Multifunctional Electronically Reconfigurable and Tunable Fractional-Order Filter

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Abstract—In this paper the authors present a multifunctional reconfigurable fractional-order filter performing a low-pass, high-pass, band-pass and band-reject transfer function. The filter is based on two types of active elements, OTA (Operational Transconductance Amplifier) and ACA (Adjustable Current Amplifier). It provides pole frequency control, depending on the values of the transconductance of the OTA elements. The quality factor is also electronically controlled, depending on the amplification of the ACA element. The order of the filter can be changed by switching the fractional-order capacitor having various values of the order. The circuit was implemented as a PCB (Printed Circuit Board) and measured in laboratory conditions. Measurement results are compared with the simulation results. The behavioural models were used for the purpose of the simulation.

Index Terms—Fractional-order element; Analogue filter; Quality factor control; Pole frequency control; DACA; OTA.

I. INTRODUCTION

Fractional-order systems have become an interesting topic as they expand the possibilities of the integer-order electronic systems [1]–[8]. Analog systems are less limited than digital systems when it comes to the range of possible values or its possible settings. An integer order is usually given for the analogue circuits (filters, oscillators, generators...). Fractional-order systems can find use in biochemistry [6], [9], agriculture [9], control systems [9] and electronics [8], [10]–[15].

Fractional-order filters provide fine changing of the order in comparison with integer-order filters. The order of the fractional-order filters is given by $(n + \alpha)$, where n is non-zero positive integer number and parameter α is a real number in range $0 < \alpha < 1$ [1], [4]. In point of view of the magnitude and phase responses, the fractional-order filters have magnitude slope of attenuation in stop-band region equal to $20(n + \alpha)$ dB/decade. The phase shift is given by $90(n + \alpha)$ degrees [8], [12], [15].

There are three possibilities how to create a fractional-order circuit. The first possible way is to use a passive element with fractional-order character referred to as FOE (Fractional-Order Element) [16], capacitor in most cases, and use it as a dominant capacitor in filtration structure [1], [17], [18]. Though this solution may seem straightforward, there is no industrial manufacturing of FOE yet.

The second approach to the proposal of the fractional-order filters is an approximation of a FOE using RC structure [1], [5], [11], [19]. There are several types of RC structures which can be used for approximation of the FOE. The most often used is an approximation by RC ladder structure [5], [11]. RC ladder is an option available for both modelling and measurement, which is an advantage. However, RC structure can only approximate a fractional-order passive element in phase, magnitude and frequency range, making it a limited immitance [17]–[19]. A potential drawback of this method is the hardware form of the fractional-order RC ladder - a required change of alpha may require a change of hardware. This drawback may be fixed by using switched topology for example, however, switched device may bring in an interference and it also makes the circuit more complex.

The last method is an approximation of the fractional-order transfer function by higher-integer-order filtering structure [4], [8], [12]–[14]. The higher-integer-order structure is obtained using approximation of a Laplacian term s^α which is in a fractional-order transfer function replaced by the integer-order function. Most frequently, a second-order function is used [4]. On one hand, no manufactured or approximated fractional element is needed. On the other hand, to emulate a filter of a 1.5 order, originally a third-order filter is required for example [4], [7], [12]. Third-order filter is a more complex structure in comparison with a second-order filter. It may be seen as disadvantageous to create a complex structure and use it to underperform in terms of the filter order. The advantage of this method is that it is possible to use commercially available active elements with adjustable parameters and standard passive components.

The fractional-order filter containing these active elements allows the electronical controlling of the filter parameters (order, pole frequency, quality factor or pass-band gain for example).

Table I summarizes some relevant previously reported works. The fractional-order low-pass filters without electronic controllable parameters are most commonly proposed [1], [4], [7], [15]. The electronic reconfiguration between two or more transfer functions, without topology modification and with electronically adjustable filter parameters, is presented only in the papers [8], [14]. However, none of the reported articles contains an electronically controlled quality factor.

