Capacity Analysis of Channel Inversion Using Space-Time Block Coding with Transmit Power Control

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***Abstract*—In this paper, we analyze channel inversion with transmit diversity in terms of capacity and bit error rates in correlated fading. We obtain the Shannon capacity of Rayleigh fading channel under channel inversion using spatially correlated dual branch orthogonal space-time block coding, in comparison with the optimal power and rate adaptation policy, and provide a simple fixed-rate power adaptation policy to invert the Rayleigh fading into additive white Gaussian noise channel, and obtain the bit error rates using binary phase-shift keying modulation with/without channel inversion. It is found that the correlation generally causes a loss in capacity, which becomes more distinct with channel inversion relative to the optimal power and rate adaptation policy, especially when power correlation is greater than 0.5, and hinders otherwise possible full inversion. Transmit power truncation introduces power gain over full inversion, which manifests itself as an increase in capacity and a decrease in bit error probability. This gain increases with decreasing truncation level of the allowed peak transmit power. Inverting with truncation, a gain of about 19 dB in signal-to-noise ratio is obtained over classical single-antenna transmission without inversion. This gain may be further increased by a combination of truncation and correlation levels, as the correlation surprisingly turns to advantage when the normalized permitted peak-power is below 2 dB. However, the gain is normally accompanied by lower throughput or bandwidth expansion for a fixed data rate due to outage.**

***Index Terms*—MISO Rayleigh channel; Channel capacity; Channel inversion; Power control; Space-time block coding; Transmit diversity.**

I. INTRODUCTION

Being inherent to radio channels, multipath fading places detrimental effects on the performance of wireless communication systems, unless countermeasures are taken [1], [2]. A way to overcome the effects of multipath fading is to use channel inversion [1], [2]. In this way, the transmitter power is adaptively changed inversely proportional to the channel gain to provide the receiver with a constant received power, through which the channel appears in the transmitted signal as a time-invariant additive white Gaussian noise (AWGN) channel regardless of the type of fading [1]–[3]. It is common in cellular spread spectrum systems, which suffer from near-far interference imbalances [4], [5], and is also particularly useful in applications such as channel coding, where fading statistics imposes a specific design tailored to the channel. For instance, a designed coding scheme based on the assumption of a fixed fading type may actually be sub-optimum most of the time, as the channel statistics may virtually change over time. For such cases, the inversion process significantly simplifies the code design to that of AWGN channel by eliminating the time-varying nature of channel response in an adaptive manner so that a constant signal-to-noise ratio (SNR) is maintained at the receiver. However, the shortcoming of this method is its suitability for only slow fading channels with moderate levels of Doppler spread [2], and the requirement of increased peak-power levels compared to the average due to deep fades occurring with some probability depending on channel statistics.

As an adaptive transmission technique along with optimal power and rate adaptation, channel inversion (CI) has received considerable attention in the literature [6], and references therein. However, most work considers channel inversion with receiver diversity and multiple-input multiple-output (MIMO) transmission as the spatio-temporal variant of it. Transmitter diversity has not been explored much except in [7], [8]. In this paper, which being of practical importance due to ease of implementation compared to other adaptive transmission methods in [1], [2], [4], channel inversion with transmit diversity is investigated in terms of channel capacity and bit error rates.

A. Related Works

The capacity analysis of channel inversion was studied for single-antenna systems [4], and with a multiple receive antenna diversity combining system assuming uncorrelated [9] and correlated [10] reception for the Rayleigh fading channel. A form of transforming Rayleigh fading into an AWGN channel was reported in [11]. Channel inversion was used in [12] in the context of spatial equalization in MIMO systems. In [13], [14], the transmission of MIMO eigenmode is considered under the channel inversion power allocation scheme. Due to its simplicity, channel inversion was also investigated for interference cancellation in multi-user MIMO downlink (or equivalently broadcast) channels

