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Impact of Symmetrical Phase Shifter on Power-System State Estimation

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***Abstract*—The paper deals with Power System State Estimation (PSSE) that plays essential role in modern dispatcher centres. The aim of the paper is to present results of original investigations on change of properties of PSSE when a symmetrical phase shifter is installed in a Power System (PS). The mentioned phase shifter is one of the types of phase shifters. Such the phase shifter is installed in the Polish Power System. In the paper, the following indices characterizing properties of PSSE are considered: the number of iterations in PSSE calculation, the conditionality of the solved equations and accuracy of estimation. The investigations are made using the IEEE 14-bus test system. The properties of PSSE are analysed when there is the phase shifter and when there is not such the device in PS for the polar and rectangular coordinate systems. The investigations are so organized to take into account possibly all space of states of PS. Results of the investigations show that there is noticeable change of the properties of PSSE as an effect of installation of the phase shifter in PS. In the paper, also such the cases are indicated, in which those changes are the smallest.**

***Index Terms*—Model; Phase shifter; Power flow control; Power system; State estimation.**

I. INTRODUCTION

One of devices, which are utilized for power flow control in modern Power Systems (PSs), is a Phase Shifter (PhS). Its characteristic feature is possibility of control of phase angles of voltages in PS. Changing the phase shift between the terminal voltages of a power line implies changes of the power flow through this line [1]–[3]. In consequence, PhS changes power flows in PS, what can lead to elimination of undesirable power flows.

Presence of PhS in PS should be taken into account in calculations made for this system, in particular, in State Estimation (SE) [4]. The Power-System State Estimation (PSSE) is important part of the real-time PS modelling [5], which should ensure the reliable estimate of the PS state vector.

The paper presents results of original investigations on change of properties of PSSE when PhS is installed in PS. The considered type of PhS is a symmetrical one [1]–[3]. A symmetrical PhS create an output voltage with an altered phase angle compared to the input voltage, but with the same magnitude [3]. In this way one forms the voltage at the beginning of a power line, of which phase is different than the phase of the voltage before PhS.

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There is relatively small number of papers presenting the problem of evaluation of properties of SE realized for PS with PhS [6], [7]. In [6], [7], the following indices characterizing properties of SE are considered: the number of iterations in PSSE calculation, the conditionality of the solved equations and accuracy of estimation. In [6], it is assumed that PhS is modelled as ideal transformer with a complex tap ratio in series with two-port π circuit which represents transformer losses. PSSE is considered in the polar and rectangular coordinate systems. [7] is devoted to one of the type of PhSs, which is a quadrature booster. In [7], the results of investigations of PSSE in the rectangular coordinate system are presented. In [7], significance of specific information on the quadrature booster from the point of view of properties of PSSE is investigated.

This paper contains the results of a next part of the investigations of the authors to be focused on properties of SE realized for PS with the symmetrical PhS. One takes into account such indices characterizing properties of SE as in [6], [7]. In this paper, PSSE is considered in the rectangular and polar coordinate systems. Up to now, such the investigations have not been considered in existing papers. It should be underlined that the mentioned investigations are essential from the point of view of SE realized for PS in which there is the above-pointed PhS. Such situation is in the Polish Power System. Now, in that system there are the symmetrical PhSs. Knowledge of properties of different solutions of SE for PS with PhS enables to choose the best one for the aim of the real-time PS modelling in dispatch centre.

In the paper, the symmetrical PhS is modelled with use of two real voltage sources. One of them is in a shunt branch and second one is in a series branch. The assumed model is other than models of PhSs that can be found in other papers [2], [8], [9]. The here-considered model can be used for modelling different types of PhSs, otherwise than models from the earlier-mentioned papers.

In the further part of the paper, the considered method for PSSE and indices characterizing its properties are presented. Next, a model of the symmetrical PhS is formulated. A main part of the paper is description of the investigations, whose aim is to show differences between properties of SE for PS with the symmetrical PhS and without this device. At the end, the most important conclusions from the conducted investigations are given.

II. Considered State Estimation Method

A. An Idea of the Method

In the described investigations, the weighted least squares PSSE method is considered. An objective function of that method can be written as follows [4]

$$J(\mathbf{x}) = \frac{1}{2}[\mathbf{z} - \mathbf{h}(\mathbf{x})]^{\mathrm{T}} \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\mathbf{x})], \quad (1)$$

where: $\mathbf{x}$ is a PS state vector; $\mathbf{z}$ is a vector of measurements; $\mathbf{h}(\mathbf{x})$ is a vector of functions (also nonlinear), representing dependence of measured quantities from the state vector $\mathbf{x}$; $\mathbf{R}$ is a diagonal matrix of measurement covariances.

State vector $\mathbf{x}$ in the rectangular coordinate system is defined as $\mathbf{x} = [e_1, e_2, \ldots, e_n, f_2, f_3, \ldots, f_n]^{\mathrm{T}}$, where: e_i $i = 1, 2, \ldots, n$ are real parts of voltages at buses $1, 2, \ldots, n$; f_i $i = 2, 3, \ldots, n$ are imaginary parts of voltages at buses $2, 3, \ldots, n$. State vector $\mathbf{x}$ in the polar coordinate system is defined as $\mathbf{x} = [\delta_2, \delta_3, \ldots, \delta_n, V_1, V_2, \ldots, V_n]^{\mathrm{T}}$, where: δ_i $i = 2, 3, \ldots, n$ - phase angles of voltages at buses $2, 3, \ldots, n$; V_i $i = 1, 2, \ldots, n$ – magnitudes of voltages at buses $1, 2, \ldots, n$.

