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Deadbeat Controller with Two Additional Steps

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***Abstract*—An n^{th}-order closed-loop hybrid system will exhibit a deadbeat response if the system output approaches the reference input in *n* steps. This paper studies the problem of obtaining a deadbeat response for an *n*th-order single-input single-output plant (process) in *n* + 2 steps with a minimum deadbeat controller output deviation from the steady state value. We identify the requirements for the design of a deadbeat controller with two additional steps and present worked examples for the problem. A deadbeat controller is then used to control a piezoelectric micro-robot model. The second order continuous transfer function is used to simulate the robot's piezoelectric actuator. This research presents the advantages of a deadbeat controller with two additional steps when applied to a robot control problem.**

***Index Terms*—Control design; deadbeat control; digital control; robot control.**

I. INTRODUCTION

A deadbeat control algorithm is used when a rapid settling time is required. The deadbeat controller design is presented in the z-domain. A deadbeat controller replaces the poles of the system with poles at the origin of the z-domain [1]. That is the reason why deadbeat controllers should be used only for the control of stable plants (processes). The main drawback of the deadbeat controller is that the sampling period T_0 is the only design parameter that influences the magnitude of the manipulated variable $u(0)$. As the magnitude of the manipulated variable $u(0)$ increases, the sampling period decreases. It is possible to increase the number of design parameters by designing an extended order deadbeat controller [1], [2].

In this article, the presented n + 2 order design for the deadbeat controller is based on the continuous object *z*-transfer function. The overall structure of the control system is shown in Fig. 1.

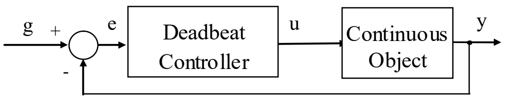

Fig. 1. Structure of the deadbeat control system: *g* – reference input; *e* – an error variable; *u* – manipulated variable (controller output); *y* – system output.

II. DEADBEAT CONTROLLER WITH TWO ADDITIONAL STEPS

The extended order deadbeat controller transfer function $W_{DC}(z)$ can be written [3] as follows

$$W_{DC}(z) = \frac{Q(z)C(z)}{1 - P(z)C(z)}, \quad (1)$$

where $Q(z)$ and $P(z)$ are the polynomials of the transfer function of the deadbeat controller, and $C(z)$ is an additional polynomial. Let $C(z) = 1 + c_1 z^{-1} + c_2 z^{-2}$. Then the coefficients of the deadbeat controller can be found using the following:

$$q_0 = \frac{1}{(b_1 + b_2 + ... + b_n)(1 + c_1 + c_2)}, \quad (2)$$

$$q_i = q_0 \times (a_i + a_{i-1} \times c_1 + a_{i-2} \times c_2), \quad (3)$$

$$p_i = q_0 \times (b_i + b_{i-1} \times c_1 + b_{i-2} \times c_2), \quad (4)$$

where $i = 1,...,n+2$, a_i and b_i are coefficients of the polynomials of the continuous object's *z*-transfer function $W_{CO}(z) = \frac{B(z)}{A(z)}$; *n* – order of the object transfer function.

The number of a deadbeat controller's steps increases by two compared with a simple deadbeat controller. The deadbeat controller's properties depend on the values of the sampling period T_0, coefficients c_1 and c_2.

A number of approaches are available for designing such a deadbeat controller, which revolve around varying the sampling period T_0 along with the coefficients c_1 and c_2. We present two different approaches below.

First approach. By changing the sampling period T_0 we design a simple deadbeat controller, such that $u(0) = q_0 \approx u_{max}$, where u_{max} is the maximum allowable magnitude of the manipulated variable *u*. It then follows from (2)–(4) that coefficients c_1 and c_2 are zero.

We then fix the sampling period T_0, where $u(0) = q_0 \approx u_{max}$.

We denote the sum of coefficients c_1 and c_2 in (2) as C

$$c_1 + c_2 = C. \quad (5)$$

We simulate the response of a deadbeat control system by gradually increasing the value of C, starting at zero. The value of c_2 is then increased from zero up to C (from (5) we have $c_1 = C - c_2$) allowing us to calculate the root mean squared deviation (RMSD) [4], [5] that represents the mean deviation of the values of the manipulated variable $u(i)$ with

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respect to the steady state value of the manipulated variable *u(m)*

$$\mathrm{RMSD}=\sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(u(i)-u(m))^2}, \qquad (6)$$

where m is the number of simulation points.