In this paper, we present a reconfigurable fractional-order frequency filter with FOE which operates in the current mode. The filter provides electronic reconfigurability between Fractional Low-Pass Filter (FLPF), High-Pass Filter (FHPF), Band-Pass Filter (FBPF) and Band-Reject Filter (FBRF) transfer functions. The pole frequency and quality factor of the filter are controlled without disturbing each other by adjustable parameters of the used active elements. The second design option was chosen because it is effective, the parasitics do not devaluate the resulting circuit and the circuit complexity is sufficient. In this paper both simulations and experimental measurements are presented.

TABLE I. COMPARISON OF FEATURES OF THE RECENTLY INTRODUCED FRACTIONAL-ORDER FILTERS.

Reference	Available transfer functions	Electronic tuning	Type of electronic tuning	Number of active elements	Approximation of the FOE	Transfer function reconfigurability	Type of verification
[1]	LP	No	–	3	Yes	No	Both
[4]	LP, HP, BP	No	–	2	No	No	Both
[7]	LP	No	–	4	No	No	Sim
[8]	LP, HP, BP, BR, AP	Yes	f_0, order	11	No	Yes	Sim
[11]	BP	No	–	2	Yes	No	Both
[14]	LP, HP, BP, BR	Yes	order	8	No	Yes	Sim
[15]	LP	No	–	2	Yes	No	Sim
Fig. 2	LP, HP, BP, BR	Yes	f_0, Q	6	Yes	Yes	Both

Note: sim – simulated, exp – experimental measurement, both – simulated and measured.

II. DESIGN OF THE CIRCUIT

This section describes the design of the multifunctional electronically reconfigurable and tunable fractional-order frequency filter. The filter, originally with OTAs, ACAs and two capacitors is proposed based on using the FOE in the circuit structure instead of capacitor C_1 (C_2 remains capacitor) as will be shown later.

The fractional-order capacitor is approximated using RC ladder structure which is shown in Fig. 1.
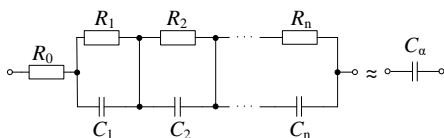

Fig. 1. The RC ladder structure used for approximation of the fractional-order capacitor.

The values of the particular passive parts in the structure

were computed in MATLAB using Oustaloup algorithm [19] for approximating the FOE with sufficient magnitude and phase accuracy in a specific frequency band. The FOE is approximated for use in the frequency band specified from 2 kHz to 1 MHz approximately.

The Butterworth-like approximation was used for design of the filter with maximally flat pass-band response [3], [4]. General transfer function of the LP filter is given by [3]

$$TF_{1+\alpha}(s) = \frac{\omega_0^{1+\alpha} k_1}{s^{1+\alpha} + s^\alpha \omega_0 k_2 + \omega_0^{1+\alpha} k_3}, \quad (1)$$

where $\omega_0 = 2\pi f_0$ rad/s and coefficients k_1, k_2 and k_3 are used to influence the shape of the obtained transfer function. The values of the coefficients k_1, k_2 and k_3 are adopted from [15]. Thanks to selection of these coefficients, quality factor of the filter can be tuned as is described in [15].

The proposed filter, which is depicted in Fig. 2, has been designed using three types of the active elements.

The first active element is Balanced-Output Transconductance Amplifier (BOTA) with current outputs. The function of the BOTA element is described as follows: $I_{OUT+} = -I_{OUT-} = g_m(V_{IN+} - V_{IN-})$, where the parameter g_m is electronically controllable transconductance.

Operation of Adjustable Current Amplifier (ACA) is described by the following equations: $I_{OUT} = BI_{IN}$, where parameter B is controllable current gain of the ACA [20], [21]. If necessary, the number of the current outputs of the ACA active element can be easily expanded (as in case of ACA4 in Fig. 2, where there are two outputs, $I_{OUT+} = -I_{OUT-} = BI_{IN}$).
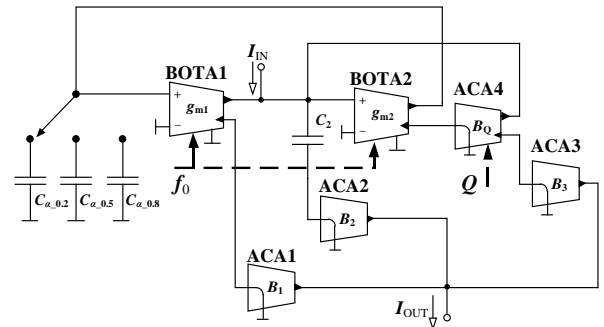

Fig. 2. Topology of the designed filter containing an approximated fractional-order capacitor (three variants).

