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Automated Knowledge Based Filter Synthesis Using Gegenbauer Approximation and Optimization of Pole-Q Factors

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***Abstract*—This work gives an original algorithm that combines design and synthesis step with the optimization for targeted implementation technology. The approximation step is based on Gegenbauer polynomial and the corresponding cost function. The proposed methodology is an example of usage of computer algebra system as an alternative to classic numeric computing.**

***Index Terms*—Analog integrated circuits, circuit analysis computing, design for manufacture, elliptic design.**

I. INTRODUCTION

Analog filters are frequency-selective electrical circuits that are used to amplify or attenuate the band-limited signal frequency spectrum [1]. Many different physical components can be used for implementation. In practice, the values of filter components diverge from the ideal and the results of filtering may be different from the expected frequency response. The different technologies are impaired with different types of errors in the components. An exhaustive research on active components is presented in [2]. It is important to choose a filter structure with a low sensitivity to the expected dominant errors in the intended implementation technology. For example, the elliptic digital filter is the most efficient filter because there is no other filter of the lower order that can fulfill the same filter specification [1]. The EMQF (Elliptic Minimum Q Factor) filters are the most efficient filters of the elliptic filters [3]. The design procedure of the EMQF digital filters is presented in [4]. The similar design strategy was implemented for analog filters implemented in SC (Switched Capacitor) technology [5].

The design of high performance digital signal processing circuits can be very efficient using digital programmable circuits as it is shown in [4], [6], and [7]. Quite different design procedure should be followed when analog programmable circuits are used with on-chip tuning [8]. The most important strategy is the optimization of the second order sections [9].

In this paper, a new synthesis strategy is presented based on knowledge inputted into CAS (Computer Algebra System) [10]. The method is based on all-pole approximation technique and formulation of the cost function that minimizes the difference of the maximal pole-Q factor of the transfer function from the set of preferred *Q* factors available in programmable analog circuits.

II. APPROXIMATION

The even-order low-pass prototype all-pole filter transfer function can be represented using the pole frequencies, $\omega_{p,k}$ (or the corner frequency, $f_{ck} = \omega_{p,k} / 2\pi$), the pole Q-factors, $Q_{p,k}$, and the filter order n

$$H_n(s) = \frac{\prod_{k=1}^{n/2} \omega_{p,k}^2}{\prod_{k=1}^{n/2} (s^2 + \frac{\omega_{p,k}}{Q_{p,k}} s + \omega_{p,k}^2)}. \tag{1}$$

The squared magnitude response can be expressed using the even-order polynomial approximation, $A_{2n}(\omega)$, and the ripple factor, ε

$$H_n(j\omega)H_n(-j\omega) = \frac{1}{1+\varepsilon^2 A_{2n}(\omega)}, A_{2n}(\omega) > -1, \varepsilon \le 1. \tag{2}$$

The synthesis means to find the procedure for computing the squared pole frequencies, $\omega_{p,k}$, and the pole Q-factors, $Q_{p,k}$, for $k = 1, 2, 3, ..., n/2)$, starting from (2) and to achieve parameters of the programmable integrated circuits with the minimal error of the designed transfer function for the specified filter specification. We choose all-pole transfer function in order to avoid an additional element for implementing zeros of the transfer function. Actually, we will try to optimize the maximal Q factor because it is the most critical element, and at the same time to fulfill filter specification and the implementation requirements of the

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targeted technology.

The squared magnitude response of the proposed filter class is based on the even-order Gegenbauer orthogonal polynomials, $G_{2r}^{\nu}(\omega)$ [11], and it is normalized to the unity value at the pass-band edge frequency, $A_{2n}(1)=1$

$$A_{2n}(\omega)=\frac{\sum_{r=0}^{2n} b_{2r}G_{2r}^{\nu}(\omega)}{\sum_{r=0}^{2n} b_{2r}G_{2r}^{\nu}(1)}. \qquad (3)$$