The elements of $\mathbf{h}(\mathbf{x})$ can be determined as follows [10]:

$$V_i = V_i\ , \quad (2)$$

$$P_i - jQ_i = \overline{\mathbf{V}}_i^{*} \mathbf{Y}_{rowi} \mathbf{V}, \quad (3)$$

$$P_{ij} - jQ_{ij} = \left[-\left(\overline{\mathbf{y}}_{si} + \overline{\mathbf{y}}_{ij}\right) \quad \overline{\mathbf{y}}_{ij} \right] \cdot \left[V_i^2 \quad \overline{\mathbf{V}}_j \cdot \overline{\mathbf{V}}_i^{*} \right]^T, \quad (4)$$

where P_i, Q_i are an active and reactive power injection at i-th bus, respectively; P_{ij}, Q_{ij} are an active and reactive power flow, respectively, between i-th and j-th bus, measured at i-th bus; $\overline{\mathbf{V}}_i$ is a voltage at i-th bus; $\overline{\mathbf{y}}_{ij}$ is an admittance of the series branch connecting i-th and j-th bus; $\overline{\mathbf{y}}_{si}$ is an admittance of the shunt branch at i-th bus; $\mathbf{Y}_{row\,i}$ is i-th row of an admittance matrix $\mathbf{Y}_{\mathrm{row}\,i} = \left[\overline{\mathbf{Y}}_{i1}, \overline{\mathbf{Y}}_{i2}, \ldots, \overline{\mathbf{Y}}_{in}\right]$, $\overline{\mathbf{Y}}_{ik}$ i, $k = 1, 2, \ldots, n$ are elements of the admittance matrix; $\mathbf{V}$ is a vector $\mathbf{V} = \left[\overline{\mathbf{V}}_1, \overline{\mathbf{V}}_2, \ldots, \overline{\mathbf{V}}_n\right]^{\mathrm{T}}$.

In the described investigations, it is assumed that a solution of the PSSE problem is achieved by solving the normal-equation set

$$\mathbf{G}\left(\mathbf{x}^k\right) \times \left(\mathbf{x}^{k+1} - \mathbf{x}^k\right) = -\mathbf{g}(\mathbf{x}^k), \quad (5)$$

where k is a number of iteration, $\mathbf{x}^k$ is a solution vector at k-th iteration, $\mathbf{G}\left(\mathbf{x}^k\right) = \mathbf{H}^T\left(\mathbf{x}^k\right) \times \mathbf{R}^{-1} \times \mathbf{H}\left(\mathbf{x}^k\right)$, $\mathbf{H}(\mathbf{x}) = \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}$, $\mathbf{g}(\mathbf{x}) = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} = -\mathbf{H}^T(\mathbf{x})\mathbf{R}^{-1}[\mathbf{z} - \mathbf{h}(\mathbf{x})]$.

$\mathbf{G}(\mathbf{x})$ is called as a gain matrix, to be a symmetric, sparse, and positive determined matrix for a fully-observable PS.

B. Indices Characterising the Method

In the paper, properties of PSSE are characterized by the following indices [10]: (i) number of iterations in calculation process (n_{it}), (ii) a condition number of the gain matrix (cond($\mathbf{G}$)), (iii) ratio J_e/J_m ($A = J_e/J_m$).

Number of iterations n_{it} affects the time of calculations.

Condition number cond($\mathbf{G}$) is a measure of conditioning of a SE process [11]. In the paper, the following definition of condition number cond($\mathbf{G}$) is taken into account: $\mathrm{cond}(\mathbf{G}) = \lambda_M / \lambda_m$, where λ_m, λ_M are the minimal and maximal eigenvalues of matrix $\mathbf{G}(\mathbf{x})$, respectively.

The larger the condition number is, the worse the conditioning of the estimation process is. The ill-conditioning of the SE process often leads to a worse convergence of the SE process or even to lack of the convergence of this process.

In the ratio $A = J_e/J_m$, J_e and J_m are calculated as $J_e = \frac{1}{m}\sum_{i=1}^{m}\left[\left(\hat{z}_i - z_i^r\right)/\sigma_i\right]^2$ and $J_m = \frac{1}{m}\sum_{i=1}^{m}\left[\left(z_i - z_i^r\right)/\sigma_i\right]^2$,

where $z_i, \hat{z}_i, z_i^r$ are the measured, estimated and real values of i-th measured quantity, respectively; σ_i is a variance of the measurement of i-th quantity; m is a number of the measured quantities. Ratio A characterizes accuracy of SE [12]. That ratio should satisfy the condition $A < 1$. If that condition is not satisfied, then accuracy of results of PSSE is assessed as insufficient.

The smaller the distinguished indices, the better the properties of SE are.

III. Description of Phase Shifter

An equivalent circuit of PhS is shown in Fig. 1. In that circuit, there are: a shunt Excitation Transformer (ET), a series Boosting Transformer (BT), which injects a series voltage ($\overline{\mathbf{V}}_{\mathrm{BT}}$) in PS, and a tap changer. Voltage $\overline{\mathbf{V}}_{\mathrm{BT}}$ is controlled by the tap changer.

