total voltage stability index of 28.0289, 27.9532 and net savings of 27.2325 %, 28.2137 % for scenarios 1 and 2, respectively. For the IEEE 94-bus system, MOWCA achieves active and reactive power losses of 270.4281 kW, 373.7962 kVAr, 269.5503 kW, 373.1232 kVAr, total voltage stability index of 75.9089, 75.7675 and net savings of 23.5678 %, 23.8286 % for scenarios 1 and 2, respectively. A considerable improvement in net savings and total voltage stability index raises the security level and system performance. Comparisons with other methods considered in this paper have illustrated that MOWCA is robust and has excellent features in providing high-quality solutions.

The future work will propose an approach to simultaneously solve the optimal allocation problem of multi-type DG units and shunt capacitors in the radial distribution systems based on MOWCA-based approach. The purpose of this work is to achieve better results in terms of voltage profile, voltage stability, losses, and net savings.

## CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

## REFERENCES

- [1] A. A. A. El-Ela, R. A. El-Sehiemy, A.-M. Kinawy, and M. T. Mouwafi, "Optimal capacitor placement in distribution systems for power loss reduction and voltage profile improvement", *IET Gener. Transm. Distrib.*, vol. 10, no. 5, pp. 1209–1221, Apr. 2016. DOI: 10.1049/iet-gtd.2015.0799.
- [2] K. R. Devabalaji, T. Yuvaraj, and K. Ravi, "An efficient method for solving the optimal siting and sizing problem of capacitor banks based on cuckoo search algorithm", *Ain Shams Eng. J.*, vol. 9, no. 4, pp. 589–597, Dec. 2018. DOI: 10.1016/j.asej.2016.04.005.
- [3] M. Shahzad, Q. Shafiullah, W. Akram, M. Arif, and B. Ullah, "Reactive power support in radial distribution network using mine blast algorithm", *Elektronika ir Elektrotechnika*, vol. 27, no. 4, pp. 33–40, 2021. DOI: 10.5755/j02.eie.28917.
- [4] S. K. Injeti, V. K. Thunuguntla, and M. Shareef, "Optimal allocation of capacitor banks in radial distribution systems for minimization of real power loss and maximization of network savings using bio-inspired optimization algorithms", *Int. J. Electr. Power Energy Syst.*, vol. 69, pp. 441–455, Jul. 2015. DOI: 10.1016/j.ijepes.2015.01.040.
- [5] H. A. Ramadan, M. A. A. Wahab, A.-H. M. El-Sayed, and M. M. Hamada, "A fuzzy-based approach for optimal allocation and sizing of capacitor banks", *Electr. Power Syst. Res.*, vol. 106, pp. 232–240, Jan. 2014. DOI: 10.1016/j.epr.2013.08.019.
- [6] H.-C. Chin and W.-M. Lin, "Capacitor placements for distribution systems with fuzzy algorithm", in *Proc. of TENCON '94 - 1994 IEEE Region 10's 9th Annu. Int. Conf. on: 'Frontiers of Computer Technology'*, 1994, pp. 1025–1029. DOI: 10.1109/TENCON.1994.369135.
- [7] C.-T. Su and C.-C. Tsai, "A new fuzzy-reasoning approach to optimum capacitor allocation for primary distribution systems", in *Proc. of the IEEE Int. Conf. on Industrial Technology (ICIT'96)*, 1996, pp. 237–241. DOI: 10.1109/ICIT.1996.601580.
- [8] I. Szuvovivski, T. S. P. Fernandes, and A. R. Aoki, "Simultaneous allocation of capacitors and voltage regulators at distribution networks using genetic algorithms and optimal power flow", *Int. J. Electr. Power Energy Syst.*, vol. 40, no. 1, pp. 62–69, Sep. 2012. DOI: 10.1016/j.ijepes.2012.02.006.
- [9] L. Zhang, C. Shen, Y. Chen, S. Huang, and W. Tang, "Coordinated optimal allocation of DGs, capacitor banks and SOPs in active distribution network considering dispatching results through bi-level programming", *Energy Procedia*, vol. 142, pp. 2065–2071, Dec. 2017. DOI: 10.1016/j.egypro.2017.12.503.
- [10] M. Ihsan, M. Shahzad, and N. Ullah, "Analytical method for optimal reactive power support in power network", in *Proc. of 2019 2nd Int. Conf. on Computing, Mathematics and Engineering Technologies (iCoMET)*, 2019, pp. 1–6. DOI: 10.1109/ICOMET.2019.8673418.