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[15]. In this case, channel inversion involves precoding the data stream with the inverse/pseudoinverse of the channel matrix depending on the number of transmit and receive antennas. The regularization of the channel matrix was proposed in [16] to address the problem that arises in the inversion of the channel matrix when the channel matrix is ill-conditioned due to deep fading. However, regularizing induces some amount of co-channel interference to the users. Regularized channel inversion was further investigated in [17]–[19] in various scenarios. The optimal use of transmitter power in fading channels and its connection to diversity was investigated in [7], and a power adaptation policy was considered using space-time block coding (STBC) to minimize the bit error rate (BER) in [8]. The optimized transmission in [8] departs from STBC and uses only one of the two antennas with the greater channel gain to transmit, which results in transmit antenna selection diversity combining (SDC). STBC is only considered with only one/two-bit feedback from receiver rather than using complete channel information, then producing relatively poor performance. The capacity of STBC in MIMO spatially uncorrelated Rayleigh fading using channel inversion was studied in [20] with Gaussian distributed channel estimation errors, assuming correlation between the estimated and actual channel coefficients. However, the impact of spatial transmit antenna correlation on capacity has not been considered under channel inversion and other power control policies.

B. *Contribution*

In this paper, we consider spatially correlated 2Tx1Rx orthogonal STBC and obtain the capacity of the Rayleigh fading channel under channel inversion based on a single correlation parameter to provide useful insight into the behavior of capacity and bit error rates, for the sake of analytical and intuitive tractability through compact non-matrix expressions, unlike those in [21], [22], compared to the optimal power and rate adaptation policy. We also provide a simple fixed-rate power adaptation policy to invert the Rayleigh fading into the AWGN channel and obtain the bit error rates using binary phase-shift keying (BPSK) modulation with/without channel inversion.

The presentation of the paper is organized as follows. The definition of the system is given for the first time in Section II. Capacity analysis with three different adaptation policies is given in Section III. The analysis of the proposed channel inversion with STBC is given in Section IV. The discussion and conclusions are given in Sections V and VI, respectively.

II. SYSTEM DEFINITIONS

STBC is well-understood in the literature and has found its place in communication standards. Assuming slow and flat Rayleigh fading transmission, the STBC signal received in k -th block is given by:

$$\begin{bmatrix} r_{2k-1} \\ r_{2k}^* \end{bmatrix} = \sqrt{\frac{E_s}{2}} \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^* & -\alpha_1^* \end{bmatrix} \begin{bmatrix} s_{2k-1} \\ s_{2k} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}, \quad (1)$$

where E_s is the energy per transmitted symbol, s_{2k-1} and s_{2k} are the unit energy symbols from the signal constellation, n_1 and n_2 are the zero-mean complex Gaussian noise samples of variance N_0, and α_1 and α_2 are the correlated channel coefficients known to the transmitter. Assuming a single receive antenna, the correlation model used in the paper has the form of

$$\alpha_2 = \rho\alpha_1 + \sqrt{1-\rho^2}\; w, \quad (2)$$

as in [23], where w and α_1 are the independent zero-mean complex Gaussian random variables of unit variance and $\rho = E[\alpha_1 \alpha_2^*]$ is the correlation coefficient between transmit antennas. Using (2), the joint pdf of the two Rayleigh fading powers $\gamma_1 = |\alpha_1|^2$ and $\gamma_2 = |\alpha_2|^2$ is found as

$$p(\gamma_1,\gamma_2) = \frac{1}{1-\rho^2} \exp\left(-\frac{\gamma_1+\gamma_2}{1-\rho^2}\right) I_0\left(\frac{2\rho}{1-\rho^2}\sqrt{\gamma_1\gamma_2}\right), \quad (3)$$

in which I_0 is the zero-order modified Bessel function of the first kind. From a practical measurement point of view, the correlation coefficient may be preferred in terms of power (ρ_P) or envelope (ρ_E) correlation, and can be found, respectively, using (2) as:

$$\rho_P = \frac{cov(\gamma_1,\gamma_2)}{\sqrt{var(\gamma_1)}\sqrt{var(\gamma_2)}} = \rho^2, \quad (4)$$

$$\rho_E = \frac{cov(h_1,h_2)}{\sqrt{var(h_1)var(h_2)}} = \frac{\left(1-\rho^2\right)^2 {}_2F_1\left(\frac{3}{2},\frac{3}{2};1;\rho^2\right) - 1}{\left(\frac{4}{\pi}-1\right)}, \quad (5)$$

where ${}_2F_1(\alpha,\beta;\gamma;z)$ is the Gaussian Hypergeometric function, $h_1 = |\alpha_1|$, and $h_2 = |\alpha_2|$. The joint pdf of the Rayleigh envelopes used to obtain (5) can easily be obtained directly from (2) or from (3) through a simple transformation as

$$p(h_1,h_2) = \frac{4h_1h_2}{1-\rho^2} \exp\left(-\frac{h_1^2+h_2^2}{1-\rho^2}\right) I_0\left(\frac{2\rho h_1 h_2}{1-\rho^2}\right). \quad (6)$$

In this paper, the results are presented in terms of power correlation ρ_P.