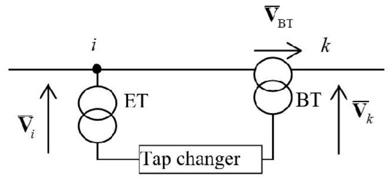

Fig. 1. An equivalent circuit of the phase shifter.

A symmetrical PhS create an output voltage whose magnitude is equal to the magnitude of voltage $\overline{\mathbf{V}}_i$. The phasor diagram for the symmetrical PhS is in Fig. 2.

In the carried out investigations, PhS is represented by the model, to be shown in Fig. 3, containing two controllable voltage sources with internal impedances $\overline{\mathbf{z}}_{\mathrm{ET}}$ and $\overline{\mathbf{z}}_{\mathrm{BT}}$, which represent impedances of ET and BT, respectively.

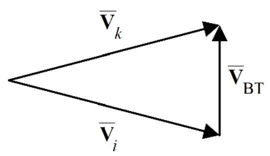

Fig. 2. Phasor diagram for the symmetrical phase shifter.

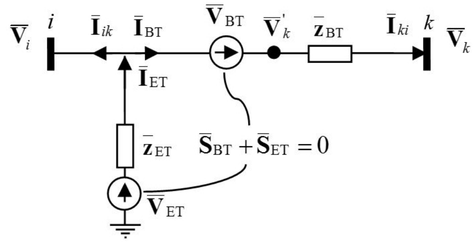

Fig. 3. The assumed model for the considered phase shifter.

For the considered model, the following equations can be derived:

$$\overline{\mathbf{S}}_{ik}^{*} = -V_i^2\left(\overline{\mathbf{y}}_{\mathrm{ET}} + \overline{\mathbf{y}}_{\mathrm{BT}}\right) + \\ + \overline{\mathbf{V}}_k \overline{\mathbf{V}}_i^{*} \overline{\mathbf{y}}_{\mathrm{BT}} - \overline{\mathbf{V}}_{\mathrm{BT}} \overline{\mathbf{V}}_i^{*} \overline{\mathbf{y}}_{\mathrm{BT}} + \overline{\mathbf{V}}_{\mathrm{ET}} \overline{\mathbf{V}}_i^{*} \overline{\mathbf{y}}_{\mathrm{BT}}, \quad (6)$$

$$\overline{\mathbf{S}}_{ki}^{*} = -V_k^2 \overline{\mathbf{y}}_{\mathrm{BT}} + \overline{\mathbf{V}}_i \overline{\mathbf{V}}_k^{*} \overline{\mathbf{y}}_{\mathrm{BT}} + \overline{\mathbf{V}}_{\mathrm{BT}} \overline{\mathbf{V}}_k^{*} \overline{\mathbf{y}}_{\mathrm{BT}}, \quad (7)$$

$$\overline{\mathbf{S}}_{\mathrm{BT}}^{*} = V_{\mathrm{BT}}^2 \overline{\mathbf{y}}_{\mathrm{BT}} + \overline{\mathbf{V}}_i \overline{\mathbf{V}}_{\mathrm{BT}}^{*} \overline{\mathbf{y}}_{\mathrm{BT}} - \overline{\mathbf{V}}_k \overline{\mathbf{V}}_{\mathrm{BT}}^{*} \overline{\mathbf{y}}_{\mathrm{BT}}, \quad (8)$$

$$\overline{\mathbf{S}}_{\mathrm{ET}}^{*} = V_{\mathrm{ET}}^2 \overline{\mathbf{y}}_{\mathrm{ET}} - \overline{\mathbf{V}}_i \overline{\mathbf{V}}_{\mathrm{ET}}^{*} \overline{\mathbf{y}}_{\mathrm{ET}}, \quad (9)$$

where $\overline{\mathbf{y}}_{\mathrm{ET}} = 1/\overline{\mathbf{z}}_{\mathrm{ET}}$, $\overline{\mathbf{y}}_{\mathrm{BT}} = 1/\overline{\mathbf{z}}_{\mathrm{BT}}$, V_{ET}, V_{BT} are voltage magnitudes of $\overline{\mathbf{V}}_{\mathrm{ET}}$ and $\overline{\mathbf{V}}_{\mathrm{BT}}$, respectively.

One can state that PhS cannot absorb and injects complex power, i.e. the complex power supplied to transformer ET equals to the complex power demanded by transformer BT

$$\overline{\mathbf{S}}_{\mathrm{ET}} + \overline{\mathbf{S}}_{\mathrm{BT}} = 0. \quad (10)$$

The powers at buses i and k are now determined as follows:

$$\begin{cases} \overline{\mathbf{S}}_i = \overline{\mathbf{S}}_{i-\mathrm{AC}} - \overline{\mathbf{S}}_{ik}, \\ \overline{\mathbf{S}}_k = \overline{\mathbf{S}}_{k-\mathrm{AC}} - \overline{\mathbf{S}}_{ki}, \end{cases} \quad (11)$$

where $\overline{\mathbf{S}}_{i-\mathrm{AC}}$ and $\overline{\mathbf{S}}_{k-\mathrm{AC}}$ are bus powers determined by formula (3) for buses i and k, respectively, when there is no PhS in PS.