- [11] A. Sadollah, H. Eskandar, and J. H. Kim, "Water cycle algorithm for solving constrained multi-objective optimization problems", *Appl. Soft Comput.*, vol. 27, pp. 279–298, Feb. 2015. DOI: 10.1016/j.asoc.2014.10.042.
- [12] S. Mirjalili, P. Jangir, and S. Saremi, "Multi-objective ant lion optimizer: A multi-objective optimization algorithm for solving engineering problems", *Appl. Intell.*, vol. 46, no. 1, pp. 79–95, 2017. DOI: 10.1007/s10489-016-0825-8.
- [13] M. A. Elhameed and A. A. El-Fergany, "Water cycle algorithm-based economic dispatcher for sequential and simultaneous objectives including practical constraints", *Appl. Soft Comput.*, vol. 58, pp. 145–154, Sep. 2017. DOI: 10.1016/j.asoc.2017.04.046.
- [14] A. Messac, A. Ismail-Yahaya, and C. A. Mattson, "The normalized normal constraint method for generating the Pareto frontier", *Struct. Multidisc. Optim.*, vol. 25, pp. 86–98, 2003. DOI: 10.1007/s00158-002-0276-1.
- [15] G. Sauvanet and E. Néron, "Search for the best compromise solution on multiobjective shortest path problem", *Electronic Notes in Discrete Mathematics*, vol. 36, no. 1, pp. 615–622, Aug. 2010. DOI: 10.1016/j.endm.2010.05.078.
- [16] S. R. Gonzalez, H. Jalali, and I. V. Nieuwenhuys, "A multiobjective stochastic simulation optimization algorithm", *Eur. J. Operational Res.*, vol. 284, no. 1, pp. 212–226, Jul. 2020. DOI: 10.1016/j.ejor.2019.12.014.
- [17] J. Wang, P. Du, T. Niu, and W. Yang, "A novel hybrid system based on a new proposed algorithm—Multi-objective whale optimization algorithm for wind speed forecasting", *Appl. Energy*, vol. 208, pp. 344–360, Dec. 2017. DOI: 10.1016/j.apenergy.2017.10.031.
- [18] C. Jariyatantiwait and G. G. Yen, "Fuzzy multiobjective differential evolution using performance metrics feedback", in *Proc. of 2014 IEEE Congress on Evolutionary Computation (CEC)*, 2014, pp. 1959–1966. DOI: 10.1109/CEC.2014.6900533.
- [19] S. Mirjalili, "Dragonfly algorithm: A new meta-heuristic optimization technique for solving single-objective, discrete, and multi-objective problems", *Neural Comput. & Applic.*, vol. 27, no. 4, pp. 1053–1073, May 2016. DOI: 10.1007/s00521-015-1920-1.

- [20] K. Hosseini, S. Araghi, M. B. Ahmadian, and V. Asadian, "Multi-objective optimal scheduling of a micro-grid consisted of renewable energies using multi-objective ant lion optimizer", in *Proc. of 2017 Smart Grid Conf. (SGC)*, 2017, pp. 1–8. DOI: 10.1109/SGC.2017.8308867.
- [21] C. A. Coello Coello and M. S. Lechuga, "MOPSO: A proposal for multiple objective particle swarm optimization", in *Proc. of the 2002 Congress on Evolutionary Computation. CEC'02 (Cat. No. 02TH8600)*, 2002, pp. 1051–1056, vol. 2. DOI: 10.1109/CEC.2002.1004388.
- [22] Z. Hongwu, Z. Jinya, L. Yan, and Y. Chun, "Multi-objective optimization of helico-axial multiphase pump impeller based on NSGA-II", in *Proc. of 2009 Second Int. Conf. on Intelligent Computation Technology and Automation*, 2009, pp. 202–205. DOI: 10.1109/ICICTA.2009.515.
- [23] L. Pappula and D. Ghosh, "Planar thinned antenna array synthesis using multi-objective binary cat swarm optimization", in *Proc. of 2015 IEEE Int. Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting*, 2015, pp. 2463–2464. DOI: 10.1109/APS.2015.7305620.
- [24] L. Chen *et al.*, "A multi-objective cuckoo search algorithm based on decomposition", in *Proc. of 2019 Eleventh Int. Conf. on Advanced Computational Intelligence (ICACI)*, 2019, pp. 229–233. DOI: 10.1109/ICACI.2019.8778450.