The circuit presented in Fig. 2 contains the fractional-order capacitor C_α (instead of C_1) and provides the following transfer function

$$TF_{1+\alpha}(s) = \frac{s^{1+\alpha} B_2 + s^\alpha B_3 \frac{B_Q g_{m2}}{C_2} + B_1 \frac{g_{m1} g_{m2}}{C_\alpha C_2}}{s^{1+\alpha} + s^\alpha \frac{B_Q g_{m2}}{C_2} + \frac{g_{m1} g_{m2}}{C_\alpha C_2}}. \quad (2)$$

From the transfer function of the proposed filter (2) can be seen that filter provides LP, HP, BP and BR responses from one output by switching the current gains B_1, B_2 and B_3 of the ACA elements. The particular configurations of the transfer function of the designed filter are summarized in Table II.

The designed filter offers ability to control its pole frequency f_0 by changing values of the transconductances. The circuit also possesses the quality factor Q control by setting of value of the current gain B_Q. The order of the filter is changed by switching the RC structures approximating the C_α-s with different values of the α ($0 < \alpha < 1$).

TABLE II. CONFIGURATIONS OF THE TRANSFER FUNCTIONS.

Transfer function	B_1	B_2	B_3
FLPF	1	0	0
FHPF	0	1	0
FBPF	0	0	1
FBRF	1	1	0

The filter parameters can be computed by comparing the coefficients of denominators in (1) and (2):

$$g_{m1} = \frac{k_3 \omega_0^\alpha C_\alpha B_Q}{k_2}, \quad (3)$$

$$g_{m2} = \frac{k_2 \omega_0 C_2}{B_Q}. \quad (4)$$

From (3) and (4) the equation can be derived for pole frequency control

$$\omega_0 = \left(\frac{g_{m1} g_{m2}}{C_\alpha C_2 k_3} \right)^{\frac{1}{1+\alpha}}. \quad (5)$$

III. MEASUREMENT AND SIMULATION RESULTS

The features of the proposed fractional-order filter from Fig. 2 were verified by OrCAD PSpice simulations with behavioural models of the BOTA and ACA elements and also by experimental measurements.

The experimental measurements of the filter were performed with help of network analyser Agilent 4395A, using V/I, I/V converters and implemented filter in the Printed Circuit Board (PCB) form. The BOTA elements were replaced by one UCC N1B 0520 [22] chip with external resistors (R) connected to X terminal representing a transconductance value $g_m = 1/R$. The UCC N1B 0520 chip contains one Universal Current Conveyor (UCC) and one Second-generation Current Conveyor (CCII+/-). The dual-output ACA (ACA4 in filter topology) was implemented on PCB by DACA (Digitally Adjustable Current Amplifier) chip. The current gain B of the DACA is controlled by 3-bit word in range from 1 to 8 with step 1 [20], [21]. Both these chips were developed at Department of Telecommunications of the Brno University of Technology in cooperation with ON Semiconductor. For implementation of the ACA elements were used the EL2082 chips [23]. Note that the current gain of the EL2082 chips is controlled using external DC voltage in range from 0 to 2.5V [23].

The simulations and experimental measurement were performed with the following initial values of the filter parameters: pole frequency $f_0 = 10$ kHz, quality factor $Q = 0.7$ (current gain $B_Q = 4$), order of the filter 1.5 ($\alpha = 0.5$) and capacitors $C_1 = 6.8$ nF and $C_2 = 100$ nF. The value of the

fractional-order capacitor C_α is based on the chosen value of the C_1, parameter α and pole frequency f_0. The C_α is given by: $C_\alpha = C_1 \omega_0^{1-\alpha}$ Fs$^{\alpha-1}$. For above mentioned initial parameters is the value of the capacitor $C_{\alpha\_0.5} = 1.7$ µF/s$^{0.5}$. The calculated values of the transconductances are: $g_{m1} = 1.89$ mS ($R_1 = 1/g_{m1} = 531$ Ω) and $g_{m2} = 1.44$ mS ($R_2 = 1/g_{m2} = 695$ Ω).