We plot a diagram where $RMSD=f(c_2)$ (Fig. 2), and select a value for c_2 that corresponds to the smallest RMSD and obtain c_1 from (5).

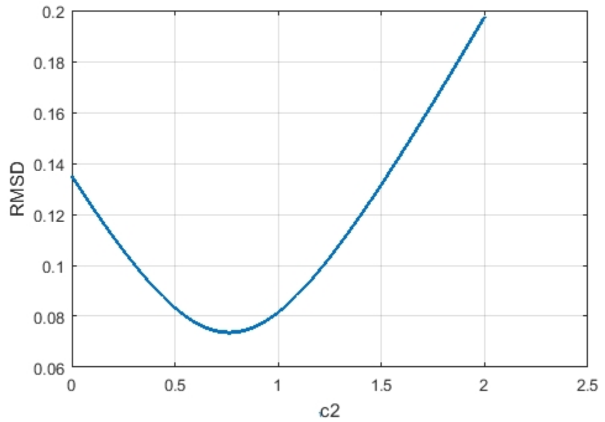

Fig. 2. RMSD dependency on c_2.

We plot a diagram where $RMSD_{\min}=f(C)$ (Fig. 3), and select a value for C and c_2 that corresponds to the smallest RMSD and obtain c_1 from (5).

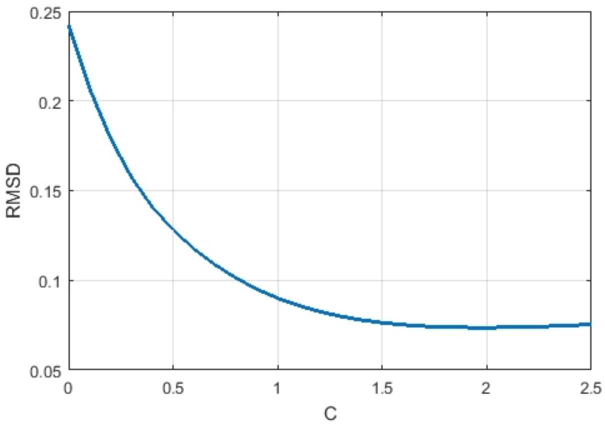

Fig. 3. RMSD dependency on C.

This guarantees that minimal resources will be required to control the object.

Second approach. By changing the sampling period T_0 we design a simple deadbeat controller (coefficients c_1 and c_2 set to zero in (2)–(4)) such that $u(0) = q_0 \approx u_{max}$. By gradually decreasing the sampling period T_0 and increasing the value of C from zero, we aim to maintain in (2)–(4) that $u(0)=q_0\le u_{\max}$. By simulating the response of a deadbeat control system with fixed values for the sampling period T_0 and the coefficient C, we increase the value of c_2 from zero up to C (then $c_1=C-c_2$), allowing us to obtain the root mean squared deviation (RMSD). We then obtain a value for c_2 that corresponds to the minimal RMSD and obtain c_1 from (5).

III. DEADBEAT CONTROLLER FOR PIEZO ELECTRIC ROBOT CONTROL

It can be shown [6] that in order to simulate the behaviour of the piezo robot, models are needed for the piezoelectric actuators. When placed in a closed loop, piezoelectric actuators are subject to significant ringing, especially when driven near their natural frequencies. To better account for this behaviour, we use the second order model of the piezoelectric actuator that is based on the frequency characteristic of the actuator [6]

$$\mathrm{W_{CO}(z)}=\frac{90000}{s^2+30s+90000}. \qquad (7)$$

This transfer function correctly places the natural frequency at 300 Hz. To drive the piezo robot's actuators, a controller with three 0 to 150 V inputs must be used [6]. The characteristic equation of the transfer function (7) has two poles at -0.1500 ± 2.9962i that ensure stability of the piezoelectric actuator.

The object (7) response – piezo output *y,* as simulated in Matlab, to the step input *u*, is depicted in Fig. 4.