For the symmetrical PhS the following assumption should be taken into account:

– for the polar coordinate system

$$V_i - V_k = 0, \quad (12)$$

– for the rectangular coordinate system

$$e_i^2 + f_i^2 - e_k^2 - f_k^2 = 0. \quad (13)$$

The above assumptions apply to an ideal symmetrical PhS. In practice, relations (12) and (13) are fulfilled only when PhS is not loaded. Therefore, for the real symmetrical PhS, the following assumptions must be taken into account:

$$t_{ik} = \frac{V_i}{\left|\overline{\mathbf{V}}_i + \overline{\mathbf{V}}_{BT}\right|} = 1, \quad (14)$$

and

$$\alpha_{ik} = \angle\left(\overline{\mathbf{V}}_i\right) - \angle\left(\overline{\mathbf{V}}_i + \overline{\mathbf{V}}_{BT}\right). \quad (15)$$

IV. INVESTIGATIONS

It is not possible to perform analytical investigation of the considered indices characterizing properties of SE in all space of operational states of PS. In this situation, there were carried out: (i) original calculations of the considered indices for representative operational states of the test PS when SE is performed in the rectangular and polar coordinate systems and in PS there is and there is no PhS, (ii) original statistical analyses of the calculated indices, (iii) original discussion on the causes of observed regularities.

A. Assumptions

The investigations are so organized to ensure representativeness of the considered cases of realization of SE for a test system.

1. The IEEE 14-bus test system (Fig. 4) is used.
2. One considers the test system without and with the real symmetrical PhS.
3. If the PhS is in the test system, then it is on the line between bus 5 and bus 4, at bus 5.
4. One considers PSSE in the rectangular and polar coordinate systems. SE_R_S and SE_P_S stand for SE for the test system with PhS in the first and second case, respectively. SE_R_S- and SE_P_S- stand for SE for the test system without PhS in the rectangular and polar coordinate systems, respectively.
5. Each of impedances $\overline{\mathbf{z}}_{\mathrm{ET}}$ and $\overline{\mathbf{z}}_{\mathrm{BT}}$ in the model of PhS is equal to 0.01 + j 0.1 pu.
6. 11 load variants are considered. For the given variant, each active and reactive load and also power injection is defined as $W = 0.5W_b + l \times W_b$, where W, W_b are the calculated and base values of the mentioned quantity; $l \in \{0, 0.1, 0.2, \ldots, 1\}$. $V^{0.5+l}$ stands for the variant associated with l.
7. For each load variant, the phase shift introduced by PhS is in the range [-20°, 20°] (as it is for Polish PhSs).
8. One takes into account the following numbers of Measurement Data (MD): $m_1 = 34$, $m_2 = 53$, $m_3 = 68$ and $m_4 = 104$, respectively.
9. For each MD number, 100 locations of measurement systems is randomly generated.
10. Each item of MD is burden with a small error characterized by the Gaussian distribution with a mean equal to zero and standard deviation σ, defined as [13], [14]: $\sigma = 1/3\left[\left(0.001 + 0.0025\right)FS + 0.02\,M\right]$ for active power, $\sigma = 1/3\left[\left(0.001 + 0.005\right)FS + 0.02\,M\right]$ for reactive power, and $\sigma = 1/3\left[\left(0.0005 + 0.0025\right)FS + 0.003\,M\right]$ for voltage magnitude, where FS is a measurement scope, M is a measured value.
11. To determine differences between properties of SE_P_S and SE_P_S- or SE_R_S and SE_R_S-, the investigations of these SEs are made for the same load variants, MD numbers, locations of measurement systems and characteristics of small errors burdening MD.

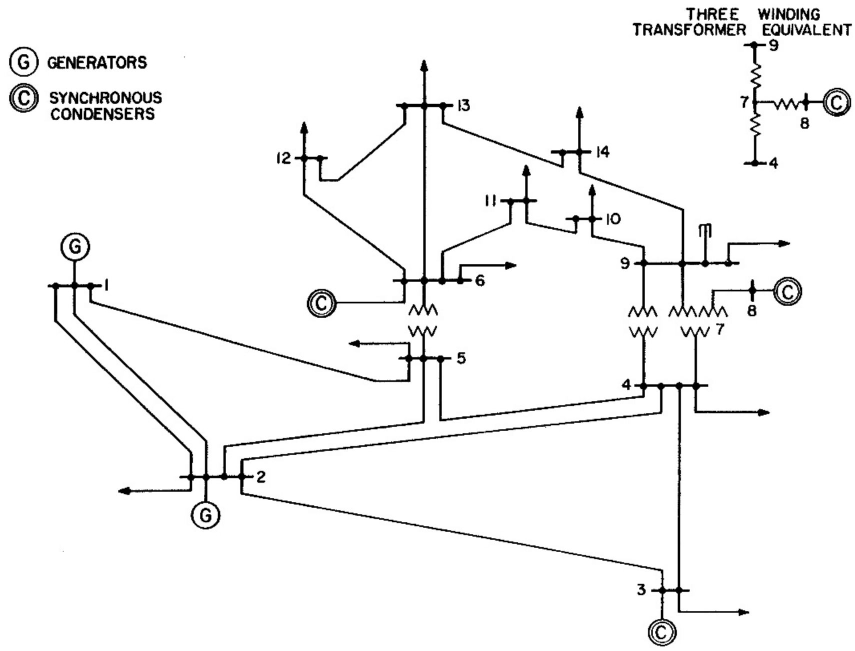

Fig. 4. The IEEE 14-bus test system [15].

