Ant System with Distributed Values of Pheromone Evaporation

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***Abstract*—The paper presents some preliminary results on distribution of pheromone evaporation values among ants in Ant System. Two cases are studied, one with uniform distribution of pheromone evaporation values among cities and other with Gaussian distribution. Experimental analysis is performed by comparing behavior of Ant System solving Traveling Salesman Problem. Minimum mean error found and number of near optimal solutions found are used as main indicators of Ant System performance evaluation. Pheromone evaporation coefficient values distribution showed very little to no impact on convergence speed. Preliminary experimental results confirm that by introducing a pheromone evaporation asymmetry, Ant System minimum mean error decreases up to 8 % and the number of near optimal solutions increases up to 25 % without sacrifice of convergence speed and without much change in complexity.**

***Index Terms*—Ant colony optimization, computational efficiency, gaussian distribution, optimization, parameter estimation.**

I. INTRODUCTION

One of the most noticeable behaviors of certain ant species is the ability to find shortest paths by exploiting communication based on pheromone, chemical substance that ants can deposit and smell. Biological background is used as a mathematical model for a group of optimization algorithms known as Ant Colony Optimization (ACO) [1]. These algorithms also rely on heuristic information, which is usually presented as a set of general rules which can reduce the search space.

ACO group is growing rapidly as each year new modifications and algorithms are proposed. However in the majority of cases the main generic ACO algorithms are used as a base for modification. The most popular generic algorithms are: Ant System (AS) [2], Elitist Ant System (EAS), Rank based Ant System (ASrank), MAX-MIN Ant System (MMAS), Ant Colony System (ACS).

ACO algorithms could be applied in different areas, where combinational problems arise. Most straightforward application of ACO is for the Traveling Salesman Problem (TSP) [3], which is used as a benchmark for different optimization algorithms. In the case of TSP heuristic information for AS is based on the distance between cities of interest. Similar application is AntNet [4], an algorithm used for network routing. There exist more sophisticated problems, where the application of ACO is not very straightforward. Image pre-processing [5] could be a good example of problem often requiring an intelligent approach [6]. The results of ACO algorithm are very parameter dependent, so a lot of hybrid ACO systems were proposed with adaptation of various parameters values during simulation by the use of other optimization techniques [7], like genetic algorithms [8] and taboo search [9]. However such an approach increases the complexity of the algorithm and is not suitable for a rapidly growing field of embedded systems, because they are limited on computational resources. As an alternative the distribution of initial parameters values was tested and proved reasonable choice when exact values are not known.

The goal of this work was to check if the introduction of the asymmetry by the distribution of evaporation coefficient and without change to the general algorithm structure could decrease the minimum mean error and improve the number of near optimal solutions found.

For experimental testing, AS was chosen as one of the simplest ACO algorithms, providing moderate convergence speed with quite high minimum mean error compared to other ACO algorithms.

II. ANT SYSTEM

When speaking about AS it is assumed that the ant-cycle based version is in question. Tour construction and pheromone update are the two main steps in the ant-cycle based version of AS. Initial pheromone trails are set according to recommendations [1]

$$\tau_{ij} = \tau_0 = m / C^{\mathrm{nn}}, \quad \forall (i, j), \tag{1}$$

where τ_{ij} is pheromone value on an arc from city i to city j; τ_0 is the initial pheromone value; m is a number of ants; C^{nn} is the length of a tour generated by the nearest-neighbor algorithm.

For AS the number of ants m is usually taken to be equal

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to the number of cities n, and during the initialization step each city contains exactly one ant. During tour construction step, ants are moved from city to city until tours are fully constructed. Movement decision depends on heuristic information $\eta_{ij} = 1/d_{ij}$ and the pheromone information τ_{ij}. Here d_{ij} is the distance between cities i and j. Ant movement probability rule is

$$p_{ij}^k = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l\in N_i^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}, \quad \text{if } j \in N_i^k, \tag{2}$$

where α determines influence of pheromone trail; β determines influence of heuristic information; N_i^k is the feasible neighborhood made of non-visited cities of ant k while at city i.