III. CAPACITY ANALYSIS

After STBC decoding in (1), the decision variable for the symbol s_k is

$$y_k = \sqrt{\frac{E_s}{2}} \left(|\alpha_1|^2 + |\alpha_2|^2\right) s_k + \alpha_1^* n_1 + \alpha_2 n_2^*, \quad (7)$$

and given the channel coefficients, the signal-to-noise ratio

(SNR), γ, is then

$$\gamma = \left(|\alpha_1|^2 + |\alpha_2|^2 \right) \frac{\bar{\gamma}}{2}, \tag{8}$$

where $\bar{\gamma}$ is the average received SNR. Using (8) and (3), the pdf of γ is found to be

$$p(\gamma;\rho) = \frac{1}{\rho\bar{\gamma}} \left[\exp\left(-\frac{2}{1+\rho} \frac{\gamma}{\bar{\gamma}} \right) - \exp\left(-\frac{2}{1-\rho} \frac{\gamma}{\bar{\gamma}} \right) \right], \quad \gamma \geq 0,\ \rho \neq 0. \tag{9}$$

For uncorrelated branches, (9) reduces to

$$p(\gamma) = \lim_{\rho \to 0^+} p(\gamma;\rho) = \left(\frac{2}{\bar{\gamma}} \right)^2 \gamma \exp\left(-\frac{2\gamma}{\bar{\gamma}} \right), \gamma \geq 0. \tag{10}$$

In the following, we obtain the capacity of Rayleigh fading channel per unit bandwidth (C/B) under different power and rate adaptation policies assuming

$$E\left[\frac{P_s(\gamma)}{P_s} \right] = 1, \tag{11}$$

in which $P_s(\gamma)$ denotes the variable transmitter power as a function of instantaneous SNR and $P_s = E[|s_k|^2]$ is the average transmit power per symbol.

A. *Full Inversion (Zero-Outage)*

Given the power constraint in (11), the constant received SNR with channel inversion is given by $1/E[1/\gamma]$ [2], [4]. Since $E[1/\gamma] = \infty$ for the SISO Rayleigh fading, the capacity under channel inversion is zero. It is found using (9) that it is also true when using dual branch transmitter diversity through STBC, if there exists any degree of spatial correlation between transmit antennas. It is seen that the presence of correlation renders the channel useless in terms of capacity, regardless of its amount. However, when correlation is not involved, i.e., $\rho = 0$, $E[1/\gamma]$ is found to be finite by using (10), and hence a nonzero capacity exists with channel inversion using STBC, as given by

$$\frac{C}{B} = \log_2\left(1 + \frac{1}{E[1/\gamma]} \right) = \log_2\left(1 + \frac{\bar{\gamma}}{2} \right) \text{ b/s/Hz}, \tag{12}$$

where B is the transmission bandwidth.

B. *Truncated Inversion (Capacity with Outage)*

Although satisfying the average power constraint in (11), the need for high transmit power may also arise in time depending on the channel state. The necessity of increased peak transmit powers compared to the average, due to low channel gain with some probability, may impose the channel inversion to be used only when the SNR is above a given threshold γ_0, and declare outage otherwise. Now relaxing the zero-outage constraint [4], the capacity with outage through truncated inversion is given by

$$\frac{C}{B} = \max_{\gamma_0} \log_2\left(1 + \frac{1}{E_{\gamma_0}[1/\gamma]} \right) P(\gamma \geq \gamma_0), \tag{13}$$

where $E_{\gamma_0}[x] \int_{\gamma_0}^{\infty} x p(x) dx$. Then, substituting $E_{\gamma_0}[1/\gamma]$ into (13), capacity is found as

