

# The Performance of the Modified GCC Technique for Differential Time Delay Estimation in the Cooperative Sensor Network

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**Abstract**—In this paper, a modified Generalized Cross-Correlation (GCC) technique (or FPT) for time delay estimation (TDE) is described. A flipped parameter (FP) can be defined for any sensor and is estimated during the observation period. Unlike classical TDE methods that evaluate the cross-correlation function, FPT requires flipped correlation evaluation based on each single sensor signal. The proposed technique gives accurate results for both coherent and non-coherent signals with symmetrical spectrum around the central frequency. The use of FPT method is especially significant for distributed sensor networks. The amount of data that needs to be transferred to the fusion center is significantly reduced than in case of classical pairwise algorithms. The main features of the FPT are tested in a noisy simulation environment and the results are compared with classical methods.

**Index Terms**—Generalized Cross-Correlation, differential time delay estimation, Hilbert transformation, sensor signal.

## I. INTRODUCTION

Time delay estimation (TDE) problem, although studied for several years [1], now still remains as an open problem [2]. This is due to the fact that TDE has found applications in diverse fields, including radar, sonar, seismology, geophysics, ultrasound, acoustic, hands-free communications and wireless sensor networks [1]–[3]. Standard techniques for TDE are based on the evaluation of generalized cross-correlation (GCC) function [4] between signals from spatially located sensors. The sensor signals should be first preprocessed in the continuous domain [5]. Several estimators have been proposed based on prefiltering prior to the evaluation of cross-correlation function; these include ROTH, PHAT and SCOT [4], [6]. Classic GCC methods require a priori knowledge of signal parameters for high quality TDE. This condition is not satisfied in practice and thus the reliability of evaluation decreases. Also, reliable TDE is not possible in the presence of small SNR. The quality of TDE is assessed by comparison with the theoretical Cramér-Rao bound (CRB), that is, against a minimum variance TDE evaluation error [7].

Depending on the application requirements, the sensor networks can contain from a few to even a thousand of (wireless) sensors. When the sensor network consists of a large number of sensors, it is usually divided into distributed wireless sensor subnetworks. In the cellular architecture of wireless sensor networks implementation of intelligent central node requires optimization of battery power consumption [8]. The cooperative sensor network designs, in addition to central intelligence (the fusion center) often contain local intelligence (a local processor) which combines and controls work of several sensors.

If the sensors are positioned at a distance from one another, which is significantly smaller than the signal wavelength, they are considered coherent signals, for which the information about time delay is contained in the signal phase. On the other hand, when sensors are very far apart from one another, the information about time delay lies in the signal envelope (non-coherent signals). The classic GCC method for real non-coherent signals does not give satisfying results, and in order to employ modified GCC (for us Flipped Parameter Technique FPT) [6]. The flipped parameter that characterizes each sensor must be obtained at every observation period. This is achieved based on the maximum cross-correlation function between the sensor signal and an appropriate analytic virtual signal [9], [10]. Such a virtual analytic sensor signal is obtained by flipping the sensor signal within the observation period. The proposed modified GCC technique provides good results for both coherent and non-coherent signals. This paper examines the performance of the recommended method in the presence of AWGN noise.

The paper is organized as follows. Section II shows the model of the signal used in this paper, while the theoretical principles of the modified cross-correlation method, as well as the differential delay algorithms in both frequency and time domain are given in Section III. Evaluating of the proposed method was performed in a simulation environment, with SNR as a variable. Performance of modified (FPT) and standard GCC methods is shown in separated Section IV. Discussions of the results, as well as the benefits of proposed method are shown in concluding

Section V.

## II. SIGNAL MODEL

Consider a signal  $s(t)$  originating from remote source and picked up by  $M$  sensors in a noisy environment. Assume that  $s_i(t)$  are spatially separated sensor outputs which can be mathematically modeled as

$$s_i(t) = a_i s(t - D_i) + n_i(t), \quad i = 1, 2, \dots, M, \quad (1)$$

where  $s(t)$  is a real band-limited source signal. The noise  $n_i(t)$  is a real jointly stationary random Gaussian processes. The parameter  $a_i$  is the attenuation factor, and  $D_i$  is the value of the time delay. The statistical properties of signal  $s_i(t)$  and noise  $n_i(t)$  are assumed to be uncorrelated for a finite observation time. The signal  $a_i s(t - D_i)$  is thus a shifted and scaled version of the signal  $s(t)$ . The basic approach to estimate the differential time delay  $D_{ji} = D_j - D_i$  is to shift the signal  $s_i(t)$  with respect to signal  $s_j(t)$ , and look for similarities between them [4]. The best match will occur at a shift equal to  $D_{ji}$ . The differential time delay  $D_{ji}$  can be estimated with respect to the first (referent) sensor signal  $s_1(t)$ :  $D_{ji} = D_{j1} - D_{i1}$ ,  $j, i = 1, 2, \dots, M$ .