The cost function φ can be represented using the weight function $p(\omega)=(1-\omega^2)^{\nu-1/2}$, the design parameters (the filter coefficients b_0, b_2, b_4, $b_{6,}\ldots,b_{2n}$, and the two free normalizing parameters, λ_1 and λ_2), and the even-order Gegenbauer orthogonal polynomials

$$\varphi(b_0, b_2, b_4, b_6, \ldots b_{2n}, \lambda_0, \lambda_1) =$$
$$=\int_0^1 (1-\omega^2)^{\nu-1/2}\Big[\sum_{r=0}^{r=n} b_{2r}G_{2r}^{\nu}(\omega)\Big]^2 d(\omega) -$$
$$-\lambda_0\Big[\sum_{r=0}^{r=n} b_{2r}G_{2r}^{\nu}(0)\Big]-\lambda_1\Big[\sum_{r=0}^{r=n} b_{2r}G_{2r}^{\nu}(1)-1\Big]. \qquad (4)$$

The optimal values are computed by solving the system of equations in which the first partial derivatives of the cost function with respect to the design parameters are equal to 0, that is $d\varphi/db_{2i}=0$, and $d\varphi/d\lambda_i=0$.

The closed-form solution is rather complicated even for experienced users, and the whole procedure is implemented using computer algebra system [10].

The squared norm, h_r, of the orthogonal Gegenbauer polynomial $G_{2r}^{\nu}(\omega)$, $r=0, 1, 2, 3,\ldots$ is defined

$$h_r=\int_0^1 p(\omega)G_{2r}^{\nu}(\omega)G_{2r}^{\nu}(\omega)d(\omega), r=0, 1, \ldots \qquad (5)$$

The integration gives the values of h_r in terms of r, the weighting parameter ν and the Pochamer symbol $(2\nu)_{2r}$

$$h_r(\nu)=\frac{(2\nu)_{2r}\Gamma(\frac{1}{2})\Gamma(\nu+\frac{1}{2})}{(2r)!(\nu+2r)\Gamma(\nu)}, \ r=0,1,2,3,\ldots \qquad (6)$$

The design procedure is explained by the example in the next section. Notice that filter specification can be given using the reflection factor ρ

$$\varepsilon^2=\frac{\rho^2}{1-\rho^2}. \qquad (7)$$

The design algorithm is implemented in software Mathematica [10].

III. Building Knowledge For Synthesis

Some software tools already have special functions that are required for the filter design. Since we are going to prepare template notebook, we present the code in Mathematica [10] software as one of the best CAS software. Firstly, the formulas are inputted into CAS software.

The first cells contain the minimal number of parameters such as the number of biquads (`nBiquads=5`) and auxiliary weighting factor (`v0=1/2`) for computing all other values:

```
nBiquads = 5;
n = 2 nBiquads;
b = Table[ToExpression[StringJoin["b",
ToString[r]]], {r, 0, 2 n, 2}]
v0 = 1/2
```

The list of coefficients **b** is automatically generated:

```
{b0,b2,b4,b6,b8,b10,b12,b14,b16,b18,b20}
```

The code of the squared magnitude response is similar to (3):

$$\mathtt{a2} = \frac{\left(\sum_{r=0}^{n} (\mathtt{b[[r+1]]\ GegenbauerC[2\,r,\ v0,\ w]})\right)}{\left(\sum_{r=0}^{n} (\mathtt{b[[r+1]]\ GegenbauerC[2\,r,\ v0,\ 1]})\right)}$$

The code for the cost function is also very similar to the definition (4), except that the name of the special function is not the Gegenbauer polynomial but unknown function **pP**:

$$\phi = \int_0^1 \left((1-w^2)^{v-\frac{1}{2}}\left(\sum_{r=0}^{n}\mathtt{b[[r+1]]\ pP[2\,r,\ v,\ w]}\right)\right)^2 dw -$$
$$\mathtt{L0}\sum_{r=0}^{n}(\mathtt{b[[r+1]]\ pP[2\,r,\ v,\ 0]}) -$$
$$\mathtt{L1}\left(-1+\sum_{r=0}^{n}(\mathtt{b[[r+1]]\ pP[2\,r,\ v,\ 1]})\right);$$

