ISSN 1392 – 1215 ELECTRONICS AND ELECTRICAL ENGINEERING 2011. No. 4(110)
ELEKTRONIKA IR ELEKTROTECHNIKA

T 180 TELECOMMUNICATIONS ENGINEERING
TELEKOMUNIKACIJŲ INŽINERIJA

Optimization of Sensor Grid aided by Modelling of Sensor Relationships with Digital Filters

J. Gorecki, B. Miedzinski
Wroclaw University of Technology,
Wybrzeze Wyspianskiego 27, 50-370 Wroclaw, Poland, phones: +48603848411, +48713203693,
e-mails: jan.gorecki@onet.eu, bogdan.miedzinski@pwr.wroc.pl

Introduction

This paper presents the optimization method of sensor grid which identifies the cause-effect relations between sensors based on measurements.

Optimization of sensor grid allows to decrease number of sensors in the grid what leads to decrease of costs related to commissioning and maintenance of measurement infrastructure. Unfortunately upon removal of particular sensors from the grid it may decrease the quality of measurements. That is why it is crucial to remove only those sensors which can be replaced by others. Otherwise the area armed with the sensor grid cannot be covered thoroughly.

This paper explains optimization method in three steps – without post-processing, with post-processing calculations and with modeling sensor relationships using digital filter to perform causality detection.

Coverage probability in sensor grid

Setting up a sensor grid, the coverage problem of measured field is an important issue which received considerably high research attention [1–5]. This paper deals with inter-sensor coverage, i.e. how particular sensor can cover the area of other sensor.

To classify whether particular point *Sx* can be covered by sensor *S1* performing measurements in particular point, there is introduced a point-to-point coverage probability ($P2P\xi$) [1].

P2Pξ is defined as a probability that sensor *S1* (located in the grid) indicates a value which is within a pre-defined range with respect to actual value in point *Sx* (not necessarily covered by any sensor

$$P2P\xi(S1, Sx, Range)|_{Range=a} = \\ = P(Sx \in (S1 - a, S1 + a). \quad (1)$$

To estimate coverage probability that sensor *S1* covers an area assigned to sensor S2, it is needed to use the following formula

$$\hat{\xi}_{S1\rightarrow S2} = \hat{\xi}_{S2}\ x\ P2P\xi(S1, S2, a), \quad (2)$$

where $\hat{\xi}_{S2}$ is a coverage probability that sensor *S2* covers own area [1].

Due to simplification reasons, it is assumed that each sensor in the grid covers perfectly its own area, i.e. $\hat{\xi}_{S2} = 1$, i.e.

$$\hat{\xi}_{S1\rightarrow S2} \approx P2P\xi(S1, S2, a). \quad (3)$$

It is considered that sensor *S2* is covered by sensor S1 when coverage probability between those sensors $\hat{\xi}_{S1\rightarrow S2}$ is greater than certain predefined value ξ_{min}.

The optimization algorithm proposed in this paper is composed of the following steps:

1. Create an empty sensor grid (GRID2) having the same dimensions as grid under optimization (GRID1). Whole area of GRID2 is UNCOVERED.
2. Select the sensor having the greatest coverage (i.e. can replace the greatest number of sensors) among UNCOVERED area of GRID2.
3. Put selected sensor on the GRID2 classifying area covered by this sensor as COVERED.
4. Go to step 2, unless whole area of GRID2 is COVERED.

To verify how above described algorithm is executed in practice, there have been performed measurements on the sensor grid. The grid is charactered by:

- Measured physical quantity: Temperature
- Environment: Floor equipped with heating (an under floor system of pipes with water acting as a heat exchanger). The temperature of the water is set to 35°C. The temperature on the floor surface is

measured at various points under steady state conditions.

- Number of sensors: 132 (grid: 11x12)
- Distance between sensors *d*: 23 cm
- Area of the grid: 5.82 m^2
- Number of measurement series: 30
- Total number of measurements: 3960 (30 series)

Applying the optimization algorithm defined above, the number of kept sensors is 74 of 132 (56.1%) for ξ_{min} equal 0.68 and accuracy range equal $\pm 0.25^0$C. Table 1 shows optimized sensor grid indicating removed sensors (by gray area). Additionally Table 1 indicates covered area by particular sensor (1 unit means that sensor covers area corresponding to one sensor coverage).