To determine the differential time delays of signals using the flipped parameter technique [6], we shall form a virtual sensor network, consisting of  $M$  virtual sensors. This way every real sensor has a corresponding virtual sensor. Real sensors provide real sensor signals  $s_i(t)$ , whereas virtual sensors form the complex analytic flipped signal  $s_i^F(t)$ . The virtual sensor signal,  $s_i^F(t)$ , corresponding to the  $i$ -th real sensor, is therefore given by

$$s_i^F(t) = s_i^A(T - t), \quad i = 1, 2, \dots, M, \quad (2)$$

where  $s_i^A(t)$ .

$$s_i^A(t) = s_i(t) + jH[s_i(t)], \quad i = 1, 2, \dots, M \quad (3)$$

is the analytic signal [9], [10].

Here,  $s_i(t)$  is a signal corresponding to the  $i$ -th real sensor,  $H[\bullet]$  denotes the Hilbert transform operator,  $T$  is the observation period and  $j = \sqrt{-1}$ . The analytic signal  $s_i^A(t)$  is obtained by adding the real signal  $s_i(t)$  and its Hilbert transform [9]. The pair  $s_i(t)$  and  $s_i^F(t)$  form the basis for the proposed flipped parameter technique. From the definition of a complex analytic flipped signal  $s_i^F(t)$ , refer to (2), a virtual sensor signal is formed by flipping of the

analytic signal  $s_i^A(t)$ , for the duration of the observation window  $T$ .

## III. MODIFIED GCC TECHNIQUE

### A. Time domain

In previous paper [6], we propose a flipped parameter technique based on the computation of numerical value  $DF$  of the flipped parameter. This parameter is to be observed for each sensor during the observation time. The flipped parameter of  $i$ -th sensor,  $DF_i$ , for an observation period  $T$ , is computed based on estimated flipped cross-correlation function  $\hat{R}_{s_i s_i^F}(\tau)$

$$DF_i = \arg \max_{\tau} \left| \hat{R}_{s_i s_i^F}(\tau) \right|, \quad i = 1, 2, \dots, M. \quad (4)$$

The flipped parameter  $DF_i$  corresponds to a differential delay between real sensor signal  $s_i(t)$  and the virtual analytic flipped signal  $s_i^F(t)$ . Let us define a flipped cross-correlation  $R_{s_i s_i^F}(\tau)$  as

$$R_{s_i s_i^F}(\tau) = E \left\{ s_i(t) s_i^F(t + \tau) \right\} = a_i R_{ss}(\tau - DF_i), \quad i = 1, 2, \dots, M, \quad (5)$$

where  $\tau$  is a correlation lag,  $E\{\bullet\}$  is the statistical expectation operator and  $R_{ss}(\bullet)$  is the autocorrelation function of  $s(t)$ . Because of the finite observation time  $T$ , the estimated flipped correlation function is given by [4]

$$\hat{R}_{s_i s_i^F}(\tau) = \frac{1}{T} \int_0^T s_i(t) s_i^F(t + \tau) dt. \quad (6)$$

The differential time delay between sensor signals  $s_j(t)$  and  $s_i(t)$  can be obtained using (4) and (6), which yields

$$D_{ji} = \frac{1}{2} (DF_j - DF_i). \quad (7)$$

### B. Frequency domain

Estimation of the differential time delay can also be achieved effectively in the frequency domain. First, during an observation period it is necessary to determine spectra  $G_{s_i}(\omega)$  of real signals  $s_i(t)$  from a real source and spectra  $G_{s_i^F}(\omega)$  of a flipped analytic signal  $s_i^F(t)$  from the virtual source. We can compute the flipped cross-power spectrum (or flipped spectrum)  $\hat{G}_{s_i s_i^F}(\omega)$ , with no prefiltering, between a real signal  $s_i(t)$  and a virtual sensor signal  $s_i^F(t)$  based on

$$\hat{G}_{s_i s_i^F}(\omega) = G_{s_i}(\omega) \text{ conj} \left[ G_{s_i^F}(\omega) \right]. \quad (8)$$