During pheromone update step, pheromone is evaporated and then deposited. Pheromone evaporation coefficient $0 < \rho < 1$ determines how quick ants can forget found paths and avoid unlimited pheromone accumulation. Evaporation is performed by

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij}, \quad \forall(i, j). \tag{3}$$

After evaporation the deposition of pheromone is performed by

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^k, \quad \forall(i, j), \tag{4}$$

where $\Delta\tau_{ij}^k$ is the amount of pheromone deposited by ant k:

$$\Delta\tau_{ij}^k = \begin{cases} 1/C^k, & \text{if arc } (i, j) \in T^k, \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where C^k is the length of the tour T^k, calculated as a sum of all arc's lengths belonging to the tour constructed by ant k.

III. Distribution of Evaporation Coefficient Values

In this case all TSP are symmetrical, so the cost of moving in both directions is the same. The pheromone trail is also symmetrical, so it's level on $\text{arc}(i, j)$ and $\text{arc}(j, i)$ is the same. The main idea of distributed values of pheromone evaporation coefficient is to assign different values of evaporation to different cities, affecting the symmetry of pheromone trail on arcs. This introduced asymmetry in pheromone trail is expected to improve the minimum mean error and the number of near optimal solutions found, by forcing ants to explore different routes. Evaporation coefficient placed in one city should affect all the arcs linking this city to other cities, this way the pheromone trail on $\text{arc}(i, j)$ is not necessary the same as on $\text{arc}(j, i)$.

First distribution to try was the uniform distribution. Uniform distribution of evaporation coefficient values could be expressed by

$$\rho^i = \rho_{\min} + \Delta\rho \cdot (i-1), \tag{6}$$

where ρ^i is the pheromone evaporation coefficient belonging to city i; and $\Delta\rho = (\rho_{\max} - \rho_{\min})/n$.

Another distribution to try was of Gaussian type. In this case the approximate expression is a little bit more complex

$$\rho^k = \left(\ln\left(\frac{d^i}{1-d^i} \right) / 9.2 + 0.5 \right) \cdot (\rho_{\max} - \rho_{\min}) + \rho_{\min}, \tag{7}$$

where $d^i = (i-1)/(n-1) \cdot 0.98 + 0.5$.

According to recommendations [1], only one pheromone evaporation coefficient value is provided for TSP problem. This makes the task of selection $\rho_{\max}$ and $\rho_{\min}$ parameters for AS an experimental choice.

IV. Experimental Setup

TSPLIB [10] database contains different TSP together with known optimal solutions. It is known that AS is not suitable for large TSP [1]. So two average symmetrical TSP were chosen: berlin52, att48; and also one small TSP – burma14.

During the first experiment two parameters' sets for each problem were chosen: the first one – for the highest number of near optimal solutions found and the second one – for the lowest minimum mean error. In this experimental setup it is assumed that near optimal solutions are those that differ from the optimal tour length by no more than 1 %.

The second experiment was to determine if the pheromone evaporation distribution could improve the number of near optimal solutions found, by trying various distributions and comparing results.

Third experiment was intended to check if the distribution of the pheromone evaporation coefficient could improve the minimum mean error.

After each experiment, solutions were compared to those obtained with the nearest neighbor algorithm and the iteration at which the nearest neighbor algorithm's solution was exceeded by the AS solution was noted for convergence speed comparison.

During experiment e, the iteration t error was calculated according to

$$E_t^e = \left(1 - \frac{C^{\text{opt}}}{C_t^{\text{fnd}}} \right), \tag{8}$$

where C^{opt} is the known shortest tour from TSPLIB database; $\forall(k),\ C_t^{\text{fnd}} = \min C_t^k$ is the iteration best distance found by AS during iteration t. As for each parameter's set

100 experiments were performed, the mean error of iteration could be expressed by

$$\bar{E}_t = \frac{1}{100}\sum_{e=1}^{100} E_t^e \ . \tag{9}$$

For each problem 300 iterations were performed. And for each iteration the mean error and minimum mean error $\forall(t)$, $\bar{E}_{\min} = \min\left(\bar{E}_t\right)$ were calculated.

For initial experiment these parameters values were chosen according to recommendations [1]: $\alpha = 1$; β could be from 2 to 5; $\rho = 0.5$; $m = n$; $\tau_0 = m / C^{nn}$. For all parameters fixed values were provided, except for the β. So the values from the provided range were tested. For each problem two values of β were selected: one with the largest number of near optimal solutions found and other with the minimum mean error.