After defining the cost function φ in CAS, the next step is to find the first partial derivatives of the cost function with respect to the design parameters. The Gegenbauer polynomials can be treated as constants and the integration can be performed after derivation. One of the key features of CAS is that it is not necessary to specify exact values of variables on the right side of an equation, but the symbolic name can be used instead. Since we have three different expressions of the Gegenbauer polynomials, the same number of substitution rules is specified:

```
sub0= pP[x_,v_,0] → GegenbauerC[x, v, 0];
sub1= pP[x_,v_,1] → GegenbauerC[x, v, 1];
subw= pP[x_,v_,w_] → GegenbauerC[x, v, w];
```

Now, we can create a set of equations that are generated using partial derivatives (actually only left side of equations):

```
eqB=Table[D[ ϕ , b[[r+1]]],{r,0,n}]/. sub0 /. sub1
/. subw /. v → v0;
eqL = {D[ ϕ ,L0]/. sub0/. sub1/. subw/. v→v0,
       D[ ϕ ,L1]/. sub0/. sub1/. subw/. v→v0};
```

The whole processing time is much larger when the substitution of the Gegenbauer polynomials is performed before derivation. The two matrices are joined in one and symbolic description for equating to 0 is added to the each row:

```
eqLeft = Join[eqB, eqL];
eqs=Thread[eqLeft==0];
parameters=Join[b,{L0,L1}]
```

After forming the system of equations **eqs** and the list of all parameters **parameters**, the solutions are computed using built-in command **Solve**. The solutions are in the form of replacement rules **sol1**, which can be used in earlier defined squared magnitude function **aw**:

```
sol1 = Solve[eqs, parameters]
aw = a2/.sol1;
```

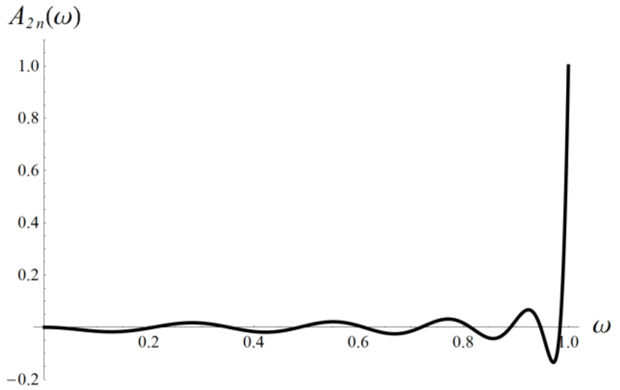

Fig. 1. Polynomial approximation for $n=10$, $\nu=0.5$, $A_{2n}(\omega) =$ -2.47× ×ω^2 + 111ω^4 + 1712.6$\omega^6 \cdots +$22318ω^{20}.

Figure 1 illustrates the computed squared magnitude function. It is important to notice that only the number of biquads is defined and the value of weighting factor. For any other number of biquads or another value of weighting factor, the same notebook can be evaluate after changing the two numeric values in the first input cell.

The rest of the notebook can be prepared in a similar way. The denominator of (2) is defined for the specified ripple factor or the preferred reflection factor, say ρ =1/10. The roots are in quadruplet and only the left half plane roots are selected to form the poles of the transfer function. Multiplying the two consecutive complex-conjugate roots the squared pole magnitude is computed, and using the definition of the pole Q factor [1] the Q factors of each biquad can be computed.

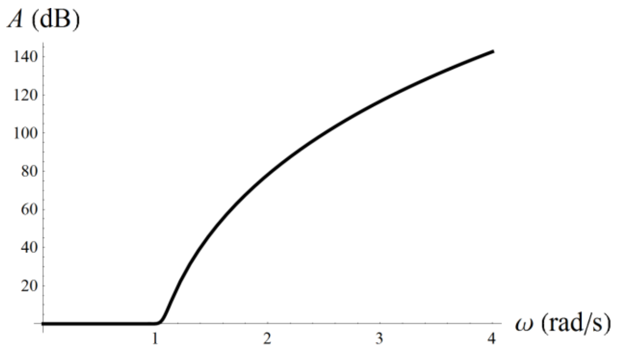

Fig. 2. Attenuation of the filter $A=-10\log_{10}\left|H_n(j\omega)\right|^2$ in dB for $n=10$, $\nu=0.5$, ρ =0.1.