Table 1. Optimized sensor grid using application neighbour method

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1		3	1			1	1	1	1
2	1	1	1	1	9		14			1		1
3	1	1	1	2				1				1
4	1	1	1	1							2	1
5		1	1					3				1
6		1				1				1	1	1
7	2			1			2	11			1	1
8	1	2	4						2	1	1	1
9	1			7			4		3	1	1	1
10	1	1						1	1	1	1	1
11	1	1	2		2		1	1	1	1	2	

It can be seen that majority of sensors that have been removed is in the centre of sensor grid. The edges however, where there is no heating floor, require better sensor penetration.

This method keeps penetration of sensor pretty high. It identifies properly similar measurement areas and ensures that each fragment of sensor grid is properly covered.

Optimization method based on application neighbour identification aided by post-processing calculations

In this chapter, the optimization method presented in previous chapter is aided by post-processing calculations. It means that measured values of absent sensors can be estimated as linear combination of other sensors that are kept in the sensor grid.

Coverage probability ξ is calculated not between 2 sensors, but between the sensor and linear representation of other sensor resulting in linear regression calculations

$$\hat{\xi}_{S1' \to S2} = P2P\xi \ (S1', S2, Range) = \\ = \mathrm{P2P}\xi(mS1 + b, S2, Range), \quad (4)$$

where *S1'* is linear representation of *S1* sensor measurements which coefficients m and b are derived from linear regression approximating *S2* sensor values.
Because linear regression coefficients m, b minimize the average difference between *S1'* and *S2* series, the following inequity is correct:

$$P2P\xi \ (S1, S2, Range) = \\ = \begin{cases} P2P\xi(1xS1 + 0, S2, Range), \\ \leq P2P\xi(mS1 + b, S2, Range). \end{cases} \quad (5)$$

It means that aiding the optimization method presented in chapter 2 by linear regression calculations always increases the coverage probability between 2 sensors, i.e. number of sensors covering the grid can be smaller.

Applying exactly the same optimization algorithm as presented in previous section, the number of kept sensors is 39 of 132 (29.5%). Exact results are shown in Table 4.

Table 2. Optimized sensor grid using application neighbour method aided by post-processing

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1			9		1	5			1
2	1	1	1	1								1
3	1	2										2
4			5									
5			1	3	1			2				
6						1				1		
7							2	59			1	1
8	1									5	1	1
9	1											1
10	2		8					2				
11							1	1	1	1		

Modelling of neighbour relationships with digital filters

Presented optimization methods have one serious drawback. They look into measurement value similarities, however do not analyse the time series similarity. It is not taken into account which sensors are measuring the phenomenon cause and which are projection of phenomenon effect. Linear regression just shows relation between measured values, but do not identify the causal connection between them. It is not correct to remove from the grid the sensor being located at phenomenon cause (cause sensor) and calculate its value basing on measurements of the sensor measuring just an effect (effect sensor).

Therefore to detect causality [5], it is proposed to estimate the dependency between 2 sensors using digital filter as shown in Fig. 1.

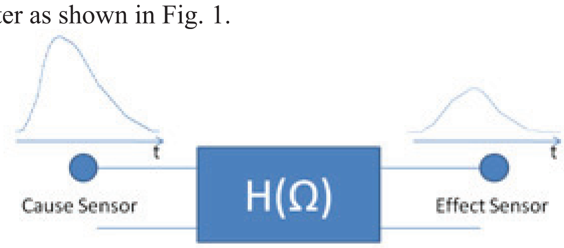

Fig. 1. Visualization of modelling the sensor neighbour dependencies with digital filters

Majority of physical phenomena in the environment can be charactered by following statements:

- Causality, the cause must happen earlier than effect is observed.

- Stability, there are no phenomena in the macro scale giving infinite amplitude effect triggered by bounded cause.
- Low-passing filter properties, phenomena observed in the function of time tends to stability, i.e. no matter how strong phenomenon is, its effects will tend to average in the function of time.

The above statements create a BIBO stability condition (Bounded Input – Bounded Output) used in the electrical engineering.

Assuming that 2 sensors are measuring distribution of certain phenomenon, the space between those sensors can be modelled as filter with transmittance *H(Ω)*.

