

Complex Fourier Series Mathematical Model of a Three-Phase Inverter with Improved PWM Output Voltage Control

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Introduction

A three phase voltage source inverter permits to product an alternative three-phase voltage of variable or constant frequency on the basis of continuous input source voltage. The load of the inverter can be either passive, which does not contains any source of the voltage, or active containing some source of voltage.

The applications of the three-phase inverters are various and their utilisations are increasing more and more. They are used in most cases as a supply device for asynchronous and synchronous motors where frequency and voltage are controlled.

In the Fig.1 there the three-phase bridge connected inverter using the insulated gate bipolar transistor (IGBT) has been drawn. This device is being increasingly used in the both the single-phase and three-phase inverters [1, 2].

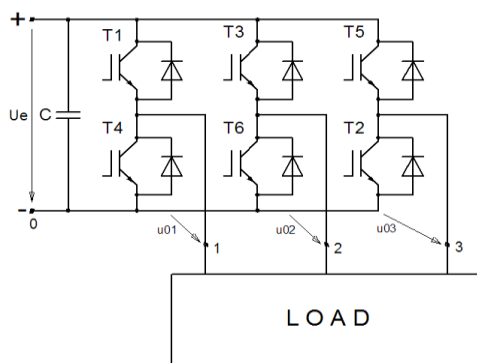


Fig. 1. Three-phase bridge connected inverter

Mathematical model of the inverter

For inverter's operation study at steady state we consider following idealized conditions:

- Power switch, that means the switch can handle unlimited current and blocks unlimited voltage;
- The voltage drop across the switch and leakage current through switch are zero;
- The switch is turned on and off with no rise and fall times;
- Sufficiently good size capacity of the input voltage capacitors divider, to can suppose converter input DC voltage to be constant for any output currents.

This assumption helps us to analyze a power circuit and helps us to build a mathematical model for the inverter at steady state.

An improvement to the notched waveform is to vary the on and off periods such that the on-periods are longest at the peak of the wave. This form of control is known as pulse-width modulation (PWM).

It can be observed that area of each pulse corresponds approximately to the area under the sine-wave between the adjacent mid-points of the off-periods. The pulse width modulated wave has much lower order harmonic content than the other waveforms.

If the desired reference voltage is sine-wave, two parameters define the control:

- *Coefficient of the modulation* m - equal to the ratio of the modulation and reference frequency;
- *Voltage control coefficient* r - equal to the ratio of the desired voltage amplitude and the DC supply voltage.

Generally to control the inverter numeric control device is used. The turn on (α) and turn off (β) angles are calculated by the discredit of the reference sine-wave. That means the reference sine-wave is by a values discreet replaced. If the coefficient of modulation m is sufficiently great, the difference between real values and discrete values is negligible.

The phase's branches are control to create the output voltages as seen in (1):

$$\begin{cases} u_{01} = \frac{U_e}{2} + r \frac{U_e}{2} \sin(\theta), \\ u_{02} = \frac{U_e}{2} + r \frac{U_e}{2} \sin\left(\theta - \frac{2\pi}{3}\right), \\ u_{03} = \frac{U_e}{2} + r \frac{U_e}{2} \sin\left(\theta + \frac{2\pi}{3}\right), \end{cases} \quad (1)$$

where U_e -is a DC inverter's input voltage value.

To calculate a turn on (α) and turn off (β) angles we compare the DC impulse area with the requested voltage area, as depicted on the Fig. 2.

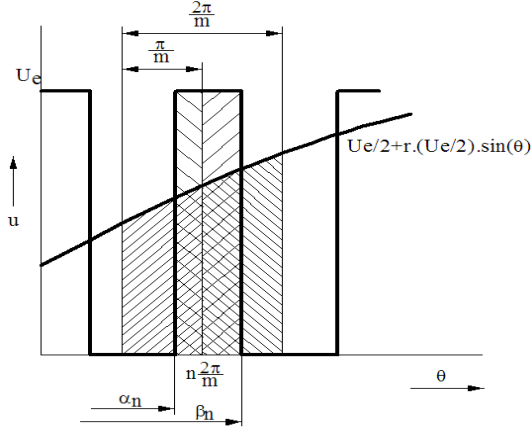


Fig. 2. Comparison of the voltages area

For the left and right crosshatched areas of the first output transistors branch the following equations are valid:

$$\begin{cases} \int_{\alpha_n}^{n \frac{2\pi}{m}} \left(\frac{U_e}{2} + r \frac{U_e}{2} \sin \theta \right) d\theta = U_e \left(n \frac{2\pi}{m} - \alpha_{01n} \right), \\ \int_{n \frac{2\pi}{m}}^{\beta_n} \left(\frac{U_e}{2} + r \frac{U_e}{2} \sin \theta \right) d\theta = U_e \left(\beta_{01n} - n \frac{2\pi}{m} \right). \end{cases} \quad (2)$$