For other experiments, ρ was distributed using uniform and Gaussian distributions around recommended center value $\rho = 0.5$. Three sets for each distribution were created and tested during experiment:

$$[\rho_{\min} = 0.4 : \rho_{\max} = 0.6], \tag{10}$$

$$[\rho_{\min} = 0.3 : \rho_{\max} = 0.7], \tag{11}$$

$$[\rho_{\min} = 0.2 : \rho_{\max} = 0.8]. \tag{12}$$

V. RESULTS

Each problem requires a different parameters set to achieve a minimum mean error or a maximum number of near optimal solutions. The only exception in this case is Berlin52 problem, where the minimum mean error (0.0666) and the maximum number of near optimal solutions (33) found are achieved with the same value of sensitivity to heuristic information: $\beta = 3.5$. For Att48 problem the largest number of near optimal solutions found (4) was when $\beta = 3.5$ and the minimum mean error (0.0713) was reached when $\beta = 5.0$. For Burma14 problem these values are: $\beta = 2.0$ for the near optimal solutions found (50); $\beta = 4.0$ for the minimum mean error (0.0267). It is natural that by increasing the sensitivity to heuristic information convergence speed is also increasing as the algorithm behavior is getting closer to the nearest neighborhood algorithm.

For the next experiment, parameters' sets with the highest number of near optimal solutions found were taken to evaluate pheromone evaporation coefficient distribution.

Results presented in the Table I prove the assumption that introduced asymmetry may increase the number of near optimal solutions found. In this case the increase of near optimal solutions found was 25 % for Att48 and 14 % for Burma14 problems; however decrease for about 3 % was noted for Berlin52 problem. Also the pheromone evaporation coefficient distribution has almost no impact on AS convergence speed.

For the last experiment, parameters' sets with the minimum mean error were taken for evaluation. From the Table II it is clear, that distribution of evaporation coefficient leads to improvement of the minimum mean error for all the problems: about 3 % for Berlin52 and Att48, and about 8 % for Burma14. Convergence speed remained constant.

TABLE I. EVAPORATION COEFFICIENT DISTRIBUTION (PARAMETERS WITH THE HIGHEST NUMBER OF NEAR OPTIMAL SOLUTIONS).

Problem	**Berlin52**		**Att48**		**Burma14**	
Pheromone evaporation coefficient, ρ	**Minimum mean error reached**	**Near optimal solutions found**	**Minimum mean error reached**	**Near optimal solutions found**	**Minimum mean error reached**	**Near optimal solutions found**
0.5	0.0666	33	0.0742	4	0.0345	50
Unif[0.4:0.6]	0.0659	29	0.0713	5	0.0325	51
Unif[0.3:0.7]	0.0684	27	0.0735	0	0.0322	47
Unif[0.2:0.8]	0.0647	28	0.0737	0	0.0335	57
Gauss[0.4:0.6]	0.0668	22	0.0738	2	0.0343	49
Gauss[0.3:0.7]	0.0660	27	0.0737	3	0.0333	51
Gauss[0.2:0.8]	0.0650	32	0.0743	3	0.0344	49

TABLE II. EVAPORATION COEFFICIENT DISTRIBUTION (PARAMETERS WITH THE LOWEST MINIMUM MEAN ERROR).

Problem	**Berlin52**		**Att48**		**Burma14**	
Pheromone evaporation coefficient, ρ	**Minimum mean error reached**	**Near optimal solutions found**	**Minimum mean error reached**	**Near optimal solutions found**	**Minimum mean error reached**	**Near optimal solutions found**
0.5	0.0666	33	0.0713	2	0.0267	7
Unif[0.4:0.6]	0.0659	29	0.0727	4	0.0262	9
Unif[0.3:0.7]	0.0684	27	0.0695	2	0.0255	12
Unif[0.2:0.8]	0.0647	28	0.0722	2	0.0258	10
Gauss[0.4:0.6]	0.0668	22	0.0713	4	0.0268	12
Gauss[0.3:0.7]	0.0660	27	0.0706	3	0.0254	11
Gauss[0.2:0.8]	0.0650	32	0.0720	5	0.0247	13

V. CONCLUSIONS

After experimental testing these conclusions could be made:

- Pheromone evaporation value distribution provided from -3 % to 25 % improvement in the number of near optimal solutions found.
- In the case of the minimum mean error with distributed pheromone evaporation coefficient values, from 3 % to 8 % improvement was noted.

- Distribution of the pheromone evaporation coefficient values has no or very little effect on AS algorithm convergence speed.
- The proposed modification is reasonable to use on small to average TSP without much increase of AS algorithm complexity.

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