To calculate the transmittance it is needed to take measurement series of 2 sensors measuring particular phenomenon into account:

$$s1(n) = s1_m(n) - s1_m(0), \tag{6}$$

$$s2(n) = s2_m(n) - s2_m(0), \tag{7}$$

where $s1_m(n)$, $s2_m(n)$ – value measured by sensor1, sensor2 in n-slot, $s1(n)$, $s2(n)$ – measured series by sensor1, sensor2 referenced to initial measurement.

To calculate the frequency spectrum of signals created by measurement series, it is needed to apply Fast Fourier Transform (FFT), i.e. [4]:

$$S1(\Omega) = FFT\big(s1(n)\big), \tag{8}$$

$$S2(\Omega) = FFT\big(s2(n)\big). \tag{9}$$

The dependency between input and output signal is defined as follows

$$S_{out}(\Omega) = H_{in,out}(\Omega) S_{in}(\Omega). \tag{10}$$

Depending whether *S1/S2* or *S2/S1* is input/output the transmittance is defined in below equations:

$$H_{1,2}(\Omega) = \frac{S_2(\Omega)}{S_1(\Omega)} = \frac{1}{H_{2,1}(\Omega)}, \tag{11}$$

$$H_{2,1}(\Omega) = \frac{S_1(\Omega)}{S_2(\Omega)} = \frac{1}{H_{1,2}(\Omega)}. \tag{12}$$

Finally the impulse response h(n) is calculated as Inverse FFT, i.e.:

$$h_{in,out}(n) = IFFT\left(H_{in,out}(\Omega)\right). \tag{13}$$

Having the impulse response calculated it is possible to estimate the output signal basing on input signal and transmittance which is the convolution function.

$$\hat{s}_2(\mathrm{n}) = \big(s1 * h_{1,2}\big)(n), \tag{14}$$

$$\hat{s}_1(\mathrm{n}) = \big(s2 * h_{2,1}\big)(n). \tag{15}$$

To verify how dependency modelling using filter is working, there have been performed measurements at 2 points of following properties:

- Observed phenomenon: turning on the floor heating and then turning it off observing how particular grid points are warming and then cooling.
- *s1* point is located in the middle of the grid.
- *s2* point is located close to the grid edge.

It can be seen from Figure 2 that *s1* series is followed by *s2* series. Assuming that s1 series is input to the filter and *s2* series is its output, the amplitude characteristics of filter has low-pass character with dominating frequency. Thus it is expected that $\hat{s}_2$ will give a good approximation of *s2*. This is confirmed in figure where *s2* and $\hat{s}_2$ almost perfectly overlap.

Taking the opposite assumption into consideration, i.e. *s2* series is an input and *s1* is an output, the filter amplitude characteristics passes almost all frequencies, however amplifying the highest frequencies. Additionally $H_{2,1}$ has much greater amplitude values than $H_{1,2}$ except dominating frequency of $H_{1,2}$. As causality is not kept, the filter is not able to predict s1 values. It would be only possible if h(n)>0 for negative n. Thus $\hat{s}_1$ does not fit at all to s1 what can be seen in Figure 2.

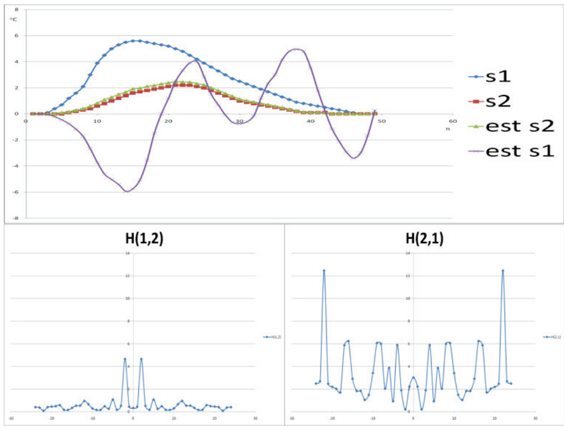

Fig. 2. Measured S1 and S2 series (referenced to initial value) and estimates of those signal basing on filter calculations (est S2, est S1) and amplitude characteristics of transmittance

It is pointless to apply linear regression calculations to derive *s1* values based on *s2* series despite the series correlation is very high. It is concluded that linear regression calculations can be applied only to selected sensor pairs.

To filter out the sensor pairs not fulfilling the causality criteria it is proposed to calculate average error caused by estimation and keep only the sensor relationships with relatively low *avgError* (equation 16).