After the calculus we obtain for the turn-on and turn-off angles of the first transistors branch the following expressions:

$$\begin{cases} \alpha_{01n} = \frac{\pi}{m} \left(2n - \frac{1}{2} \right) + \frac{r}{2} \left[\cos n \frac{2\pi}{m} - \cos \frac{\pi}{m} (2n - 1) \right], \\ \beta_{01n} = \frac{\pi}{m} \left(2n + \frac{1}{2} \right) + \frac{r}{2} \left[\cos n \frac{2\pi}{m} - \cos \frac{\pi}{m} (2n + 1) \right]. \end{cases} \quad (3)$$

It will be similarly for the second and third transistor branch.

The inverter's output voltage of the first branch can be mathematically expressed as a complex Fourier series of the form [3–6]

$$u_{01} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^m c_{01n} e^{jk\theta}, \quad (4)$$

where the Fourier coefficients take a form

$$\begin{aligned} c_{01n} &= \frac{1}{j2k\pi} \left(e^{-jk\alpha_{01n}} - e^{-jk\beta_{01n}} \right) \quad (\text{for } k \neq 0) \quad \text{and} \\ c_{01n} &= \frac{\beta_{01n} - \alpha_{01n}}{2\pi} \quad (\text{for } k = 0). \end{aligned}$$

The Fig. 3 depicts a branch voltage waveform for supply voltage of $U_e = 100V$, output frequency $f = 50Hz$ and $r = 1; m = 10$;

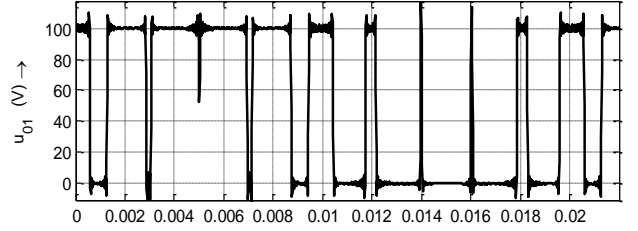


Fig. 3. The branch voltage waveform U_{01}

Similarly for the other two phases the following equations are valid:

$$\begin{cases} u_{02} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^m c_{02n} e^{jk\theta}, \\ u_{03} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^m c_{03n} e^{jk\theta}. \end{cases} \quad (5)$$

The terminal voltages calculation

Assuming, that the load is balanced and wye connected, as given in the Fig. 4. Consider the load of each phase consists of series connected RL elements ($R = 2\Omega; L = 50mH$).

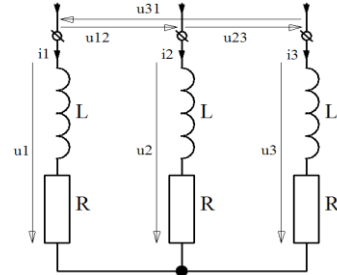


Fig. 4. Balanced wye connected load

The line voltages are given by a difference between two voltages of the branches as follow [1]:

$$\begin{cases} u_{12} = u_{01} - u_{02} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (c_{01n} - c_{02n}) e^{jk\theta}, \\ u_{23} = u_{02} - u_{03} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (c_{02n} - c_{03n}) e^{jk\theta}, \\ u_{31} = u_{03} - u_{01} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (c_{03n} - c_{01n}) e^{jk\theta}. \end{cases} \quad (6)$$

There is in the Fig. 5 shown the waveform of the output line-to-line voltage U_{12} (similarly for other line-to-line voltages).