$$\frac{C}{B} = \max_{\gamma_0} \log_2\left(1 + \frac{\rho\bar{\gamma}}{E_1\left(\frac{2}{1+\rho} \frac{\gamma_0}{\bar{\gamma}} \right) - E_1\left(\frac{2}{1-\rho} \frac{\gamma_0}{\bar{\gamma}} \right)} \right) \times P(\gamma \geq \gamma_0), \tag{14}$$

where $E_1(.)$ is the exponential integral of first order defined as

$$E_1(z) = \int_1^{+\infty} \frac{e^{-zx}}{x} dx \tag{15}$$

and

$$P(\gamma \geq \gamma_0) = \frac{1}{2\rho} \left[(1+\rho) \exp\left(-\frac{2}{1+\rho} \frac{\gamma_0}{\bar{\gamma}} \right) - (1-\rho) \exp\left(-\frac{2}{1-\rho} \frac{\gamma_0}{\bar{\gamma}} \right) \right]. \tag{16}$$

In the case of uncorrelated branches $(\rho = 0)$, either using (10) in (13) or directly taking the limit of (14) as $\rho \to 0^+$, the capacity is found to be

$$\frac{C}{B} = \max_{\gamma_0} \log_2\left(1 + \frac{\bar{\gamma}}{2} \exp\left(\frac{2\gamma_0}{\bar{\gamma}} \right) \right) \left(1 + \frac{2\gamma_0}{\bar{\gamma}} \right) \times \exp\left(-\frac{2\gamma_0}{\bar{\gamma}} \right). \tag{17}$$

The channel capacity of STBC in Rayleigh fading given in (14) and (17) is illustrated in Fig. 1.

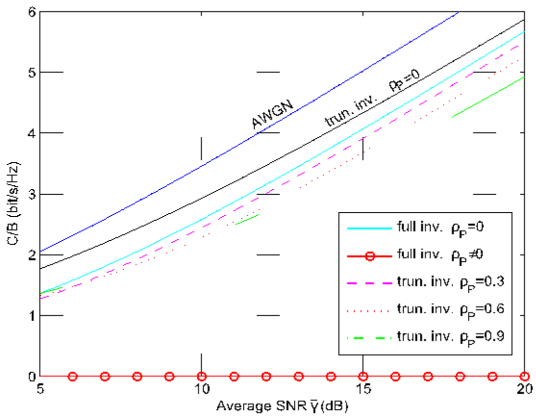

Fig. 1. Truncated channel capacity in Rayleigh fading with space-time block coding, illustrating the impact of transmit antenna power correlation.

It is seen from the figure that correlation causes capacity loss as it increases, and its impact on spectral efficiency becomes more evident for $\rho_P > 0.5$, causing a relatively more distinctive loss. However, truncation introduces capacity gain compared to full inversion, due to preserving power rather than wasting to compensate for deep fades.

C. Optimum Power and Rate Adaptation (OPRA)

To see the channel inversion capacity compared to OPRA [4], we also find the OPRA capacity as

$$\frac{C}{B} = \frac{1}{2\rho\ln(2)}\left[(1+\rho)E_1\left(\frac{2}{1+\rho}\frac{\gamma_0}{\bar{\gamma}}\right) - (1-\rho)E_1\left(\frac{2}{1-\rho}\frac{\gamma_0}{\bar{\gamma}}\right)\right],\ \rho \neq 0, \tag{18}$$

for which γ_0 as the optimum cut-off SNR value here satisfies

$$\int_{\gamma_0}^{\infty}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right)p(\gamma)\,d\gamma = 1, \tag{19}$$

along with (11), resulting in

$$E_0\left(\frac{2\gamma_0}{(1+\rho)\bar{\gamma}}\right)+E_1\left(\frac{2\gamma_0}{(1-\rho)\bar{\gamma}}\right) - E_0\left(\frac{2\gamma_0}{(1-\rho)\bar{\gamma}}\right)-E_1\left(\frac{2\gamma_0}{(1+\rho)\bar{\gamma}}\right) = \rho\bar{\gamma}, \tag{20}$$

where $E_0(z) = e^{-z}/z$. Taking the limit of (18) as $\rho \to 0^+$, the capacity for the uncorrelated transmit antennas is found to be

$$\frac{C}{B} = \frac{1}{\ln(2)}\left[\exp\left(-\frac{2\gamma_0}{\bar{\gamma}}\right)+E_1\left(\frac{2\gamma_0}{\bar{\gamma}}\right)\right], \rho = 0, \tag{21}$$

which satisfies $2\gamma_0 + \ln(\gamma_0) = 0$ as reduces form (20). The capacity results with OPRA policy is shown in Fig. 2. As seen from the figure, the capacity loss due to correlation is relatively insignificant compared to that of truncated inversion, for which it was shown to cause severe losses.