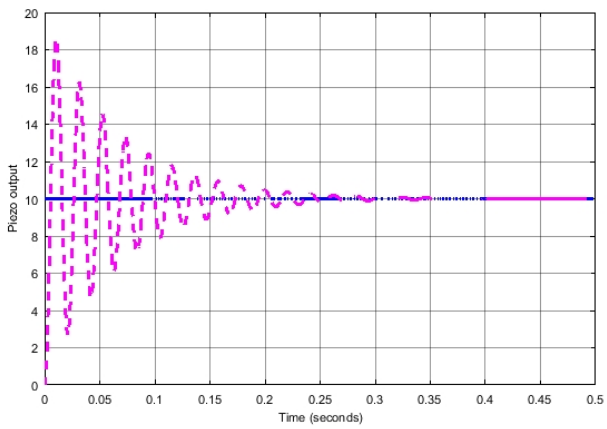

Fig. 4. Object response (magenta curve - *y*) to the step input (blue curve - *u*).

Figure 4 shows that the object response to the bounded input is bounded and has a finite settling time. The response oscillates with a transient time of 0.3 s (the response remains within 1 % of its final value) and has an overshoot of 85.4 %.

The piezoelectric actuator transfer function given in (7) will henceforth be used to design the deadbeat controllers. The design of the deadbeat controllers is done in Matlab. The continuous object transfer function (7) is converted to discrete time assuming a zero order hold on the input.

We then apply the first approach to design the extended order deadbeat controller. By using (2)–(4), with the values of c_1 and c_2 set to zero and while holding that $u(0) = q_0 \approx$ 3.0 V, we get the object (7) z-transfer function at the sampling period $T_0 = 0.002$ s

$$\mathrm{W_{CO}(z)}=\frac{0.1712z^{-1}+0.1678z^{-2}}{1-1.6027z^{-1}+0.9418z^{-2}}. \qquad (8)$$

By using (1), (2)–(4) and (6), with the values of c_1 and

c_2 set to zero, we get the transfer function of the simple deadbeat controller

$$W_{DC}(z) = \frac{2.9494 - 4.7270z^{-1} + 2.7776z^{-2}}{1 - 0.5051z^{-1} - 0.4949z^{-2}}, \quad (9)$$

where $T_0 = 0.002s$. In the time domain, this can be represented as

$$u(n) = 0.5051u(n-1) + 0.4949u(n-2) + \\ +2.9494e(n) - 4.7270e(n-1) + 2.7776e(n-2). \quad (10)$$

Using Matlab, we obtain the response y of the deadbeat control system (Fig. 5) to the unit step input reference signal g.

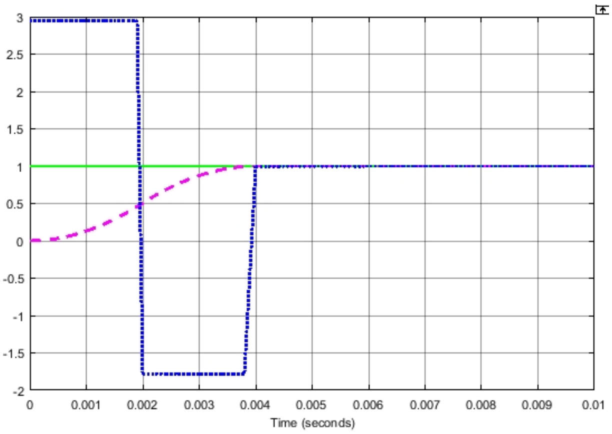

Fig. 5. Simulated deadbeat control system response (magenta curve – y) to unit step reference input signal (green curve – g), blue curve – controller output u.

Figure 5 shows that the system response has a finite settling time. The response ends in 0.00375 s (the response remains within 1 % of its final value) after two steps of the control signal, as this is a second order object. The system response does not show any signs of overshooting. The main drawback of this controller is that the controller output is below 0 V.