B. Results

Results of the investigations are shown in Table I – Table III and in Fig. 5–Fig. 10.

In Table I – Table III, there are gathered results of analysis of parameters of the indices, which describe properties of PSSE for different cases. The mentioned parameters are the minimum, maximum, mean values and standard deviations of the considered indices. Relative changes of parameters of the considered indices for SE_R_S with respect to the suitable parameters of indices for SE_R_S- are given in Table I. Similarly, in Table II, there are relative changes of parameters of the considered indices when SE is realized in the polar coordinate system. The mentioned changes of the parameters of the considered indices characterizing properties of PSSE are calculated using formula: $\Delta p_X = 100\left(p_{X,S} - p_{X,S-}\right)/p_{X,S-}$, where $p_{X,S}$, $p_{X,S-}$ are parameters of index X, when SE is performed for PS with or without PhS, respectively.

TABLE I. RELATIVE CHANGES OF PARAMETERS OF THE INDICES FOR SE_R_S WITH RESPECT TO SUITABLE PARAMETERS OF INDICES FOR SE_R_S- (IN PERCENTAGE).

number of MD	Load variant $V^{0.5}$				Load variant $V^{1.5}$			
	m_1	m_2	m_3	m_4	m_1	m_2	m_3	m_4
number of iterations n_{it}								
min	20.0	20.0	20.0	20.0	0	16.7	16.7	16.7
max	14.3	14.3	16.7	16.7	37.5	12.5	14.3	14.3
mean	6.7	9.5	8.1	9.5	13.1	11.4	12.3	13.5
std.dev.	118.2	195.2	140.8	49.2	21.5	-8.7	-32.3	-61.1
condition number cond(G)								
min	108.90	93.16	92.02	90.57	111.41	88.80	82.96	74.72
max	585.87	372.98	160.98	108.06	274.11	234.57	91.80	91.69
mean	260.49	132.21	107.09	96.45	254.11	129.54	104.03	92.66
std.dev.	597.47	510.29	302.71	515.36	405.54	206.95	114.01	153.80
ratio A								
min	7.25	5.20	0.89	-4.33	11.03	5.48	7.83	-3.14
max	0.90	1.17	0.17	1.49	1.20	2.02	3.79	5.42
mean	3.53	1.87	0.80	-0.60	4.63	5.64	6.31	7.19
std.dev.	-8.51	1.49	-0.97	2.76	-10.69	-1.25	2.81	7.80

Relative differences of the changes of the parameters of the considered indices determined for SE performed in the polar and rectangular coordinate system are shown in Table III. Those relative differences are calculated as $100\left(\Delta p_{X,P} - \Delta p_{X,R}\right)/\Delta p_{X,R}$, where $\Delta p_{X,P}$, $\Delta p_{X,R}$ changes of parameter p_X, calculated, when SE is performed in the polar and rectangular coordinate system, respectively.

Analysis of Table I and Table II shows, that the largest differences of the distinguished parameters determined for SE for PS with PhS and for PS without PhS are when condition number cond(G) is taken into account independently of the used coordinate system. Smaller differences of the distinguished parameters are for number of iterations n_{it} and the smallest ones for ratio A.

For all load variants and for all numbers of MD, mean values of condition number cond(G) and number of iterations n_{it}, determined for SE_R_S as well as for SE_P_S are larger than for SE_R_S- or SE_P_S-, respectively. That fact means that the properties of SE_R_S and SE_P_S are worse than the properties of SE_R_S- or SE_P_S-, respectively. The mean values of ratio A are also larger for SE_R_S and SE_P_S than for SE_R_S- or SE_P_S-, respectively, except for load variant $V^{0.5}$ and the MD number equal to m_4. One can ascertain that in most cases the mean values of ratio A are larger for SE_R_S and SE_P_S than for SE_R_S- or SE_P_S-, respectively, and from the point of view of ratio A the properties of SE_R_S and SE_P_S are worse than the properties of SE_R_S- or SE_P_S-, respectively.

In Fig. 5–Fig. 10, there are presented relative differences of percentiles of appropriate indices for SE_R_S and SE_R_S- or SE_P_S and SE_P_S- as functions of percentile rank, calculated as:

$$\Delta PC_{\text{SE\_R,X,r}} = 100\frac{PC_{\text{SE\_R\_S,X,r}} - PC_{\text{SE\_R\_S-,X,r}}}{PC_{\text{SE\_R\_S-,X,r}}}, \quad (16)$$

$$\Delta PC_{\text{SE\_P,X,r}} = 100\frac{PC_{\text{SE\_P\_S,X,r}} - PC_{\text{SE\_P\_S-,X,r}}}{PC_{\text{SE\_P\_S-,X,r}}}, \qquad (17)$$

where $PC_{\text{SE\_R\_S,X,r}}$, $PC_{\text{SE\_P\_S,X,r}}$, $PC_{\text{SE\_R\_S,-X,r}}$, $PC_{\text{SE\_P\_S-,X,r}}$ are percentiles of index X of rank r determined for SE_R_S, SE_P_S, SE_R_S-, SE_P_S-, respectively.