- [25] P. Sherubha, P. Amudhavalli, and S. P. Sasirekha, "Clone attack detection using random forest and multi objective cuckoo search classification", in *Proc. of 2019 Int. Conf. on Communication and Signal Processing (ICCSPP)*, 2019, pp. 0450–0454. DOI: 10.1109/ICCSPP.2019.8698077.
- [26] R. J. C. Gallano and A. C. Nerves, "Multi-objective optimization of distribution network reconfiguration with capacitor and distributed generator placement", in *Proc. of TENCON 2014 - 2014 IEEE Region 10 Conf.*, 2014, pp. 1–6. DOI: 10.1109/TENCON.2014.7022365.
- [27] J. H. D. Onaka *et al.*, "Optimal capacitor banks placement in distribution grids using NSGA II and harmonic resonance chart", in *Proc. of 2016 17th Int. Conf. on Harmonics and Quality of Power (ICHQP)*, 2016, pp. 89–94. DOI: 10.1109/ICHQP.2016.7783363.
- [28] J. H. D. Onaka *et al.*, "Comparing NSGA-II and SPEA2 metaheuristics in solving the problem of optimal capacitor banks placement and sizing in distribution grids considering harmonic distortion restrictions", in *Proc. of 2016 17th Int. Conf. on Harmonics and Quality of Power (ICHQP)*, 2016, pp. 77–82. DOI: 10.1109/ICHQP.2016.7783313.
- [29] H. do Nascimento Alves, "Optimal placement of capacitor banks in distorted electrical distribution network based on a constrained multi-objective immune algorithm", in *Proc. of 2016 IEEE Congress on Evolutionary Computation (CEC)*, 2016, pp. 3933–3938. DOI: 10.1109/CEC.2016.7744288.
- [30] S. R. Gampa, S. Makkena, P. Goli, and D. Das, "FPA Pareto optimality-based multiobjective approach for capacitor placement and reconfiguring of urban distribution systems with solar DG units", *Int. J. Ambient Energy*, pp. 1–17, 2020. DOI: 10.1080/01430750.2020.1713887.
- [31] A. A. Saleh, A.-A. A. Mohamed, and A. M. Hemeida, "Optimal allocation of distributed generations and capacitor using multi-objective different optimization techniques", in *Proc. of 2019 Int. Conf. on Innovative Trends in Computer Engineering (ITCE)*, 2019, pp. 377–383. DOI: 10.1109/ITCE.2019.8646426.
- [32] V. Kona and R. Kollu, "A multi-objective approach for DG and capacitor placement using harmony search algorithm", in *Proc. of 2017 Ninth Int. Conf. on Advanced Computing (ICoAC)*, 2017, pp. 320–325. DOI: 10.1109/ICoAC.2017.8441380.
- [33] I. Trach and Y. Zubiuk, "A combined approach to multi-objective optimization of capacitor placement in radial distribution networks", in *Proc. of 2013 3rd Int. Conf. on Electric Power and Energy Conversion Systems*, 2013, pp. 1–5. DOI: 10.1109/EPECS.2013.6713040.
- [34] A. A. A. El-Ela, R. A. El-Sheimy, and A. S. Abbas, "Optimal placement and sizing of distributed generation and capacitor banks in distribution systems using water cycle algorithm", *IEEE Syst. J.*, vol. 12, no. 4, pp. 3629–3636, Dec. 2018. DOI: 10.1109/JSYST.2018.2796847.
- [35] H. M. Hasanien, "Transient stability augmentation of a wave energy conversion system using a water cycle algorithm-based multiobjective optimal control strategy", *IEEE Trans. Ind. Inform.*, vol. 15, no. 6, pp. 3411–3419, Jun. 2019. DOI: 10.1109/TII.2018.2871098.
- [36] X. Yang, K. Yao, W. Meng, and L. Yang, "Optimal scheduling of CCHP with distributed energy resources based on water cycle algorithm", *IEEE Access*, vol. 7, pp. 105583–105592, 2019. DOI: 10.1109/ACCESS.2019.2926803.
- [37] C. Veeramani and S. Sharanya, "Analyzing the performance measures of multi-objective water cycle algorithm for multi-objective linear fractional programming problem", in *Proc. of 2018 Second Int. Conf. on Intelligent Computing and Control Systems (ICICCS)*, 2018, pp. 297–306. DOI: 10.1109/ICCONS.2018.8662923.