By using the first approach, we plot a diagram $RMSD_{\min} = f(C)$ (Fig. 3) and select a value for C = 2.0 that corresponds to the smallest RMSD = 0.0736. Then from Fig. 2 $c_2 = 0.77$ and from (5) c_1 = 1.23. By using equations (1), (2)–(4) and (6), we get the transfer function of the deadbeat controller with two additional steps

$$W_{DC}(z) = \\ = \frac{0.9831 - 0.3664z^{-1} - 0.2552z^{-2} - 0.0744z^{-3} + 0.7129z^{-4}}{1 - 0.1684z^{-1} - 0.3721z^{-2} - 0.3326z^{-3} - 0.1270z^{-4}}, \quad (11)$$

where $T_0 = 0.002s$. In the time domain this can be represented as

$$u(n) = 0.1684u(n-1) + 0.3721u(n-2) + \\ +0.3326u(n-3) + 0.127u(n-4) + \\ +0.9831e(n) - 0.3664e(n-1) - \\ -0.2552e(n-2) - 0.0744e(n-3) + \\ +0.7128e(n-4). \quad (12)$$

We then obtain the deadbeat control system response y to the unit step input reference signal g (Fig. 6).

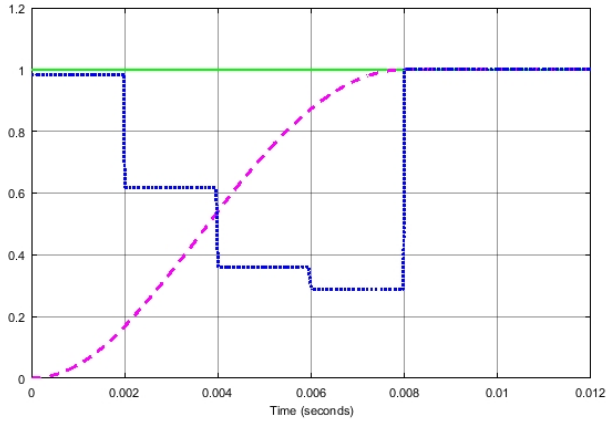

Fig. 6. Response (magenta curve – y) of a simulated deadbeat system with two additional controller steps to the unit step reference input signal (green curve – g), blue curve – controller output u.

Figure 6 shows that the system response has a finite settling time. The response ends in 0.0075 s after four steps of the control signal. The system response does not show any signs of overshooting. Including two additional coefficients c_1 and c_2 into a deadbeat controller's parameter calculation (2)–(4) reduces the variance/range of the manipulated variable from [-1.7 to +3.0 V] for a simple deadbeat controller (Fig. 5) to [0 to +1 V].

Figure 7 shows how the disturbance, affecting the object output at time 0.015 s, influences the system response and the controller's output value, which increases by the value of disturbance signal. The system response has no overshoot, while the settling time takes 4 sampling time units.

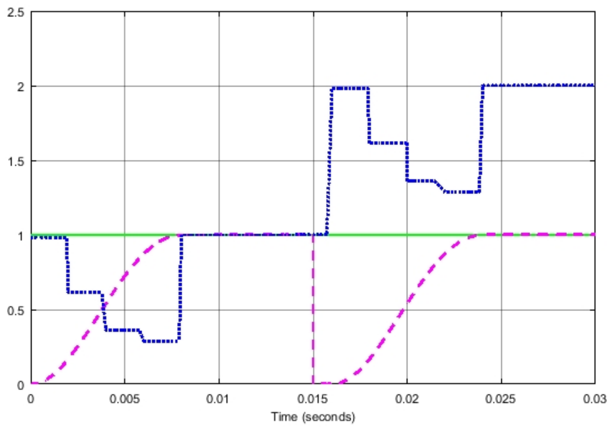

Fig. 7. Response (magenta curve – y) of a simulated deadbeat system with two additional controller steps to the unit step disturbance at the system output, reference input signal (green curve – g), blue curve – controller output u.

We then apply the second approach to design an extended order deadbeat controller. We first take the simple deadbeat controller design represented in (9). By gradually decreasing the sampling period T_0 from 0.002 s and increasing the value of C from zero, we aim to maintain in (2)–(4) that

$u(0) = q_0 \le 3.0$. The lowest value of the sampling period T_0 where the manipulated variable remains in the positive range is 0.0015 seconds.