TABLE II. RELATIVE CHANGES OF PARAMETERS OF THE INDICES FOR SE_P_S WITH RESPECT TO SUITABLE PARAMETERS OF INDICES FOR SE_P_S- (IN PERCENTAGE).

number of MD	Load variant $V^{0.5}$				Load variant $V^{1.5}$			
	m_1	m_2	m_3	m_4	m_1	m_2	m_3	m_4
number of iterations n_{it}								
min	60.0	60.0	60.0	60.0	50.0	50.0	50.0	33.3
max	214.3	214.3	266.7	250.0	212.5	200.0	228.6	214.3
mean	106.3	96.9	95.0	95.2	103.9	95.4	100.4	101.4
std.dev.	719.3	1299	1096	607.4	420.4	424.5	467.2	544.6
condition number cond(G)								
min	120.94	104.66	102.41	100.85	127.78	104.26	99.93	94.26
max	593.40	348.78	164.42	111.32	308.22	226.96	133.72	118.15
mean	261.54	136.88	114.84	106.34	292.39	154.33	129.41	110.07
std.dev.	598.95	465.10	258.45	303.65	457.52	225.66	152.02	203.29
ratio A								
min	7.54	5.15	0.79	-4.30	10.70	5.27	7.74	-3.28
max	0.92	1.25	0.21	1.59	1.16	2.08	3.76	5.41
mean	3.54	1.87	0.81	-0.58	4.63	5.64	6.30	7.16
std.dev.	-8.47	1.48	-0.99	2.83	-10.60	-1.20	2.81	7.86

TABLE III. RELATIVE CHANGES OF PARAMETERS OF THE INDICES DETERMINED FOR SE PERFORMED IN POLAR AND RECTANGULAR COORDINATE SYSTEM (IN PERCENTAGE).

number of MD	Load variant $V^{0.5}$				Load variant $V^{1.5}$			
	m_1	m_2	m_3	m_4	m_1	m_2	m_3	m_4
number of iterations n_{it}								
min	200.0	200.0	200.0	200.0	-	199.4	199.4	99.4
max	1398.6	1398.6	1497.0	1397.0	466.7	1500.0	1498.6	1398.6
mean	1486.6	920.0	1072.8	902.1	693.1	736.8	716.3	651.1
std.dev.	508.5	565.5	678.4	1134.6	1855.3	-4979	-1546	-991.3
condition number cond(G)								
min	11.06	12.34	11.29	11.35	14.69	17.41	20.46	26.15
max	1.28	-6.49	2.14	3.02	12.44	-3.24	45.66	28.86
mean	0.40	3.53	7.24	10.25	15.06	19.14	24.40	18.79
std.dev.	0.25	-8.86	-14.62	-41.08	12.82	9.04	33.34	32.18
ratio A								
min	4.00	-0.96	-11.24	-0.69	-2.99	-3.83	-1.15	4.46
max	2.22	6.84	23.53	6.71	-3.33	2.97	-0.79	-0.18
mean	0.28	0.00	1.25	-3.33	0.00	0.00	-0.16	-0.42
std.dev.	-0.47	-0.67	2.06	2.54	-0.84	-4.00	0.00	0.77

For number of iterations, $\Delta PC_{\text{SE\_P,nit,r}}$ is much larger than $\Delta PC_{\text{SE\_R,nit,r}}$, independently of r and the load variant. For such r, for which $\Delta PC_{\text{SE\_R,nit,r}}$ is not equal to 0 %, $\Delta PC_{\text{SE\_P,nit,r}}$ is from 3 to 15 times larger than $\Delta PC_{\text{SE\_R,nit,r}}$. For condition number cond(**G**), $\Delta PC_{\text{SE\_P,cond(G),r}}$ and $\Delta PC_{\text{SE\_R,cond(G),r}}$ have values which are relatively close to each other. The module of relative difference between these numbers is not larger than 10.6 % for MD number m_1, 9.7 % for MD number m_4 and load variant $V^{0.5}$ and 7.54 % for MD number m_1, 10.87 % for MD number m_4 and load variant $V^{1.5}$. Values $\Delta PC_{\text{SE\_P,cond(G),r}}$ and $\Delta PC_{\text{SE\_R,cond(G),r}}$ are relatively large. On average, each of them is at least 19 times larger than module of difference between them. Difference between $\Delta PC_{\text{SE\_P,A,r}}$ and $\Delta PC_{\text{SE\_R,A,r}}$ is small. On average, the module of it is not larger than 0.09 %. $|\Delta PC_{\text{SE\_P,A,r}}|$ and $|\Delta PC_{\text{SE\_R,A,r}}|$ are not larger than 10 %.

V. DISCUSSION

Presence of PhS in PS entails greater complexity of the model of this system. The state vector comprises of larger number of elements. Now, in the state vector are also real and imaginary parts or magnitudes and phase angles of voltages $\overline{\mathbf{V}}_{ET}$ and $\overline{\mathbf{V}}_{BT}$, i.e. the number of state variables increases by 4. A number of equations, which are taken into account in SE, increases by 3. Additional equations result from (10) and (14). Moreover, equations for P_i, Q_i, P_k, Q_k where i and k are indices of terminal buses of the branch with PhS, are more complex (see (11)).