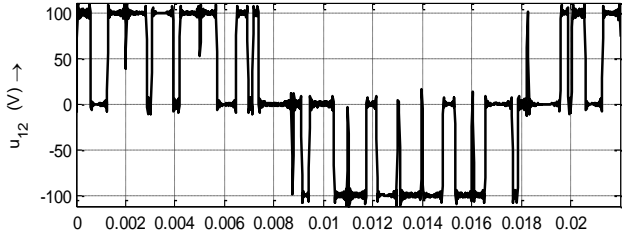


Fig. 5. Inverter's line-to-line output voltage waveform U_{12}

For the balanced load and insulated neutral node, following relations between line and phase voltages are valid:

$$\begin{cases} u_1 = \frac{1}{3}(2u_{01} - u_{02} - u_{03}) = \\ = \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{01n} - c_{02n} - c_{03n}) e^{jk\theta}, \\ u_2 = \frac{1}{3}(2u_{02} - u_{03} - u_{01}) = \\ = \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{02n} - c_{03n} - c_{01n}) e^{jk\theta}, \\ u_3 = \frac{1}{3}(2u_{03} - u_{01} - u_{02}) = \\ = \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{03n} - c_{01n} - c_{02n}) e^{jk\theta}. \end{cases} \quad (7)$$

The Fig. 6 depicts waveforms of a phase voltages, calculated on basis of (7).

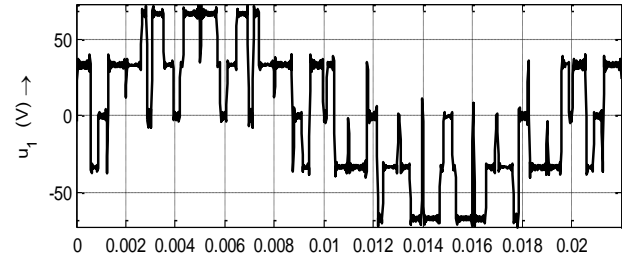


Fig. 6. The phase voltages waveform

For any of the phases the voltage equations are valid:

$$\begin{cases} \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{01n} - c_{02n} - c_{03n}) e^{jk\theta} = \sum_{k=-\infty}^{\infty} (Ri_1 + jk\omega L i_1), \\ \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{02n} - c_{03n} - c_{01n}) e^{jk\theta} = \sum_{k=-\infty}^{\infty} (Ri_2 + jk\omega L i_2), \\ \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{03n} - c_{02n} - c_{01n}) e^{jk\theta} = \sum_{k=-\infty}^{\infty} (Ri_3 + jk\omega L i_3). \end{cases} \quad (8)$$

The phase currents calculation

Based on (8) the analytical solution takes a form:

$$\begin{cases} i_1 = \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{01n} - c_{02n} - c_{03n}) \frac{e^{jk\theta}}{R + jk\omega L}, \\ i_2 = \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{02n} - c_{03n} - c_{01n}) \frac{e^{jk\theta}}{R + jk\omega L}, \\ i_3 = \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{03n} - c_{02n} - c_{01n}) \frac{e^{jk\theta}}{R + jk\omega L}. \end{cases} \quad (9)$$

In the Fig. 7 is given calculated phase load current wave-form. The waveform was calculated on a basis of (9).

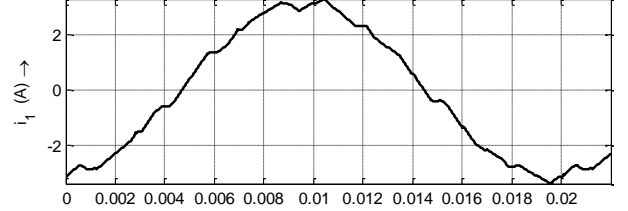


Fig. 7. The load currents waveform

In electrical engineering, the Clarke $\alpha, \beta, 0$ mathematical transformation is employed. It employment very often simplifies the analysis of three-phase circuits.

Conceptually the Clarke transformation present particular part of Park transformation. It converts ordinary 3-phase system into orthogonal 2-phase one [2, 4]. One of very useful application of the Clarke transformation is the generation of the reference signal used for space vector modulation control of the three-phase inverters.

The Clarke transformation applied to the three-phase quantities is shown below in a matrix form:

$$\begin{bmatrix} x_\alpha \\ x_\beta \\ x_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}. \quad (10)$$

Note also, that x_0 is just a scaled version of the zero sequence term from symmetrical components.