The magnitude responses of the FLPF, FHPF, FBRF and FBPF responses are demonstrated in Fig. 3. Results obtained by experimental measurement (solid lines) are compared with simulations results (dashed lines). From obtained responses it is obvious that measured results are very close to the simulations. The differences between results are mainly caused by current gain B_Q, which is for DACA element slightly lower than the theoretical assumptions (for example for theoretic gain 4 is real current gain equal to 3.6), and by parasitic properties of the used active elements. That applies for all results presented below.
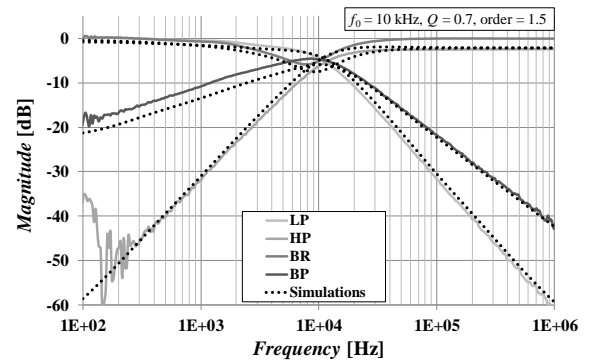

Fig. 3. Measurement results vs. simulations with behavioural models of the fractional-order multifunctional filter. Magnitude responses of FLPF, FHPF, FBRF and FBPF of the filter for f_0=10 kHz, order 1.5 and Q=0.7.

Figure 4(a)–Fig. 4(b) demonstrate the magnitude and phase responses in case of changing the order of the FLPF, for three values of the order (1.2, 1.5 and 1.8) when we choose the $f_0 = 50$ kHz and $Q = 0.7$. The values of the passive elements are summarized in Table III. The obtained values of the slopes of attenuation from measured results are: 23.9 dB/decade (order 1.19), 30.4 dB/decade (order 1.52) and 35.7 dB/decade (order 1.79). The values of the slope of attenuation obtained from simulations are: 24.1 dB/decade (order 1.2), 30.1 dB/decade (order 1.5) and 36 dB/decade (order 1.8).

TABLE III. THE FILTER PARAMETERS FOR ORDER CHANGING.

Theoretical values of the order	1.2	1.5	1.8
C_α [µFs$^{\alpha-1}$]	46.9	1.7	0.062
C_2 [nF]	10	10	10
$R_1 = 1/g_{m1}$ [Ω]	26	133	178
$R_2 = 1/g_{m2}$ [Ω]	878	174	130

The differences at high frequencies (around 1 MHz), which are seen from graphs are given by parasitic properties of the designed PCB and by bandwidth limitations of the used active elements. That actually applies for all results in the paper. The obtained values are in good agreement with simulations.

The proposed fractional-order filter provides the electronic control of the pole frequency by changing the values of the transconductances. The frequency tuning was

tested for five values for FLPF when the order of the filter is 1.5 and $Q = 0.7$. Obtained magnitude and phase responses of the measurements (solid lines) are compared with simulations (dashed lines) in Fig. 5(a)–Fig. 5(b). The calculated values of the used passive elements are stated in Table IV.
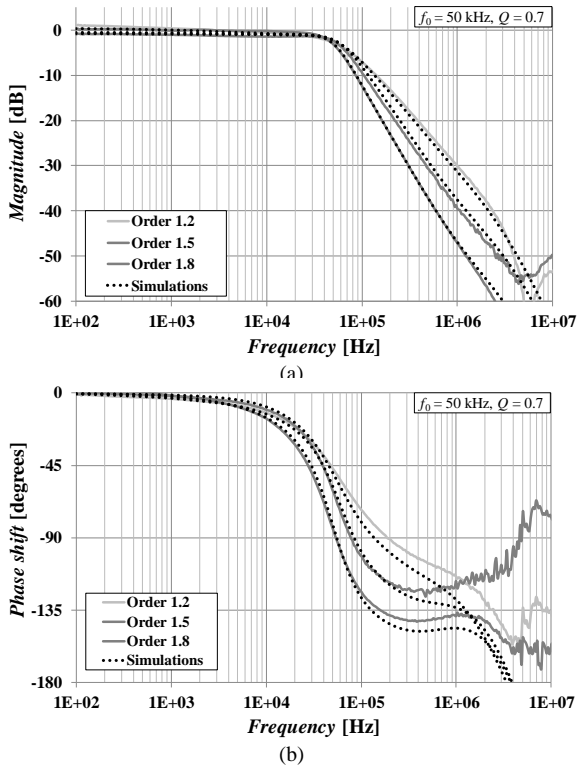