From the SE viewpoint, when configuration of measuring systems in PS does not change, situation becomes worse. A level of data redundancy, defined as $rd = n_d/n_s$, where: n_d is a number of data used in a SE process, n_s is a number of state variables, decreases. The levels of data redundancy for PS with and without PhS are gathered in Table IV.

TABLE IV. LEVELS OF DATA REDUNDANCY FOR PS WITH AND WITHOUT THE CONSIDERED PHS.

m	rd_{S-}	rd_S	rd_S - rd_{S-}	(rd_S - rd_{S-})/rd_{S-} %
34	1.172	1.121	-0.051	-4.37
53	1.828	1.697	-0.131	-7.15
68	2.345	2.152	-0.193	-8.24
104	3.586	3.242	-0.344	-9.59

Note: rd_{S-} is a level of data redundancy for PS without PhS, rd_S is a level of data redundancy for PS with the symmetrical PhS.

Calculating rd_s and rd_{s-}, where rd_s, rd_{s-} are levels of data redundancy for PS with PhS and for PS without PhS, respectively, it is taken into account that in the carried out SE, apart from measurements also virtual measurements are utilized. In this situation, $n_d = m + m_v$, where m_v is a number of the virtual measurements. For PS without PhS we have $m_v = 4$, for PS with PhS $m_v = 7$.

For PS without PhS, the virtual measurements are injections at two zero-injection nodes, i.e. at node 7 and at node 15. Node 15 is an additional node in the test system. In the investigations, when PS with PhS is taken into account, the considered PhS is between nodes 5 and 15. The power line, which is between nodes 4 and 5 in the original IEEE 14-bus test system, is between nodes 4 and 15 in the modified IEEE 14-bus test system.

For PS with PhS, the virtual measurements are injections at the earlier-mentioned zero-injection nodes, three zero right-hand sides of the equations resulting from relationships (10) (for active and reactive powers) and (14).

The consequence of decreasing the level of data redundancy is possibility of occurrence of larger values of ratio A ($A = J_e/J_m$). Such situation is observed in the performed investigations. Only for certain number of cases, when the load variant is $V^{0.5}$ and the MD number is m_4, ratio A is larger for SE_R_S- and SE_P_S- than for SE_R_S, and SE_P_S, respectively. However, for the mentioned load variant and the MD number, the relative difference of mean values of A for SE_R_S and SE_R_S- as well as SE_P_S and SE_P_S- is not larger than 0.6 %.

Larger number of equations together with greater complexity of some equations and larger number of the virtual measurements (having higher accuracy) leads to larger values of condition number cond(**G**) and also to worse convergence of the calculation process, i.e. to the larger number of iterations. The results of investigations confirm the presented ascertainment. For load variant $V^{0.5}$ as well as for load variant $V^{1.5}$ and for all considered MD numbers, the mean values and the percentiles of the condition numbers are larger for SE_R_S, and SE_P_S than for SE_R_S- and SE_P_S-, respectively.

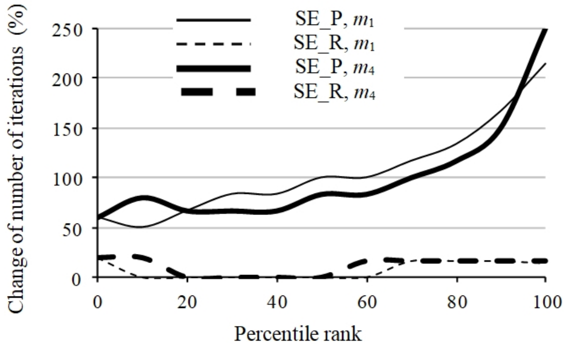

Fig. 5. Relative changes of percentiles of the number of iterations in the SE process as a function of percentile rank for load variant $V^{0.5}$.

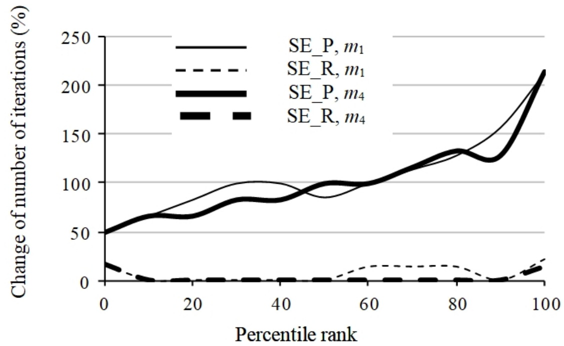

Fig. 6. Relative changes of percentiles of the number of iterations in the SE process as a function of percentile rank for load variant $V^{1.5}$.

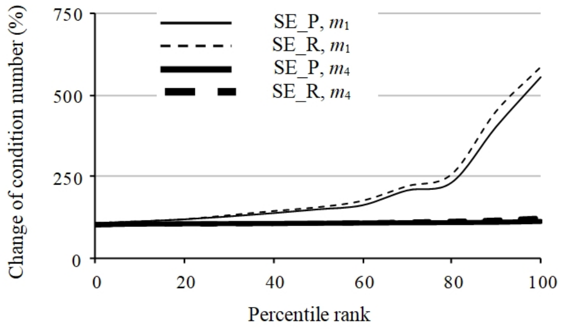

Fig. 7. Relative changes of percentiles of the condition number in the SE process as a function of percentile rank for load variant $V^{0.5}$.