The spectrum  $G_{s_i}(\omega)$  of signal,  $s_i(t)$ , is given by

$$G_{s_i}(\omega) = FFT \left\{ \Re \left[ s_i^A(t) \right] \right\} = FFT \left\{ s_i(t) \right\} \quad (9)$$

and the spectrum  $G_{s_i^F}(\omega)$  of the flipped analytic signal  $s_i^F(t)$  is given as

$$G_{s_i^F}(\omega) = FFT \left[ s_i^F(t) \right], \quad (10)$$

where symbol  $FFT[\bullet]$  denotes the fast Fourier transform operator, symbol  $\Re[\bullet]$  represents a real part of complex number operator and  $\text{conj}[\bullet]$  represents a complex conjugate operator. A set of *flipped parameter* values  $DF_i$  for a fixed observation period can be determined as

$$DF_i = \arg \max_{\tau} \left| FFT^{-1} \left[ \hat{G}_{s_i s_i^F}(\omega) \right] \right|, \quad i=1, 2, \dots, M, \quad (11)$$

where symbol  $FFT^{-1}[\bullet]$  denotes the inverse fast Fourier transform operator. Subsequently, differential delays  $D_{ij}$ ,  $i=1, 2, \dots, M$ ,  $j=1, 2, \dots, M$ ,  $i \neq j$  can be obtained directly from (7). The flipped parameter method is therefore based on special properties of the Fourier transform of the real and complex sequence. After numerical computation of the differential delays  $D_{ji}$ , the algorithms of the fusion center are identical for all two-step methods of source localization [11].

#### IV. SIMULATION RESULTS

Evaluation of the FPT method has been done in the simulational environment using model (1), for  $M=2$ . Signals which correspond to the response of the geophone to the seismic excitation  $s_i(t, D_i)$  are modeled as real deterministic signals [12]

$$s_i(t, D_i) = \Re \left\{ a_i \exp \left( j\omega_0 t - \alpha(t - D_i)^2 \right) \right\}, \quad 0 \leq t \leq T, i=1, 2. \quad (12)$$

In time domain, real signal of the  $i$ -th sensor is represented as  $s_i(t, D_i)$ , where  $D_i$  represents the delay of the signal  $i$ -th sensor with respect to  $t=0$ ,  $\alpha$  is a parameter which defines the width of the frequency spectrum of the signal,  $\omega_0$  is the central circular frequency of the signal and  $T$  is the length of the observation window. The frequency spectrum of the sensor signal (12) is symmetrical with respect to the central circular frequency of the signal  $\omega_0$ .

Generally speaking, with this type of signals, the correlation maximum does not need to correspond to the signal maximum. Therefore, these signals may be interpreted as non-coherent [4], [6]. Scaled Gaussian noise has been added to the simulated sensor signals (12), which formed the corresponding set of signals with the SNR defined in the range  $-5dB \leq SNR \leq 20dB$ .

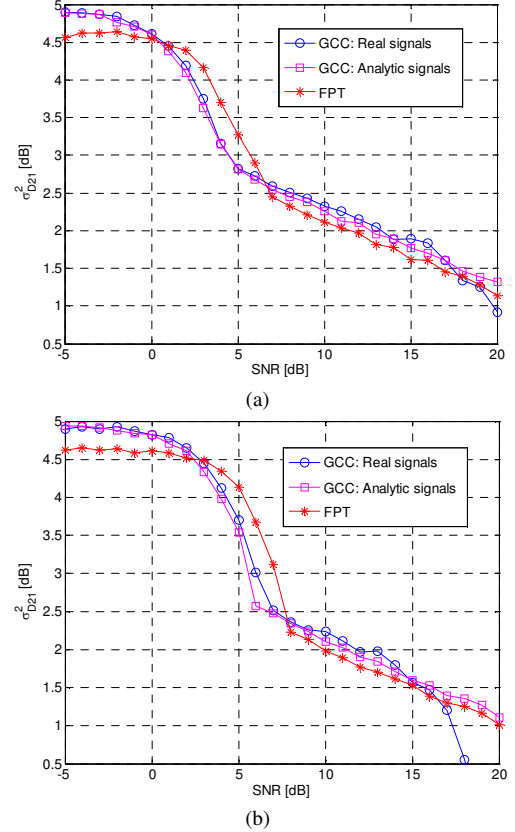


Fig. 1. Dependence of the variance  $\sigma_{D_{21}}^2$  with the SNR: GCC-real signals, GCC-analytic signals and FPT (a)  $\alpha=200$  and (b)  $\alpha=400$ .