Finally, a list of the biquad transfer functions is carried out:

```
Hbiquad = Table[wp2[[k]] / (s^2 + s Sqrt[wp2[[k]]]/Qp[[k]] + wp2[[k]]), {k, 1, nBiquads}]
```

The attenuation is computed and shown in Fig. 2 and Fig. 3.

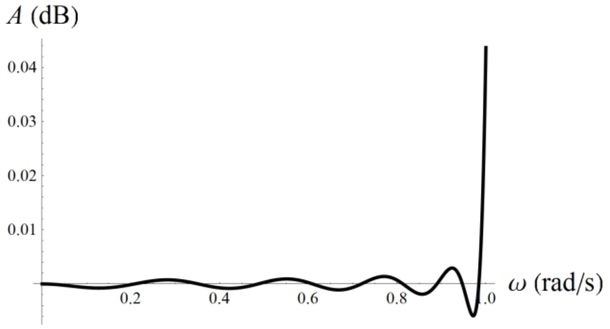

Fig. 3. Attenuation of the filter in the pass-band for $n=10$, $\nu=0.5$, ρ =0.1.

IV. Pole Q-Factor Optimization

Commonly, the filter design generates transfer function from the filter specification and at least one of the parameters is better than required [1]. We can use that parameter for the optimization and to improve some characteristics. For example, the stop-band attenuation can be higher than specified. Instead of redesigning the filter, that can be time consuming, we can optimize one of the design parameters. In this paper, we are changing the weighting factor until we have the exact value of the critical Q factor from the set of available values.

The optimization procedure is as follows: firstly, we determine the range of the weighting factors for that the pass-band and stop-band attenuations fulfill filter specification. Next, we determine the pole-Q-factor as a function of the weighting factor. Finally, by equating the function with one or more values from the available values specified by manufacturer, we find values of the weighting factor.

For example, suppose that the filter specifications are fulfilled for the weighting factor from the range of values {0.1. 0.2}. Since the filter design knowledge already exists in computer algebra system, the redesign process can be simplified. The basic idea is not to find a minimal value of the pole-Q-factor, but to design a filter that has an exact predefined value of the maximal pole-Q-factor, for example a value from the set of possible values available from programmable analog integrated circuits [7]. Using fitting function, we can derive a closed form approximation of the critical Q-factor (the largest Q-factor) in terms of the parameter ν. The closed form approximation can be presented in a polynomial form. Solving the equation $Q=Q_{set}$ we can derive the parameter ν in terms of the specified Q-factor. In this case, for the preferred value of the critical Q-factor $Q_{set}=9$, the value of the weighting factor is 0.125198. The next step is to find the optimal sequence of biquads for the maximal dynamic range. Figure 4 illustrates an example of the implementation using programmable chip (AN221E04 device) that is designed using AnadigmDesigner software. Four biquads are placed in a chip, while the fifth one is in another chip. The implemented corner frequencies and the corresponding Q factors are shown with each biquad.

The attenuations at the outputs of the cascaded connection of biquads are illustrated in Fig. 5. The maximal variation in the pass-band is lower than 12 dB.

Attenuation in the pass-band is presented in Fig. 6 for ±0.2% tolerance of Q factors and in Fig. 7 for ±0.2% tolerances of pole magnitudes. It follows that the pass-band variation is lower than 0.1 dB, except for the pole magnitude tolerances of the critical biquad. The on-chip tuning can be performed for the critical biquad so that the overall attenuation in the pass-band is lower than 0.1 dB.