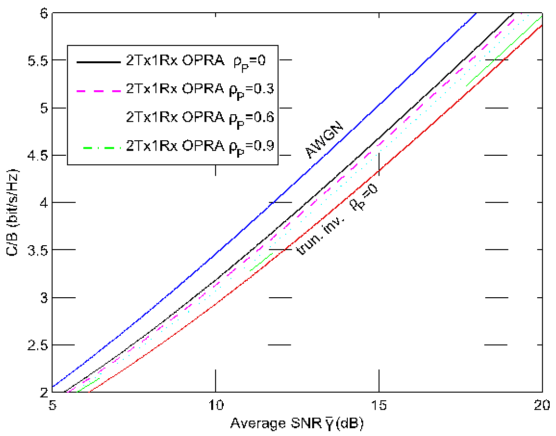

Fig. 2. Channel capacity in Rayleigh fading with space-time block coding under optimum power and rate adaptation, illustrating the impact of transmit antenna power correlation.

IV. INVERSION USING STBC WITH FIXED-RATE POWER CONTROL

Without power control, conditioned on the BPSK symbol, the pdf of y_k in (7) is found as

$$(y_k \mid s_k = +1) = \frac{1}{\rho}\left[\frac{1}{\sqrt{2\bar{\gamma}+\frac{4}{1+\rho}}}\exp\left(-\sqrt{2\bar{\gamma}+\frac{4}{1+\rho}}\,|y_k|\right) - \frac{1}{\sqrt{2\bar{\gamma}+\frac{4}{1-\rho}}}\exp\left(-\sqrt{2\bar{\gamma}+\frac{4}{1-\rho}}\,|y_k|\right)\right]\exp\left(\sqrt{2\bar{\gamma}}\,y_k\right), \tag{22}$$

from which the average BER is obtained to be

$$P_b = \frac{1}{4\rho}\left[(1+\rho)\left(1-\sqrt{\frac{\bar{\gamma}(1+\rho)}{2+\bar{\gamma}(1+\rho)}}\right) - (1-\rho)\left(1-\sqrt{\frac{\bar{\gamma}(1-\rho)}{2+\bar{\gamma}(1-\rho)}}\right)\right]. \tag{23}$$

We note that this result coincides with that of (16) in [22], if the eigenvalue μ_1 and the average SNR ρ therein are replaced by $(1+\rho)$ and $\bar{\gamma}/2$, respectively. The BER for the uncorrelated case is found by taking the limit of (23) as $\rho \to 0^+$ to be

$$P_b = \frac{1}{2}\left[1-\frac{(\bar{\gamma}+3)\sqrt{\bar{\gamma}}}{(\bar{\gamma}+2)^{3/2}}\right]. \tag{24}$$

The expression in (23), reduces to that of the classical 1Tx1Rx single-antenna transmission case for $\rho = \mp 1$, given by

$$P_b = \frac{1}{2}\left[1-\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right]. \tag{25}$$

To invert the 2Tx1Rx Rayleigh channel using full-rate STBC given in (1), the transmitted signal from each antenna is scaled by $\frac{1}{\sqrt{|\alpha_1|^2+|\alpha_2|^2}} = \frac{1}{\sqrt{\gamma_1+\gamma_2}} = \frac{1}{\sqrt{\lambda}}$ to control the transmit power before transmission. Clearly, here λ is the power control variable set as $\lambda = \gamma_1+\gamma_2$, and the transmitted energy per bit per antenna is then $\frac{E_b/2}{\lambda}$. Since $E[1/\lambda] = 1$ in the case of uncorrelated branches ($\rho = 0$), the average of the total transmitted energy per bit is $E[E_b(\lambda)] = \frac{E_b}{2}+\frac{E_b}{2} = E_b$, which satisfies the power constraint in (11). But, in the presence of correlation, full channel inversion is not possible since $E[1/\lambda]$ does not converge to a finite value. However, from a practical point of view, full inversion is virtually not preferred due to high peak-power needs depending on fading depth with some probability, imposing the use of truncated-inversion instead by allowing some amount of outage depending on the permitted peak-power level. The presence of inevitable outage naturally imposes that the transmitted signal must be scaled by $\sqrt{\varphi(X_p,\rho)}$ to meet the power constraint, where $\varphi(X_p,\rho)=\left[E_{X_p}[1/\lambda]\right]^{-1}$ and $E_{X_p}[1/\lambda]$ are found as