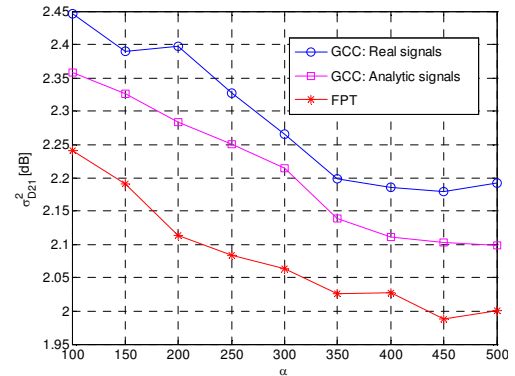


Fig. 2. Dependence of the variance  $\sigma_{D_{21}}^2$  with the  $\alpha$ : GCC-real signals, GCC-analytic signals and FPT, SNR=10 dB.

The exact value of the differential time delay  $D_{21}$  between the simulated signals  $s_1(t)$  and  $s_2(t)$  is defined as follows:  $D_{21} = D_2 - D_1$ . For the simulated sensor signal  $s_i(t)$  the corresponding analytical signal  $s_i^A(t)$  and virtual sensor signal  $s_i^F(t)$  are acquired through (3) and (2) respectively. Simulational signals formed this way are sampled with the

sampling interval ( $si$ )  $T_s = 1/f_s$ ,  $f_s = 1kHz$ , which gives us the real sequence  $s_i(nT_s)$  for signals  $s_i(t)$  and complex sequences  $s_i^A(nT_s)$  and  $s_i^F(nT_s)$  for signals  $s_i^A(t)$  and  $s_i^F(t)$ ,  $n = 1, 2, \dots, N$ ,  $N = 1024$ . The rest of the simulation parameters are as follows:  $100T_s \leq D_1, D_2 \leq 200T_s$ ,  $T = 1s$ ,  $\alpha = 200$ ,  $\omega_b = 2\pi 32 \text{rads}^{-1}$  and  $a_i = 1$ . Simulational algorithms were carried in Matlab. The experiment was repeated 500 times for each simulated SNR, while differential time delay  $D_{21}$  was the constant.

Figure 1 shows TDE error variance with SNR for all considered methods. Figure 1(a) shows the results obtained for the parameter  $\alpha = 200$ , while Fig. 1(b) shows the results for the parameter  $\alpha = 400$ . With the increasing value of the parameter  $\alpha$ , increases the width of the frequency spectrum of the signal. Comparing the graphics in Fig. 1(a) and Fig. 1(b) changes are evident in the boundaries of the range where the exponential variance (Barankin bound) translates into a linear dependence of the SNR (Cramér-Rao bound). When the signal frequency spectrum is wider, this boundary moves to higher SNR values.

In environments with higher noise levels, when SNR is in the range  $-5dB \leq SNR \leq 0dB$ , variance TDE does not depend on the value of SNR. Figure 1(a) shows that in this SNR segment FPT method provides a better error estimate than both of the GCC methods. When SNR is in range of  $0dB < SNR \leq 6dB$ , it is common for all three methods shown that the TDE error variance decreases exponentially with the increase of the SNR. In this SNR range, the FPT method has a greater variance than both of the GCC methods. However, with SNR in the range of  $6dB < SNR \leq 20dB$ , with the increase of the SNR, TDE error variance decreases linearly for all three TDE methods. This conclusion corresponds to the results given in [6]. Error variance for FPT method is the lowest in this SNR segment, therefore, the FPT method can be applied successfully, along with its benefits. A similar relationship can be seen in Fig. 1(b), except that the border shifted towards a higher SNR values.

Figure 2 shows the TDE error variance as a function of the parameter  $\alpha$ , for all three methods. Graphics in Fig. 2 are for SNR=10dB. Figure 2 clearly shows that the increase of the parameter  $\alpha$  causes the decrease of the TDE error variance in all methods. The recommended FPT method has the lowest level of the variance, which makes it optimal.