While holding the sampling period T_0 = 0.0015 s we plot a diagram $RMSD_{\min} = f(C)$ and select a value for C = 1.25 that corresponds to the smallest RMSD = 0.1477 and c_2 = 0.44. Then from (5) we get c_1 = 0.81. By using (1) and (2)–(4), we obtain the transfer function of the deadbeat controller with two additional steps

$$W_{DC}(z) = \\ = \frac{2.2828 - 2.1716z^{-1} - 0.0700z^{-2} - 0.0014z^{-3} + 0.9602z^{-4}}{1 - 0.2239z^{-1} - 0.4019z^{-2} - 0.2772z^{-3} - 0.0970z^{-4}}, \quad (13)$$

where $T_0 = 0.0015s$. We then obtain the deadbeat control system response y to the unit step input reference signal g and the unit step disturbance affecting the object output at time 0.01 seconds in the middle of the sampling period.

Figure 8 shows that the system response to the reference signal has a finite settling time. The response ends in 0.0056 s after four steps of the control signal. The system response does not show any signs of overshooting.

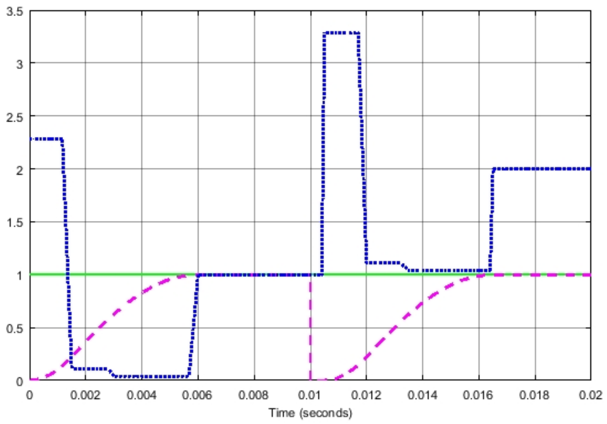

Fig. 8. Response (magenta curve – y) of a simulated deadbeat system with two additional controller steps to the unit step disturbance at the system output; reference input signal (green curve – g), blue curve – controller output u.

TABLE I. CONTROL STATISTICS FOR DEADBEAT CONTROLLERS.

Controller	Object Gain Variation	Settling Time, ms	Overshoot, %
Simple deadbeat	-10 %	7.3	0
	-5 %	6.5	0
	0 %	3.75	0
	+5 %	6.1	3.65
	+10 %	6.4	7.2
First approach - deadbeat controller with two additional steps.	-10 %	13	0
	-5 %	11.2	0
	0 %	7.5	0
	+5 %	10.7	2.93
	+10 %	11.4	5.7
Second approach - deadbeat controller with two additional steps.	-10 %	9.3	0
	-5 %	8	0
	0 %	5.6	0
	+5 %	7.5	2.3
	+10 %	8	4.6

The disturbance affecting the object output at time 0.01seconds influences the controller's output, which increases by the value of the disturbance signal. The system response has no overshoot, while the settling time takes 4 sampling time units.

To determine the robustness of deadbeat control systems to changes in the object's parameters, the values of the settling time and overshoot were obtained for an object (7) with a gain change of ±10 % from a nominal value. As shown in Table I, both of the extended order controllers performed comparably.

IV. CONCLUSIONS

In this paper we proposed a control strategy to utilise the deadbeat controllers design for the piezoelectric actuator. The proposed control design relies on the inclusion of an auxiliary polynomial into the controller's transfer function. The presented strategy for selecting the deadbeat controller parameters c_1 and c_2 allows to decrease the sampling period T_0 and the absolute value of the manipulated variable. Conversely, the time available to the controlling processor is decreased, which leads to fewer functions that need to be implemented on a processor with reduced hardware complexity.

The mathematical model used for the proposed control system is observed to involve decaying oscillations.

Three deadbeat controllers were developed for piezoelectric actuators. Controllers with two additional steps appear to be robust to output disturbances. We simulated the performance of these controllers and the techniques presented in this paper can be applied to other control tasks.

The following observations were made based on these results. The simulation results show that incorporating a two-step approach into the controller's design allows to narrow down the range of the control signal variance, compared to a deadbeat controller without any modifications. Lastly, we observe that the designed deadbeat controllers satisfy the performance requirements when the object gain has a variance of ±10 %.

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