$$\int_{\frac{1}{X_p}}^{\infty}\frac{1}{\lambda}p(\lambda)\,d\lambda = = \frac{\Gamma\left(0,\frac{1}{X_p(1+\rho)}\right)-\Gamma\left(0,\frac{1}{X_p(1-\rho)}\right)}{(1+\rho)e^{-\frac{1}{X_p(1+\rho)}}-(1-\rho)e^{-\frac{1}{X_p(1-\rho)}}}, \qquad (26)$$

in which $\Gamma(0,x)$ is the incomplete Gamma function and $X_p = max\{E_b(\lambda)/E_b\}$ is the permitted peak-power. The scaling term for uncorrelated antennas simplifies to the form $\lim_{\rho\to 0^+}\varphi(X_p,\rho)=1+\frac{1}{X_p}$. $\varphi(X_p,\rho)$ can be considered as a power gain factor over full inversion ($\varphi(X_p,\rho)=1$), and the behavior of which is shown in Fig. 3. It is seen that the less the permitted peak-power, the more the gain and the presence of correlation surprisingly turns to advantage for permitted peak-powers below 2 dB. However, gain smoothly diminishes with increasing peak-power, which is intuitively satisfying as truncated inversion approaches full inversion (zero outage). To compensate for the outage-induced loss in throughput, in other words, to maintain average data rate constant regardless of truncation level, bit period is reduced to $T_{new}=T_b(1-P_o)$, which correspondingly requires scaling the transmit power as $P_{new}=P_s/(1-P_o)$ to hold the same energy per bit. Here, P_o is the probability of an outage.

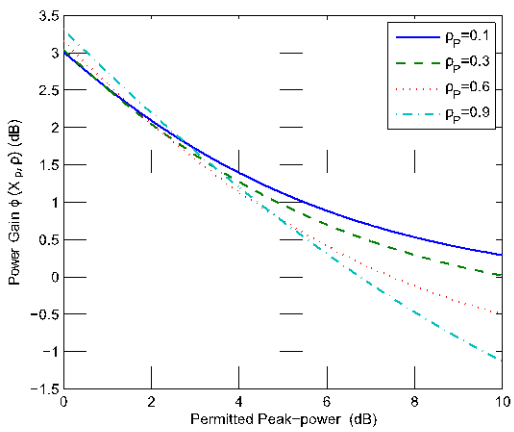

Fig. 3. Power gain due to channel inversion with power truncation in the Rayleigh fading 2Tx1Rx STBC system as a function of the maximum power allowed to transmit, in relation to the transmit antenna power correlation.

However, this compensation with fixed signal constellation size clearly comes at the cost of expansion in the transmitted signal bandwidth. This is illustrated in Fig. 4 along with the outage probability versus permitted transmit peak-power. Note that the normalized power curves in the figure also represent the resulting bandwidth expansion.

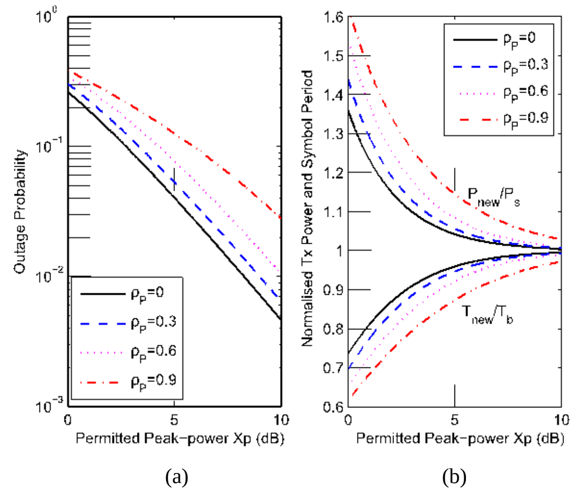

Fig. 4. (a)Outage probability and (b) normalized bit period with corresponding transmit power vs. allowed transmit peak-power for inversion, for various degrees of transmit power correlation in Rayleigh fading 2Tx1Rx STBC transmission.