- [38] A. Sadollah, H. Eskandar, A. Bahreinejad, and J. H. Kim, "Water cycle algorithm for solving multi-objective optimization problems", *Soft Comput.*, vol. 19, no. 9, pp. 2587–2603, 2015. DOI: 10.1007/s00500-014-1424-4.
- [39] A. El-Fergany, "Multi-objective allocation of multi-type distributed generators along distribution networks using backtracking search algorithm and fuzzy expert rules", *Electr. Power Compon. Syst.*, vol. 44, no. 3, pp. 252–267, 2016. DOI: 10.1080/15325008.2015.1102989.
- [40] A. A. El-Fergany and A. Y. Abdelaziz, "Capacitor placement for net saving maximization and system stability enhancement in distribution networks using artificial bee colony-based approach", *Int. J. Electr. Power Energy Syst.*, vol. 54, pp. 235–243, Jan. 2014. DOI: 10.1016/j.ijepes.2013.07.015.
- [41] V. Tamilselvan, T. Jayabarathi, T. Raghunathan, and X.-S. Yang, "Optimal capacitor placement in radial distribution systems using flower pollination algorithm", *Alex. Eng. J.*, vol. 57, no. 4, pp. 2775–2786, Dec. 2018. DOI: 10.1016/j.aej.2018.01.004.
- [42] S. R. Biswal and G. Shankar, "Optimal sizing and allocation of capacitors in radial distribution system using sine cosine algorithm", in *Proc. of 2018 IEEE Int. Conf. on Power Electronics, Drives and Energy Systems (PEDES)*, 2018, pp. 1–4. DOI: 10.1109/PEDES.2018.8707739.
- [43] A. Mujezinović, N. Turković, N. Dautbašić, M. M. Dedović, and I. Turković, "Use of integer genetic algorithm for optimal allocation and sizing of the shunt capacitor banks in the radial distribution networks", in *Proc. of 2019 18th Int. Symposium INFOTEH-JAHORINA (INFOTEH)*, 2019, pp. 1–6. DOI: 10.1109/INFOTEH.2019.8717653.
- [44] H. N. Ng, M. M. A. Salama, and A. Y. Chikhani, "Capacitor allocation by approximate reasoning: Fuzzy capacitor placement", *IEEE Trans. Power Deliv.*, vol. 15, no. 1, pp. 393–398, Jan. 2000. DOI: 10.1109/61.847279.
- [45] S. Khalilpourazari and M. Mohammadi, "Optimization of closed-loop supply chain network design: A water cycle algorithm approach", in *Proc. of 2016 12th Int. Conf. on Industrial Engineering (ICIE)*, 2016, pp. 41–45. DOI: 10.1109/INDUSENG.2016.7519347.
- [46] G. Al-Rawashdeh, R. Mamat, and N. H. B. Abd Rahim, "Hybrid water cycle optimization algorithm with simulated annealing for spam e-mail detection", *IEEE Access*, vol. 7, pp. 143721–143734, 2019. DOI: 10.1109/ACCESS.2019.2944089.
- [47] J.-H. Teng, "A direct approach for distribution system load flow solutions", *IEEE Trans. Power Deliv.*, vol. 18, no. 3, pp. 882–887, Jul. 2003. DOI: 10.1109/TPWRD.2003.813818.
- [48] C. S. Cheng and D. Shirmohammadi, "A three-phase power flow method for real-time distribution system analysis", *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 671–679, May 1995. DOI: 10.1109/59.387902.
- [49] M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing", *IEEE Trans. Power Deliv.*, vol. 4, no. 2, pp. 1401–1407, Apr. 1989. DOI: 10.1109/61.25627.
- [50] Y. M. Shuaib, M. S. Kalavathi, and C. C. A. Rajan, "Optimal capacitor placement in radial distribution system using gravitational search algorithm", *Int. J. Electr. Power Energy Syst.*, vol. 64, pp. 384–397, Jan. 2015. DOI: 10.1016/j.ijepes.2014.07.041.
- [51] A. R. Abul'Wafa, "Optimal capacitor placement for enhancing voltage stability in distribution systems using analytical algorithm and fuzzy-real coded GA", *Int. J. Electr. Power Energy Syst.*, vol. 55, pp. 246–252, Feb. 2014. DOI: 10.1016/j.ijepes.2013.09.014.
- [52] D. F. Pires, C. H. Antunes, and A. G. Martins, "An NSGA-II approach with local search for A VAR planning multi-objective problem", *INESC—Coimbra*, Jan. 2009. [Online]. Available: <https://www.researchgate.net/publication/285888261>