$AvgError_{s1,s2}$ and $avgError_{s2,s1}$ are 0.18^{0}C and 4.82^{0}C respectively what is a significant difference.

There were performed measurements for cooling phenomenon for the whole grid. Basing on avgError calculations there have been identified good "causers" and bad "causers". It was calculated that good causers are usually located in the middle of the grid and bad causers are close to edges, and mainly in the corners where impact of floor heating is the most limited

$$avgError_{s1,s2} = \sqrt{\frac{\sum_{i=0}^{n}\big(s1(i) - \hat{s}_1(i)\big)^2}{n}}. \tag{16}$$

To improve the optimization method presented in previous sections by causal sensor dependencies it is needed to apply the following condition on coverage probability

$$If\ avgError_{s1,s2} > THR\ \ then\ \mathrm{P2P}\xi\,(S1', S2, Range) = 0\ , \quad (17)$$

where THR is a pre-defined threshold to remove neighbour relationships.

As relationships are unidirectional, removing of S1→S2 sensor dependency does not remove S2→S1 dependency and opposite.

Having applied above defined condition to the exemplary sensor grid with THR=0.25^{0}C, the results of optimization have changed, as follows:

- Number of sensor increased from 39 to 50 (up to 37.9%).
- 34 of 39 sensors remained in the sensor grid.
- 5 sensors were removed.
- 16 sensors were added.

It means that there were observed 21 sensor position changes what is a significant change.

Conclusions

The paper discusses optimization method based on the coverage probability. Presented optimization method can be applied both in the symmetrical sensor grids and asymmetrical sensor grids where sensors are not distributed uniformly. The discussed exemplary measurements relate to temperature; however this method can be used successfully for any physical quantity control.

It is revealed that applying post-processing calculations like linear-regression makes incredible improvement in the optimization process. However despite of good correlation between particular sensors, there is a significant risk that causal connection between correlating and correlated sensor does not exist. It is pointless to apply linear regression calculations when causality is not kept to derive particular sensor series values, even despite the series correlation is very high.

Therefore the identification of unidirectional relationships between the sensors is desired taking into account time series similarities. Before execution of optimization process it is crucial to identify possible phenomena and simulate them with full set of sensors. Deriving causal dependency between the sensors, which is modelled by digital filter, it is possible to mitigate those sensor connections which do not fulfil causality.

Dependencies which are modelled by means of low-pass filter are fully unidirectional, i.e. there is a uniquely identified cause and effect on both sides of the filter.

Acknowledgements

This paper has been written as a result of realization of the project entitled: "Detectors and sensors for measuring factors hazardous to environment – modeling and monitoring of threats".

The project financed by the European Union via the European Regional Development Fund and the Polish state budget, within the framework of the Operational Programme Innovative Economy 2007÷2013. The contract for refinancing No. POIG.01.03.01-02-002/08-00.

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Received 2011 03 21

J. Gorecki, B. Miedzinski. Optimization of Sensor Grid aided by Modelling of Sensor Relationships with Digital Filters // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 4(110). – P. 39–42.

The paper discusses optimization method based on the coverage probability. Presented optimization method can be applied both in the symmetrical sensor grids and asymmetrical sensor grids where sensors are not distributed uniformly. It is revealed that applying post-processing calculations like linear-regression makes incredible improvement in the optimization process. However despite of good correlation between particular sensors, there is a significant risk that causal connection between correlating and correlated sensor does not exist. This paper describes how to detect causal dependency. Ill. 2, bibl. 5, tabl. 2 (in English; abstracts in English and Lithuanian).

J. Gorecki, B. Miedzinski. Jutiklio tinklelio taikymo optimizavimas modeliuojant skaitmeninių filtrų ir jutiklių sąryšį // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 4(110). – P. 39–42.

Analizuojamas tikimybinis optimizavimo metodas. Jis gali būti taikomas simetriniam ir nesimetriniam jutiklių tinkleliui, kuriame jutikliai nebūtų tolygiai paskirstyti. Nustatyta, kad taikant tiesinę regresiją ypač pagerėjo optimizavimo procesas. Būtina atkreipti dėmesį į dalinių jutiklių tarpusavio koreliaciją. Il. 2, bibl. 5, lent. 2 (anglų kalba; santraukos anglų ir lietuvių k.).