It is clear from the comparison of Fig. 3 and Fig. 4 that an increase in gain over full inversion is associated with an increase in outage probability and therefore with bandwidth expansion. However, it is assumed that the amount of bandwidth expansion does not alter the flat status of fading adopted in the paper. The decision variable for the symbol s_k after decoding is then found as

$$y_k=\sqrt{\frac{E_b}{2}\varphi(X_p,\rho)}\,s_k+\Re\left\{\frac{\alpha_1^* n_1+\alpha_2 n_2^*}{\sqrt{|\alpha_1|^2+|\alpha_2|^2}}\right\}, \qquad (27)$$

which clearly differs from (7). Since $p(y_k\,|\,\alpha_1,\alpha_2,s_k)$ is Gaussian distributed as $N\left(\sqrt{\frac{E_b}{2}\varphi(X_p,\rho)}\,s_k,\frac{N_0}{2}\right)$, controlling the transmitted power in this way, the instantaneous SNR is maintained constant over time as $\frac{E_b\varphi(X_p,\rho)}{2N_0}$, conditioned on α_1 and α_2, hence, resulting in the inversion of the channel. Therefore, the bit error probability with truncated channel inversion is simply

$$P_b=\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}\varphi(X_p,\rho)}\right), \qquad (28)$$

which reduces to

$$P_b=\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) \qquad (29)$$

with full inversion ($X_p\to\infty$ and $\rho\to 0^+$). The average bit error probability for the 2Tx1Rx STBC employing BPSK modulation with and without channel inversion is illustrated in Fig. 5, in which the results for full inversion and truncated inversion with various peak-power levels are included.

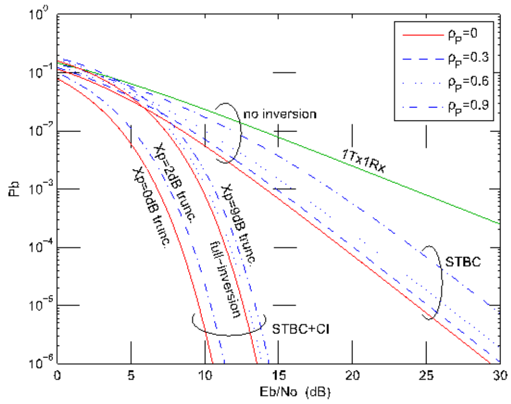

Fig. 5. Bit error rate of binary phase-shift keying modulation with/without full/truncated channel inversion in Rayleigh fading under various degrees of transmit antenna power correlation.

The results in the figure have been verified by simulation. It is seen from the figure that the dominant parameter on error probability is the permitted transmit peak-power. The impact of correlation on increase of BER is seen to be relatively less significant compared to that of increasing transmit peak-power. For instance, increasing the permitted peak-power X_p from 2 dB to 9 dB causes an SNR loss of 3 dB for a given BER, but the loss due to increase in correlation ρ_P from 0.1 to 0.9 at $X_p = 9\,dB$ is relatively insignificant and even negligible at $X_p = 2\,dB$, as can be verified from Fig. 3. For clarity of picture, only $\rho_P = 0.9$ is therefore included in the figure for $X_P = 2\,dB$.

At $X_P = 0\,dB$ peak-power with no correlation, AWGN channel performance of BPSK is obtained, since $X_P = 0\,dB$ provides 3 dB gain over full inversion as seen from Fig. 3. It is also seen from Fig. 5 that about 19 dB SNR gain can be obtained through channel inversion using STBC with BPSK modulation over its non-inverted counterpart with uncorrelated transmission, and much more if correlation exists. For instance, truncated channel inversion with $X_P = 0\,dB$ and $\rho_P = 0.9$ correlation employing STBC introduces about 24 dB gain over STBC with the same amount of correlation without inversion.

BER results involving channel inversion are also illustrated in Fig. 6 for clarity of the picture, where inversion with transmit antenna selection in [8] is also included. It is seen that the BER results of SDC in [8] and its optimized variant perform 1.5 dB and 2 dB better, respectively, compared to the full inversion with STBC. Note that the authors in [8] use weighting not only a function of channel gain but also SNR, through computationally expensive nonlinear optimization using the LambertW function. But our proposal is very simple and does not depend on SNR, yet providing comparable results. However, results of truncated versions of inversion cannot be directly compared, since they have different outage probabilities.