(a)

(b)

Fig. 4. Demonstration of the measurement and simulations results of the designed FLPF when changing the order, for the $f_0 = 50$ kHz, Q = 0.7: a) magnitude; b) phase shift.

TABLE IV. USED VALUES OF THE FILTER PARAMETERS FOR POLE FREQUENCY CONTROL.

Theoretical values of the f_0 [kHz]	5	10	25	50	150
C_α [µFs$^{\alpha-1}$]	1.7				
C_2 [nF]	10				
$R_1 = 1/g_{m1}$ [Ω]	1062	531	266	133	89
$R_2 = 1/g_{m2}$ [Ω]	1390	695	348	174	116

The obtained values of the pole frequencies from measurements are the following: 3.9 kHz, 8.9 kHz, 22.3 kHz, 54.7 kHz and 149.8 kHz. The values of the pole frequencies from simulations are: 4.4 kHz, 10.1 kHz, 24.5 kHz, 58.1 kHz and 140.5 kHz. The differences are given by inaccuracies of the passive elements as already discussed.
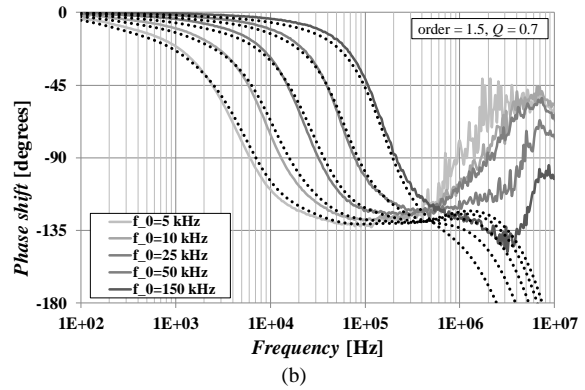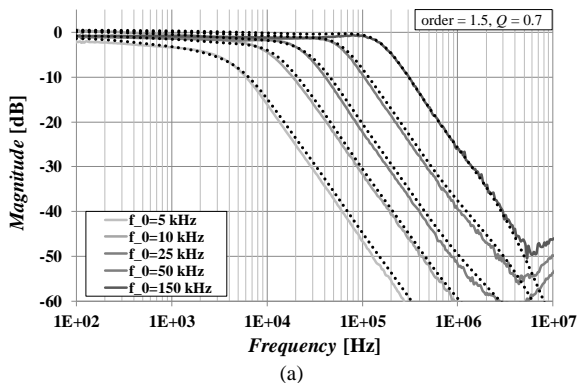

(a)

(b)

Fig. 5. Measurement vs. simulations results of the designed FLPF in case of frequency when the order was set to 1.5 and $Q = 0.7$: a) magnitude responses; b) phase shift.

From measurement results can be seen that for $f_0 = 150$ kHz, the value of the quality factor is slightly bigger than 0.7. The results are affected by small values of the $1/g_m$-s of the BOTA elements in this particular case. The observed pass-band attenuation of magnitude response for $f_0 = 5$ kHz is caused by limited frequency range of the used approximation of the fractional-order capacitor.