We can also write equation (11):

$$\begin{cases} x_\alpha = \frac{1}{3}(2x_1 - x_2 - x_3), \\ x_\beta = \frac{1}{\sqrt{3}}(x_2 - x_3). \end{cases} \quad (11)$$

Then for transformed phase voltages equations (12) are valid [3, 7, 8]:

$$\begin{cases} u_\alpha = \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{01n} - c_{02n} - c_{03n}) e^{jk\theta}, \\ u_\beta = \frac{U_e}{\sqrt{3}} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (c_{02n} - c_{03n}) e^{jk\theta}. \end{cases} \quad (12)$$

The Fig. 8 below shows the trajectory of the space vector of the transformed voltage components u_α, u_β , calculated on the basis of (12).

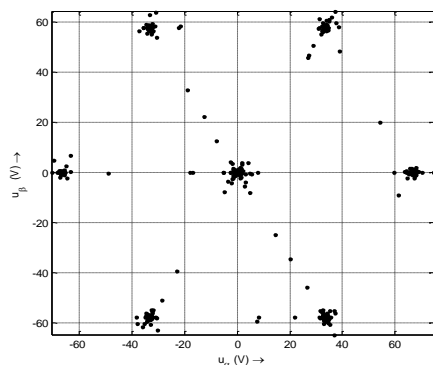


Fig. 8. Trajectory of the space vector of the phase voltages

On the basis of the (11) and (12) can be done the currents transformations. For transformed phase currents (Fig. 9) the following equations are valid:

$$\begin{cases} i_\alpha = \frac{U_e}{3} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (2c_{01n} - c_{02n} - c_{03n}) \frac{e^{jk\theta}}{R + jk\omega L}, \\ i_\beta = \frac{U_e}{\sqrt{3}} \sum_{k=-\infty}^{\infty} \sum_{n=1}^m (c_{02n} - c_{03n}) \frac{e^{jk\theta}}{R + jk\omega L}. \end{cases} \quad (13)$$

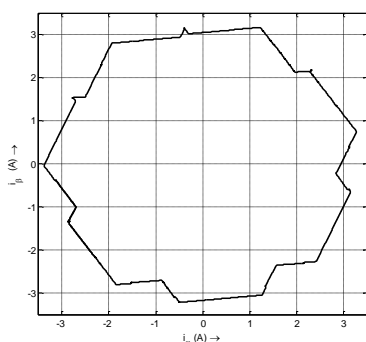


Fig. 9. Trajectory of the space vector of the phase currents

The trajectory (Fig. 9) will be a circle for infinite number of harmonics very high switching frequency.

Conclusions

In this study, an article about the more accurate

mathematical model of a three-phase voltage source inverter with PWM output voltage control has been presented.

The method is based on use of complex Fourier series calculation. The results carried-out by modelling are adequate to the presented mathematical model.

Comparison of results to those of carried-out by classical method ones will be done in next paper.

Acknowledgements

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P. Zaskalicky, B. Dobrucky. Complex Fourier Series Mathematical Model of a Three-Phase Inverter with Improved PWM Output Voltage Control // Electronics and Electrical Engineering. – Kaunas: Technologija, 2012. – No. 7(123). – P. 65–68.

The presented article steady state analysis of a three-phase voltage source inverter with PWM output voltage control is proposed. The mathematical model is built on condition of assumption of idealized semiconductor devices. A complex Fourier series approach is used to predict the waveforms of the terminal output inverter voltages. On the basis of the developed voltage formula the load current waveform for the wye load connections are calculated. Ill. 9, bibl. 8 (in English; abstracts in English and Lithuanian).

P. Zaskalicky, B. Dobrucky. Trifazio inverterio, geriau valdančio IPM išėjimo įtampą, kompleksinės Furje eilutės matematinis modelis // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 7(123). – P. 65–68.

Pasiūlytas trifazio inverterio, geriau valdančio IPM išėjimo įtampą, stabilios būsenos modelis. Matematinis modelis sukurtas priimant idealizuotų puslaidininkinių įtaisų sąlygą. Kompleksinės Furje eilutės panaudotos prognozuojant inverterio įtampos signalus. Naudojant sukurtą įtampos formulę, apskaičiuojamas apkrovos srovės signalas. Il. 9, bibl. 8 (anglų kalba; santraukos anglų ir lietuvių k.).