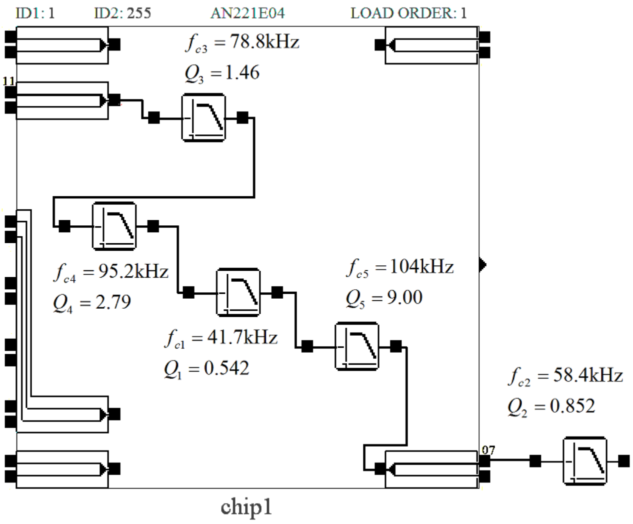

Fig. 4. Implementation with programmable chips.

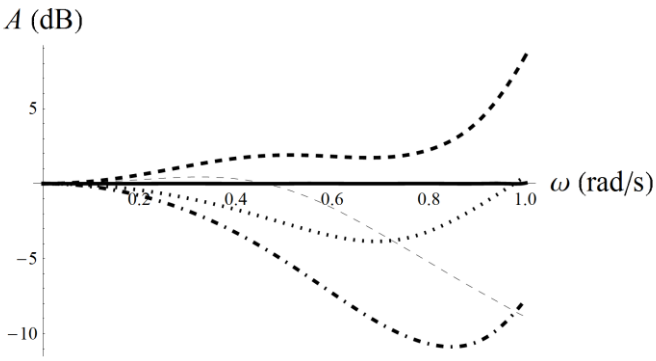

Fig. 5. Attenuation at the output of biquads for implementation with programmable chips.

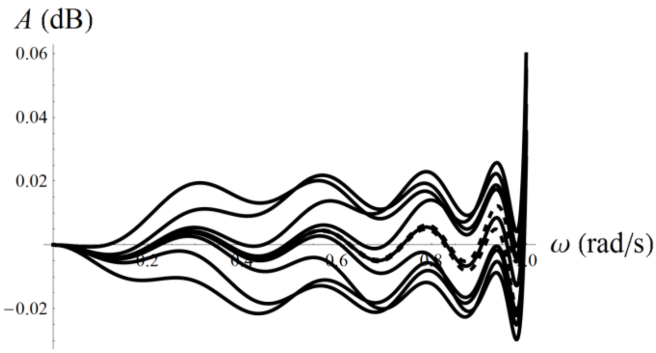

Fig. 6. Overall attenuation for ±0.2% tolerances of the critical Q factor (dashed lines) and other Q factors (solid line).

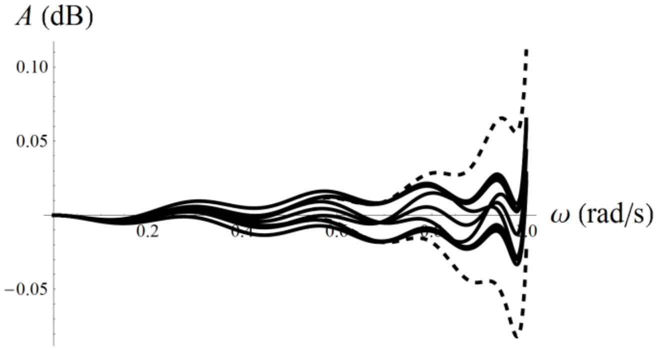

Fig. 7. Overall attenuation for ±0.2% tolerances of the pole magnitudes (biquad with critical Q factor - dashed lines, other biquads - solid line).

The design procedure is very simple because the weighting factor can be used for optimization for the minimal variation in the pass-band.

V. CONCLUSIONS

In this paper, a new class of even-order transfer functions based on Gegenbauer polynomials and the appropriate cost function is presented. The design step is a part of the synthesis step, because the free optimization parameter is selected on implementation requirements. In order to simplify the procedure, a template notebook is prepared so that the minimal numbers of input parameters are specified, such as the number of biquadratic sections and the ripple factor.

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