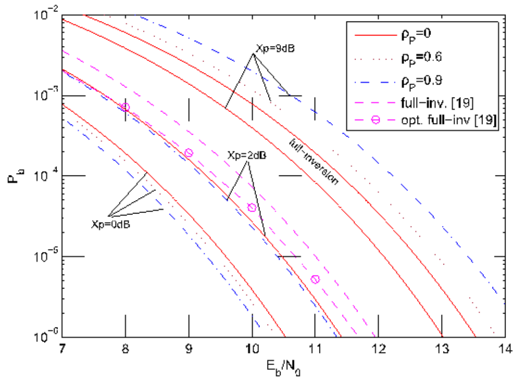

Fig. 6. Bit error rate of binary phase-shift keying modulation with full/truncated channel inversion in Rayleigh fading under various degrees of transmit antenna power correlation. For comparison, a channel inversion with 2Tx1Rx uncorrelated transmit antenna selection diversity combining [8] has also been included.

V. DISCUSSION

It is seen from the results that correlation causes a loss in capacity, whose upper bound is formed by uncorrelated transmission. This loss is more severe under channel inversion compared to optimal power and rate adaptation, appearing more evident with increasing correlation. The presence of spatial correlation prevents only otherwise possible full inversion of the channel. However, in the case of truncated inversion, the existence of correlation surprisingly turns to advantage as a power gain over full inversion when transmit peak-power is limited to levels below 2 dB, and hence contributes to decreasing the BER. When peak-power is not limited, correlation becomes destructive and ultimately incurs zero gain to prevent inversion. This result is of practical importance, since correlation may actually arise in practice in some fraction of time even if it is not assumed. However, the choice of low levels of power limit is advantageous in terms of BER since the existence of any possible correlation is taken to advantage but is traded for increased outage and hence bandwidth expansion to hold the required data rate constant if the size of signal constellation is fixed. Obviously, this expansion can be avoided (traded) by increasing the constellation size.

When comparing the capacity results with those of [20], it is found that the impact of spatial correlation on capacity is adversely related to that of the channel estimation errors, which is intuitively expected. That is, for instance, zero spatial correlation provides maximum capacity, while zero correlation in channel estimation errors produces lowest capacity. The BER obtained without channel inversion is seen to coincide with that of [22] under certain parameter settings. The BER obtained in inversion is found to be similar to that of [8] in the case of full inversion, and the results with a truncated inverse in correlation are justified by simulations.

We note that the single-user 2Tx1Rx STBC studied in this paper, which transmits the code in two consecutive intervals, may, however, be considered as two transmit antenna two-user each equipped with single receive antenna broadcast transmission with channel matrix as given in (1).

Transmit precoding as in [16] would produce the same capacity as full inversion in (12) for each user.

Regularization and truncation in essence serve the identical purpose of limiting transmit power in compensating deep fades. However, regularization can also be used in our scenario at the cost of reduction in received SNR. It is known that the use of truncation with inversion improves the capacity as shown above, as does the use of regularization in [16]. A possible extension of the study in this paper may therefore be the joint optimization of the impact of truncation and regularization on capacity and BER. Nevertheless, it would also be interesting to apply joint optimization in MIMO cases as in [16]–[19] to exploit the best attribute of both.

VI. CONCLUSIONS

The Shannon capacity of the Rayleigh fading channel has been obtained under channel inversion using correlated 2Tx1Rx orthogonal STBC, compared to the OPRA policy. A fixed-rate power adaptation has been provided, and the bit error rates using BPSK modulation have been obtained. Results indicate that capacity loss due to correlation is more distinct with channel inversion compared to OPRA, and the impact of permitted transmit peak-power on bit error rate is significant in low peak-power levels. It is found that strong correlation turns into advantage in BER for permitted peak-powers below 2 dB. Since the Rayleigh channel is converted to AWGN channel by inversion, inducing about 19 dB SNR gain, an outer channel coding scheme simply designed for AWGN channel may also be used to further improve the BER or obtain extra SNR gain.

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CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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