## V. CONCLUSIONS

In this paper, we analyze performance of a proposed FPT method for differential time delay estimation. This is achieved based on estimated numerical values of flipped parameters of sensors in presence of AWGN noise. Unlike the classical GCC methods for differential delay computation, where signals from pairs of two real sensors are used, the flipped parameter can be estimated using pairs of real sensor and virtual analytic sensor signals. We have shown that virtual sensor signals are formed based on their real counterparts, and that makes the flipped parameter

method very convenient for source localization in cooperative sensor networks. From the point of sensor network source localization, it is necessary that sensor modules compute their flipped parameters during the observation period and communicate them to the local processor or fusion center. The fusion center determines useful pairs of sensors and computes differential delays. A main advantage of this method is the fact that: 1) the numeric values of flipped parameters  $DF_i$  are sufficient to determine the differential delay; 2) the characteristics of the proposed algorithm are particularly important in cooperative sensor networks where reduces communication between sensor nodes (the amount of data that needs to be transferred is significantly reduced than in case of all pairs or pairwise algorithms); 3) FPT gives accurate results for both coherent and non-coherent signals; 4) in the case of high and low SNR, the FPT method gives better results than conventional GCC methods of, and its results are slightly worse in the medium SNR range.

The main features of the FPT are tested in a noisy simulation environment and the results are compared with classical methods.

## REFERENCES

- [1] A. H. Quazi, "An overview on the time delay estimate in active and passive systems for target localization", *IEEE Trans. Acoustics, Speech, and Signal Processing*, vol. 29, no. 3, pp. 527–533, June 1981. [Online]. Available: <http://dx.doi.org/10.1109/TASSP.1981.1163618>
- [2] S. Choi, D. Eom, "Minimizing false peak errors in generalized cross-correlation time delay estimation using subsample time delay estimation", *IEICE Trans. Fundamentals of Electronics*, vol. E96-A, no.1, pp. 304–311, 2013.
- [3] A. Bettina, "Linear elastic wave propagation in unsaturated sands, silts, loams and clays", *Transport in porous media*, vol. 86, no 2, pp. 537–557, 2011. [Online]. Available: <http://dx.doi.org/10.1007/s11242-010-9638-0>
- [4] C. H. Knap, G. C. Carter, "The generalized correlation method for estimation of time delay", *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. 24, no. 4, pp. 320–327, Aug. 1976.
- [5] D. G. Ciric, V. D. Pavlovic, "Linear phase two-dimensional FIR digital filter functions generated by applying Christoffel-Darboux formula for orthonormal polynomials", *Elektronika ir Elektrotehnika (Electronics and Electrical Engineering)*, no. 4, pp. 39–42, 2012.
- [6] V. D. Pavlović, Z. S. Veličković, "Flipped parameter technique applied on source localization in energy constraint sensor arrays", *FACTA UNIVERSITATIS, Series: Physics, Chemistry and Techn.*, vol. 7, no. 1, pp. 7–22, 2009.
- [7] X. Wen, A. B. Baggeroer, K. L. Bell, "A bound on mean-square estimation error with background parameter mismatch", *IEEE Trans. Information Theory*, vol. 50, no. 4, pp. 621–632, Apr. 2004. [Online]. Available: <http://dx.doi.org/10.1109/TIT.2004.825023>
- [8] Z. S. Velickovic, M. Jevtovic, "Adaptive cross-layer optimization based on Markov decision process", *Elektronika ir Elektrotehnika (Electronics and Electrical Engineering)*, no. 2, pp. 39–42, 2011.
- [9] S. L. Hahn, K. M. Snopek, "The unified theory of n-dimensional complex and hypercomplex analytic signals", *Bull. Pol. Academy: Tech.*, vol. 59, no. 2, pp. 167–181, 2011.
- [10] Z. S. Veličković, V. D. Pavlović, "Complex analytic signals applied on time delay estimation", *FACTA UNIVERSITATIS, Series: Physics, Chemistry and Techn.*, vol. 6, no 1, pp. 11–28, 2008.
- [11] R. Kozick, B. Sadler, "Source localization with distributed sensor arrays and partial spatial coherence", *IEEE Trans. Signal Processing*, vol. 52, pp. 601–616, Mar. 2004. [Online]. Available: <http://dx.doi.org/10.1109/TSP.2003.822354>
- [12] S. Golden, B. Friedlander, "Maximum likelihood estimation, analysis, and applications of exponential polynomial signals", *IEEE Trans. Signal Processing*, vol. 47, pp. 1493–1501, June 1999. [Online]. Available: <http://dx.doi.org/10.1109/78.765111>