The quality factor control of the designed filter is possible by changing the value of the current gain B_Q of the DACA element. The tunability of the quality factor was carried out for four values when we choose the $f_0 = 10$ kHz and order of the filter 1.8. The used values of the filter parameters are summarized in Table V. The magnitude and phase responses are demonstrated in Fig. 6.
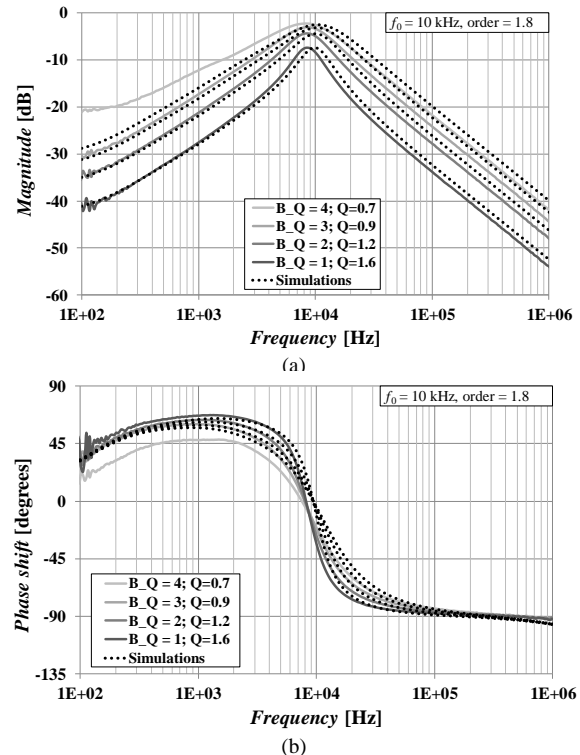

(a)

(b)

Fig. 6. Demonstration of the quality factor control of the FBPF for four values of current gain B_Q when the $f_0 = 10$ kHz and order 1.8. Measurement vs. simulations: a) magnitude; b) phase shift.

From magnitude responses can be seen that the fractional-order band-pass function has asymmetrical slopes of attenuation. Below the center frequency is the slope of attenuation 20α dB/decade and above the center

frequency is the slope 20 dB/decade. This is caused by selection of the C_1 as FOE and particular form of transfer function (2).

TABLE V. QUALITY FACTOR CONTROL.

Theoretical values of Q	0.7	0.9	1.2	1.6
C_α [μFs$^{\alpha-1}$]		0.062		
C_2 [nF]		10		
$R_1 = 1/g_{m1}$ [Ω]		712		
$R_2 = 1/g_{m2}$ [Ω]		520		
B_Q	4	3	2	1

The obtained values of the quality factor from measurements are: 0.69, 0.9, 1.14, and 1.85. The values of the quality factor from simulations are following: 0.72, 0.87, 1.13 and 1.64. The obtained values of the quality factor by experimental measurement are in good agreement with results from simulations. From graphs it is obvious that results obtained by measurement are frequency shifted. The frequency shift is caused by inaccurate values of the transconductances. It is also seen that for $B_Q = 4$ ($Q = 0.7$), bigger differences at low frequency stop band are more significant. This differences are caused mainly by inaccuracy of passive parts and current gain of the B_Q as mentioned above.

IV. CONCLUSIONS

The reconfigurable frequency filter containing the FOE is presented in this paper. The fractional-order filter provides electronic reconfigurability between LP, HP, BP and BR transfer functions without any reconnection in the filter topology. The circuit provides beneficial ability of the electronic control of the pole frequency and quality factor. The verification of the filter features was performed using simulation with behavioural models and also by measurement with real devices. The correct function of the proposed filter is supported by obtained measured results, which are discussed in the graphs. The filter was tested for three values of the order (1.2, 1.5, 1.8), obtained values from measurements were 1.19, 1.52 and 1.79. The pole frequency tuning was tested for five values of f_0 (5 kHz, 10 kHz, 25 kHz, 50 kHz and 150 kHz), values obtained from measurements were 3.9 kHz, 8.9 kHz, 22.3 kHz, 54.7 kHz and 149.8 kHz. The quality factor control was tested for four values (0.7, 0.9, 1.2 and 1.6), obtained values of Q were 0.69, 0.9, 1.14 and 1.85.

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