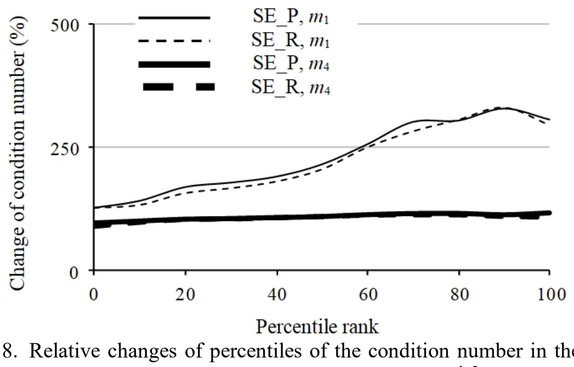

Fig. 8. Relative changes of percentiles of the condition number in the SE process as a function of percentile rank for load variant $V^{1..5}$.

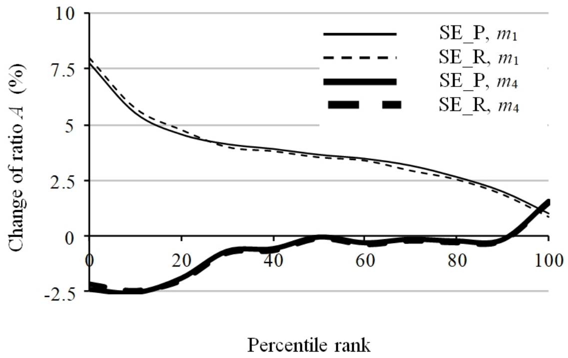

Fig. 9. Relative changes of percentiles of ratio A in the SE process as a function of percentile rank for load variant $V^{0.5}$.

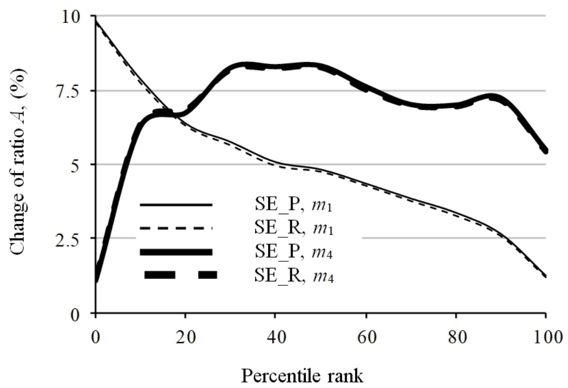

Fig. 10. Relative changes of percentiles of ratio A in the SE process as a function of percentile rank for load variant $V^{1.5}$.

When numbers of iterations are taken into account, one can observe that for PSSE in the rectangular coordinate system for all considered load variants and all considered MD numbers: (i) the mean values of numbers of iterations are larger for SE_R_S than for SE_R_S-, (ii) a certain part of percentiles of numbers of iterations is larger for SE_R_S than for SE_R_S-, and other part of percentiles of numbers of iterations is the same for SE_R_S and for SE_R_S-.

Especially, when the number of iterations is considered, one can note essential influence of the coordinate system on results of PSSE. $\Delta PC_{\mathrm{SE\_P,nit,r}}$ is essentially larger than $\Delta PC_{\mathrm{SE\_R,nit,r}}$. That fact can be explained taking into account character of functions, that are in the equations utilized in SE in different coordinate systems. In the rectangular coordinate system, one utilizes equations in which there are quadratic functions, in the polar coordinate system, one utilizes equations in which there are trigonometric functions (that can be considered as infinite series). This fact makes convergence, which is observed in SE calculations in the polar coordinate system, worse.

VI. CONCLUSIONS

The installation of PhSs in PS changes the conditions that must be taken into account designing SE calculation. In the new conditions, properties of SE are different than it was before. The paper presents original investigations aimed on those properties when in PS there is and there is not one of types of PhSs, which is the symmetrical PhS (such PhSs are in Polish Power System). In the paper, one takes into considerations such indices characterizing properties of PSSE as: (i) n_{it} – the number of iterations of SE calculations, (ii) cond(**G**) - the condition number of the gain matrix, that plays essential role in the calculations, (iii) ratio A ($A = J_e/J_m$) characterizing accuracy of SE. It is not possible to perform analytical investigation of the mentioned indices in all space of states of PS. In this situation, there were carried out: (i) original calculations of the considered indices for representative operational states of the test PS, (ii) original statistical analyses of the calculated indices, (iii) original discussion on the causes of observed regularities. Analyzing the earlier-mentioned indices, one can ascertain that properties of PSSE are worse when SE is performed for PS with the considered PhS than when it is performed for PS without such the PhS. In each case (defined by the load variant and the data redundancy) mean value and each quantile of number of iterations n_{it} as well as mean value and each quantile of condition number cond(**G**) are larger for SE_R_S or SE_P_S than for SE_R_S-, SE_P_S-, respectively. There are cases, in which the mean value and some quantiles of ratio A are smaller for SE_R_S or SE_P_S than for SE_R_S-, SE_P_S-, respectively. However, number of such cases is relatively low and does not change the earlier-given statement on general evaluation of SE for PS with the considered PhS.

Comparing PSSE for different coordinate systems, one can state, that generally, PSSE has the better properties for the rectangular coordinate system.

The obtained conclusions can be used for seeking a solution to improve the properties of SE for PS with the considered PhS.

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